

**Figure 4.** A tension diagram of concrete grade B10

Comparisons of deformation diagrams constructed by approximating formulas (1), the coefficients of which are determined by the method of least squares and by the ratios (4), (5), (6), with experimental diagrams are shown in figures 3, 4. It is visible from the figures that experimental diagrams approach the solution well enough at deformations close to the limit.

We rewrite the deformation condition for  $i$ -th layer in the form

$$\sigma_i = A_i \left( \frac{\varepsilon}{\varepsilon_{*i}} \right) \left( \varepsilon_{*i} + \frac{B_i}{A_i} (\varepsilon_{*i})^2 \left( \frac{\varepsilon}{\varepsilon_{*i}} \right) \right), \quad (7)$$

where

$$\varepsilon_{*i} = \min\{\varepsilon_{*i}^+, \varepsilon_{*i}^-\}.$$

To solve this problem, we introduce a small dimensionless parameter  $\delta$ , which is determined according to the ratio (7) as

$$\delta = \frac{B_i}{A_i} (\varepsilon_{*i})^2. \quad (8)$$

If we determine the parameter  $\delta$  at  $i = 1$  and assume that the first layer of the rod is made of concrete grade B10, then from 2 we obtain the value of the parameter  $\delta \approx 0,0038$ .

We rewrite the ratios (7) in the form

$$\sigma_i = a_i \varepsilon + c_i \delta \varepsilon^2, \quad (9)$$

where  $a_i = A_i$ ,  $c_i = \frac{A_i B_i}{A_i B_i \varepsilon_{*i}^2}$ .

Using the classical kinematic hypotheses of Kirchhoff-Lyav, for deformations we have expressions [12]

$$\varepsilon(x, y, z) = \varepsilon_0(x) - y\kappa_z(x) + z\kappa_y(x), \quad (10)$$

where

$$\varepsilon_0(x) = \frac{du_0}{dx}, \quad \kappa_y = \frac{d^2 w_0}{dx^2}, \quad \kappa_z = \frac{d^2 v_0}{dx^2}. \quad (11)$$

We consider that the values  $\sigma(x, y, z)$ ,  $u_0(x)$ ,  $v_0(x)$ ,  $w_0(x)$  depend on the specified parameter  $\delta$ . We expand them into series of degrees of the specified parameter

$$\begin{aligned} \sigma(x, y, z) &= \sum_{k=0}^{\infty} \delta^k \sigma^{(k)}(x, y, z), \quad u_0(x) = \sum_{k=0}^{\infty} \delta^k u_0^{(k)}(x), \\ v_0(x) &= \sum_{k=0}^{\infty} \delta^k v_0^{(k)}(x), \quad w_0(x) = \sum_{k=0}^{\infty} \delta^k w_0^{(k)}(x), \end{aligned} \quad (12)$$

where the components at tension and movements get the index  $(n)$ , corresponding to the degree of parameter  $\delta$ .

From ratios (10), (11), (12), we obtain the expression for deformations

$$\varepsilon(x, y, z) = \sum_{n=0}^{\infty} \delta^n \varepsilon^{(n)}(x, y, z), \quad \varepsilon^{(n)}(x, y, z) = \frac{du_0^{(n)}}{dx} + z \frac{d^2 w_0^{(n)}}{dx^2} - y \frac{d^2 v_0^{(n)}}{dx^2}. \quad (13)$$

Substituting (12), (13) in the expression (9) and equating the ratios at identical degrees of  $\delta$ , we obtain for the zero and first approximations

$$\sigma_{(i)}^{(0)} = a_i \varepsilon^{(0)}, \quad (14)$$

$$\sigma_{(i)}^{(1)} = a_i \varepsilon^{(1)} + c_i (\varepsilon^{(0)})^2. \quad (15)$$

Longitudinal forces and bending moments can be determined from the ratios

$$N = \sum_{i=1}^n \int_{S_i} \sigma_i dS, \quad M_y = \sum_{i=1}^n \int_{S_i} \sigma_i z dS, \quad M_z = - \sum_{i=1}^n \int_{S_i} \sigma_i y dS. \quad (16)$$

Equilibrium equations have the form [12]

$$\frac{d^2 M_z}{dx^2} = q_y - \frac{dm_z}{dx}, \quad \frac{d^2 M_y}{dx^2} = q_z - \frac{dm_y}{dx}, \quad \frac{dN}{dx} = -q_x, \quad (17)$$

where  $N$  is projections of the vector of distributed loading of the rod attached to the axis,  $M_z$ ,  $M_y$  are projections of the vector of distributed moments on  $z$  and  $y$  axes.

Substituting the expansion for stresses (16) in (12) we obtain

$$N = \sum_{n=0}^{\infty} \delta^n N^{(n)}, \quad M_y = \sum_{n=0}^{\infty} \delta^n M_y^{(n)}, \quad M_z = \sum_{n=0}^{\infty} \delta^n M_z^{(n)}, \quad (18)$$

where

$$N^{(i)} = \sum_{j=1}^n \int_{S_j} \sigma_j^{(i)} dS, \quad M_y^{(i)} = \sum_{j=1}^n \int_{S_j} \sigma_j^{(i)} z dS, \quad M_z^{(i)} = - \sum_{j=1}^n \int_{S_j} \sigma_j^{(i)} y dS. \quad (19)$$

From equilibrium equations (17) and ratios (18), we obtain for the zero approximation

$$\frac{d^2 M_z^{(0)}}{dx^2} = q_y - \frac{dm_z}{dx}, \quad \frac{d^2 M_y^{(0)}}{dx^2} = q_z - \frac{dm_y}{dx}, \quad \frac{dN^{(0)}}{dx} = -q_x. \quad (20)$$

From (13), (14), (19) we determine the longitudinal force and bending moments

$$\begin{aligned}
 N^{(0)} &= 2 \sum_{i=1}^n \iint_S \sigma_i^{(0)} dS = 2 \sum_{i=1}^n \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} \sigma_i^{(0)} dy = 2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} \varepsilon^{(0)} dy = \\
 &= 2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} \left( \frac{du_0^{(0)}}{dx} - y \frac{d^2 v_0^{(0)}}{dx^2} + z \frac{d^2 w_0^{(0)}}{dx^2} \right) dy = f_1 \frac{du_0^{(0)}}{dx} + f_2 \frac{d^2 v_0^{(0)}}{dx^2} + f_3 \frac{d^2 w_0^{(0)}}{dx^2}, \quad (21)
 \end{aligned}$$

where

$$f_1 = 2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} b_i(z) dz, \quad f_2 = - \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} b_i^2(z) dz, \quad f_3 = 2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} z b_i(z) dz,$$

$$\begin{aligned}
 M_y^{(0)} &= 2 \sum_{i=1}^n \iint_S z \sigma_i^{(0)} dS = 2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} z \sigma_i^{(0)} dy = \\
 &= 2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} \left( z \frac{du_0^{(0)}}{dx} - zy \frac{d^2 v_0^{(0)}}{dx^2} + z^2 \frac{d^2 w_0^{(0)}}{dx^2} \right) dy = f_3 \frac{du_0^{(0)}}{dx} + f_4 \frac{d^2 v_0^{(0)}}{dx^2} + f_5 \frac{d^2 w_0^{(0)}}{dx^2},
 \end{aligned}$$

where

$$f_4 = - \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} z b_i^2(z) dz, \quad f_5 = 2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} z^2 b_i(z) dz,$$

$$\begin{aligned}
 M_z^{(0)} &= -2 \sum_{i=1}^n \iint_S y \sigma_i^{(0)} dS = -2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} \sigma_i^{(0)} y dy = \\
 &= -2 \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} \left( y \frac{du_0^{(0)}}{dx} + zy \frac{d^2 v_0^{(0)}}{dx^2} + y^2 \frac{d^2 w_0^{(0)}}{dx^2} \right) dy = f_2 \frac{du_0^{(0)}}{dx} + f_6 \frac{d^2 v_0^{(0)}}{dx^2} + f_4 \frac{d^2 w_0^{(0)}}{dx^2},
 \end{aligned}$$

where

$$f_6 = \frac{2}{3} \sum_{i=1}^n a_i \int_{h_{i-1}}^{h_i} b_i^3(z) dz.$$

From (17), (21) we obtain a system of linear differential equations to determine the movements in the zero approximation

$$\begin{aligned}
 f_4 \frac{d^4 w_0^{(0)}}{dx^4} + f_6 \frac{d^4 v_0^{(0)}}{dx^4} + f_2 \frac{d^3 u_0^{(0)}}{dx^3} &= q_y - \frac{dm_z}{dx}, \\
 f_5 \frac{d^4 w_0^{(0)}}{dx^4} + f_4 \frac{d^4 v_0^{(0)}}{dx^4} + f_3 \frac{d^3 u_0^{(0)}}{dx^3} &= q_z - \frac{dm_y}{dx}, \\
 f_3 \frac{d^3 w_0^{(0)}}{dx^3} + f_2 \frac{d^3 v_0^{(0)}}{dx^3} + f_1 \frac{d^2 u_0^{(0)}}{dx^2} &= -q_x.
 \end{aligned} \quad (22)$$

From the last equation of the system (22), we express  $\frac{d^2 u_0^{(0)}}{dx^2}$  and substitute in the first two. We obtain a system of two equations and two unknown functions

$$\begin{aligned} g_1 \frac{d^4 w_0^{(0)}}{dx^4} + g_2 \frac{d^4 v_0^{(0)}}{dx^4} &= f_2 \frac{dq_x}{dx} - f_1 \frac{dm_z}{dx} + f_1 q_y, \\ g_3 \frac{d^4 w_0^{(0)}}{dx^4} + g_1 \frac{d^4 v_0^{(0)}}{dx^4} &= f_3 \frac{dq_x}{dx} - f_1 \frac{dm_y}{dx} + f_1 q_z, \end{aligned} \quad (23)$$

where

$$g_1 = f_4 f_1 - f_3 f_2, \quad g_2 = f_6 f_1 + f_2^2, \quad g_3 = f_5 f_1 - f_3^2.$$

From (23) we obtain

$$\frac{d^4 w_0^{(0)}}{dx^4} = g_4 \frac{dq_x}{dx} + g_5 \frac{dm_z}{dx} + g_6 \frac{dm_y}{dx} - g_5 q_y - g_6 q_z, \quad (24)$$

$$\frac{d^4 v_0^{(0)}}{dx^4} = g_7 \frac{dq_x}{dx} + g_8 \frac{dm_z}{dx} + g_9 \frac{dm_y}{dx} - g_8 q_y - g_9 q_z, \quad (25)$$

where

$$\begin{aligned} g_4 &= \frac{f_2 g_1 - f_3 g_2}{g_1^2 - g_2 g_3}, \quad g_5 = -\frac{f_1 g_1}{g_1^2 - g_2 g_3}, \quad g_6 = \frac{f_1 g_2}{g_1^2 - g_2 g_3}, \\ g_7 &= \frac{f_2 g_3 - f_3 g_1}{g_2 g_3 - g_1^2}, \quad g_8 = -\frac{f_1 g_3}{g_2 g_3 - g_1^2}, \quad g_9 = \frac{f_1 g_1}{g_2 g_3 - g_1^2}. \end{aligned}$$

Where we get from (24), (25)

$$\begin{aligned} w_0^{(0)} &= \int_0^x dx \int_0^x dx \int_0^x dx \int_0^x \left( g_4 \frac{dq_x}{dx} + g_5 \frac{dm_z}{dx} + g_6 \frac{dm_y}{dx} - g_5 q_y - g_6 q_z \right) dx + \\ &\quad + C_1 x^3 + C_2 x^2 + C_3 x + C_4, \end{aligned}$$

$$\begin{aligned} v_0^{(0)} &= \int_0^x dx \int_0^x dx \int_0^x dx \int_0^x \left( g_7 \frac{dq_x}{dx} + g_8 \frac{dm_z}{dx} + g_9 \frac{dm_y}{dx} - g_8 q_y - g_9 q_z \right) dx + \\ &\quad + C_5 x^3 + C_6 x^2 + C_7 x + C_8, \end{aligned}$$

where the values of constants  $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8$  can be determined from fastening conditions of the rod.

From the last equation of the system (22), we obtain

$$\frac{d^2 u_0^{(0)}}{dx^2} = -\frac{q_x}{f_1} - \frac{f_3}{f_1} \frac{d^3 w_0^{(0)}}{dx^3} - \frac{f_2}{f_1} \frac{d^3 v_0^{(0)}}{dx^3}. \quad (26)$$

We substitute the found values  $v_0^{(0)}, w_0^{(0)}$  into expression (26), and from the fastening conditions we get the expression for  $u_0^{(0)}$ . Then we can obtain an expression for deformations in the zero approximation

$$\varepsilon^{(0)}(x, y, z) = \frac{du_0^{(0)}}{dx} + z \frac{d^2 w_0^{(0)}}{dx^2} - y \frac{d^2 v_0^{(0)}}{dx^2}. \quad (27)$$

We substitute the obtained value (27) in (15) and have

$$\begin{aligned} \sigma_{(i)}^{(1)} = & a_i \left( \frac{du_0^{(1)}}{dx} - y \frac{d^2v_0^{(1)}}{dx^2} + z \frac{d^2w_0^{(1)}}{dx^2} \right) + c_i \left( \left( \frac{du_0^{(0)}}{dx} \right)^2 + y^2 \left( \frac{dv_0^{(0)}}{dx} \right)^2 + \right. \\ & \left. + z^2 \left( \frac{dw_0^{(0)}}{dx} \right)^2 - 2y \frac{du_0^{(0)}}{dx} \frac{d^2v_0^{(0)}}{dx^2} + 2z \frac{du_0^{(0)}}{dx} \frac{d^2w_0^{(0)}}{dx^2} - 2zy \frac{d^2v_0^{(0)}}{dx^2} \frac{d^2w_0^{(0)}}{dx^2} \right). \end{aligned} \quad (28)$$

We define the longitudinal force and bending moments in the first approximation

$$\begin{aligned} N^{(1)} = & 2 \sum_{i=1}^n \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} \sigma_i^{(1)} dy = f_1 \frac{du_0^{(1)}}{dx} + f_2 \frac{d^2v_0^{(1)}}{dx^2} + f_3 \frac{d^2w_0^{(1)}}{dx^2} + j_1(x), \\ j_1(x) = & r_1 \left( \frac{du_0^{(0)}}{dx} \right)^2 + r_4 \left( \frac{d^2v_0^{(0)}}{dx^2} \right)^2 + r_6 \left( \frac{d^2w_0^{(0)}}{dx^2} \right)^2 + 2r_2 \frac{du_0^{(0)}}{dx} \frac{d^2v_0^{(0)}}{dx^2} + \\ & + 2r_5 \frac{d^2v_0^{(0)}}{dx^2} \frac{d^2w_0^{(0)}}{dx^2} + 2r_3 \frac{du_0^{(0)}}{dx} \frac{d^2w_0^{(0)}}{dx^2}, \end{aligned} \quad (29)$$

where

$$\begin{aligned} r_1 = & 2 \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} b_i(z) dz, \quad r_2 = - \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} b_i^2(z) dz, \quad r_3 = 2 \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} z b_i(z) dz, \\ r_4 = & \frac{2}{3} \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} b_i^3(z) dz, \quad r_5 = - \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} z b_i^2(z) dz, \quad r_6 = 2 \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} z^2 b_i(z) dz. \\ M_y^{(1)} = & 2 \sum_{i=1}^n \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} z \sigma_i^{(1)} dy = f_3 \frac{du_0^{(1)}}{dx} + f_4 \frac{d^2v_0^{(1)}}{dx^2} + f_5 \frac{d^2w_0^{(1)}}{dx^2} + j_2(x), \\ j_2(x) = & r_3 \left( \frac{du_0^{(0)}}{dx} \right)^2 + r_7 \left( \frac{d^2v_0^{(0)}}{dx^2} \right)^2 + r_8 \left( \frac{d^2w_0^{(0)}}{dx^2} \right)^2 + 2r_5 \frac{du_0^{(0)}}{dx} \frac{d^2v_0^{(0)}}{dx^2} + \\ & + r_9 \frac{d^2v_0^{(0)}}{dx^2} \frac{d^2w_0^{(0)}}{dx^2} + 2r_6 \frac{du_0^{(0)}}{dx} \frac{d^2w_0^{(0)}}{dx^2}, \end{aligned} \quad (30)$$

where

$$\begin{aligned} r_7 = & \frac{2}{3} \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} z b_i^3(z) dz, \quad r_8 = 2 \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} z^3 b_i(z) dz, \quad r_9 = -2 \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} z^2 b_i^2(z) dz. \\ M_z^{(1)} = & -2 \sum_{i=1}^n \int_{h_{i-1}}^{h_i} dz \int_0^{b_i(z)} \sigma_i^{(1)} y dy = f_2 \frac{du_0^{(1)}}{dx} + f_5 \frac{d^2v_0^{(1)}}{dx^2} + f_6 \frac{d^2w_0^{(1)}}{dx^2} + j_3(x), \end{aligned} \quad (31)$$

where

$$j_3(x) = r_2 \left( \frac{du_0^{(0)}}{dx} \right)^2 + r_{10} \left( \frac{d^2v_0^{(0)}}{dx^2} \right)^2 + \frac{1}{2} r_9 \left( \frac{d^2w_0^{(0)}}{dx^2} \right)^2 + 2r_4 \frac{du_0^{(0)}}{dx} \frac{d^2v_0^{(0)}}{dx^2} +$$



$$+2r_7 \frac{d^2 v_0^{(0)}}{dx^2} \frac{d^2 w_0^{(0)}}{dx^2} + 2r_5 \frac{du_0^{(0)}}{dx} \frac{d^2 w_0^{(0)}}{dx^2},$$

where

$$r_{10} = -\frac{1}{2} \sum_{i=1}^n c_i \int_{h_{i-1}}^{h_i} b_i^4(z) dz.$$

We substitute the obtained values (29), (30), (31) in equilibrium equations

$$\frac{d^2 M_z^{(1)}}{dx^2} = 0, \quad \frac{d^2 M_y^{(1)}}{dx^2} = 0, \quad \frac{dN^{(1)}}{dx} = 0,$$

and receive a system of three differential equations to determine the movements

$$\begin{aligned} f_1 \frac{d^2 u_0^{(1)}}{dx^2} + f_2 \frac{d^3 v_0^{(1)}}{dx^3} + f_3 \frac{d^3 w_0^{(1)}}{dx^3} + \frac{dj_1(x)}{dx} &= 0, \\ f_2 \frac{d^3 u_0^{(1)}}{dx^3} + f_4 \frac{d^4 v_0^{(1)}}{dx^4} + f_5 \frac{d^4 w_0^{(1)}}{dx^4} + \frac{d^2 j_2(x)}{dx^2} &= 0, \\ f_3 \frac{d^3 u_0^{(1)}}{dx^3} + f_5 \frac{d^4 v_0^{(1)}}{dx^4} + f_6 \frac{d^4 w_0^{(1)}}{dx^4} + \frac{d^2 j_3(x)}{dx^2} &= 0. \end{aligned} \tag{32}$$

We rewrite the system (32) as

$$\begin{aligned} \frac{d^2 u_0^{(1)}}{dx^2} &= -\frac{f_2}{f_1} \frac{d^3 v_0^{(1)}}{dx^3} - \frac{f_3}{f_1} \frac{d^3 w_0^{(1)}}{dx^3} - \frac{1}{f_1} \frac{dj_1(x)}{dx}, \\ \left( f_4 - \frac{f_2 f_3}{f_1} \right) \frac{d^4 v_0^{(1)}}{dx^4} + \left( f_5 - \frac{f_3^2}{f_1} \right) \frac{d^4 w_0^{(1)}}{dx^4} + \frac{d^2 j_2(x)}{dx^2} - \frac{f_3}{f_1} \frac{d^2 j_1(x)}{dx^2} &= 0, \\ \left( f_5 - \frac{f_2^2}{f_1} \right) \frac{d^4 v_0^{(1)}}{dx^4} + \left( f_6 - \frac{f_3 f_2}{f_1} \right) \frac{d^4 w_0^{(1)}}{dx^4} + \frac{d^2 j_3(x)}{dx^2} - \frac{f_2}{f_1} \frac{d^2 j_1(x)}{dx^2} &= 0, \end{aligned} \tag{33}$$

where we get the expressions to determine the movements in the first approximation

$$\frac{d^4 v_0^{(1)}}{dx^4} = r_{12} \frac{d^2 j_1(x)}{dx^2} + r_{13} \frac{d^2 j_2(x)}{dx^2} + r_{14} \frac{d^2 j_3(x)}{dx^2} = 0, \tag{34}$$

$$\frac{d^4 w_0^{(1)}}{dx^4} = r_{15} \frac{d^2 j_1(x)}{dx^2} + r_{16} \frac{d^2 j_2(x)}{dx^2} + r_{17} \frac{d^2 j_3(x)}{dx^2} = 0, \tag{35}$$

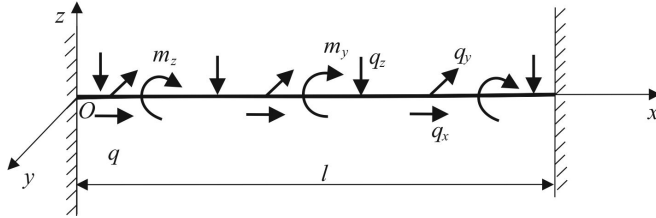
$$\begin{aligned} \frac{d^2 u_0^{(1)}}{dx^2} &= -\left( \frac{f_2 r_{12} + f_3 r_{15}}{f_1} \right) \frac{d^5 j_1(x)}{dx^5} - \left( \frac{f_2 r_{13} + f_3 r_{16}}{f_1} \right) \frac{d^5 j_2(x)}{dx^5} - \\ &\quad - \left( \frac{f_2 r_{14} + f_3 r_{17}}{f_1} \right) \frac{d^5 j_3(x)}{dx^5} - \frac{1}{f_1} \frac{dj_1(x)}{dx}, \end{aligned} \tag{36}$$

where

$$\begin{aligned} r_{11} &= (f_1 f_5 - f_3^2)(f_1 f_5 - f_2^2) - (f_4 f_1 - f_2 f_3)(f_1 f_6 - f_2 f_3), \\ r_{12} &= \frac{f_1 f_3 f_5 - f_1 f_2 f_6}{r_{11}}, \quad r_{13} = \frac{(f_1 f_6 - f_2 f_3) f_1}{r_{11}}, \quad r_{14} = \frac{(f_3^2 - f_1 f_5) f_1}{r_{11}}, \\ r_{15} &= \frac{f_5 f_1 f_2 - f_1 f_3 f_4}{r_{11}}, \quad r_{16} = \frac{f_1 f_3^2 - f_5 f_1^2}{r_{11}}, \quad r_{17} = \frac{f_4 f_1^2 - f_1 f_2 f_3}{r_{11}}. \end{aligned}$$

Having integrated the equations (34), (35), (36) and defined the integration constants from fastening conditions of the rod, we can determine the movements in the first approximation.

From equations (29), (30), (31), we can find the expressions for longitudinal force and bending moments in the first approximation.



**Figure 5.** A rod pinched at both ends

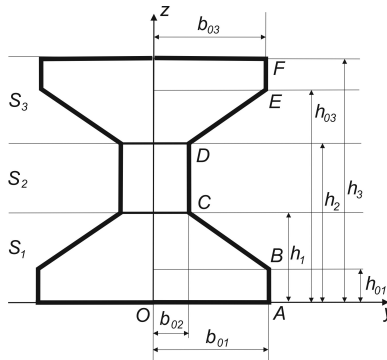
### 3 Results

As an example, we consider the problem of complex bend of a rod pinched at both ends (figure 5) by distributed forces and moments

$$\begin{aligned}
 q_x(x) &= q_{0x} \frac{x}{l}, & q_y(x) &= q_{0y}(\alpha_1 + \alpha_2 x), & q_z(x) &= q_{0z}(\beta_1 + \beta_2 x), \\
 m_y(x) &= m_{0y} \frac{x}{l}, & m_z(x) &= m_{0z} \frac{x}{l},
 \end{aligned}
 \tag{37}$$

where  $q_{0x}, q_{0z}, m_0, \alpha_1, \alpha_2$  are constants.

We assume that the cross-section of the rod has the form figure 6.



**Figure 6.** A cross-section of the rod

Coordinates of the points, according to figure 4 have the form  $A(b_{01}, 0), B(b_{01}, h_{01}), C(b_{02}, h_1), D(b_{02}, h_2), E(b_{03}, h_{03}), F(b_{03}, h_3)$ .

Then the equations of the direct line  $BC$  have the form

$$y = \frac{b_{02} - b_{01}}{h_1 - h_{01}} z + \frac{b_{01}h_1 - b_{02}h_{01}}{h_1 - h_{01}},
 \tag{38}$$

for the direct line  $DE$

$$y = \frac{b_{03} - b_{02}}{h_{03} - h_2} z + \frac{b_{02}h_{03} - b_{03}h_2}{h_{03} - h_2}.
 \tag{39}$$

Then we get

$$\begin{aligned}
 f_1 &= ((h_1 - h_{01}) b_{02} + (h_{01} + h_1) b_{01}) a_1 + (-2h_1 + 2h_2) b_{02} a_2 + \\
 &\quad + ((h_{03} - h_2) b_{02} - b_{03} (h_{03} + h_2 - 2h_3)) a_3, \\
 f_2 &= \left( -\frac{1}{3} (-h_{01} + h_1) b_{02}^2 + \frac{1}{3} b_{01} (h_{01} - h_1) b_{02} - \frac{2}{3} b_{01}^2 (h_{01} + \frac{1}{2} h_1) \right) a_1 + \\
 &\quad + (h_1 - h_2) b_{02}^2 a_2 - \frac{1}{3} \left( (h_{03} - h_2) b_{02}^2 + \frac{1}{3} b_{03} (h_{03} - h_2) b_{02} + \right. \\
 &\quad \left. + \frac{2}{3} \left( h_{03} + \frac{1}{2} h_2 - \frac{3}{2} h_3 \right) b_{03}^2 \right) a_3, \\
 f_3 &= \left( -\frac{1}{3} (h_{01} + 2h_1) (h_{01} - h_1) b_{02} + \frac{1}{3} b_{01} (h_{01}^2 + h_{01} h_1 + h_1^2) \right) a_1 + \\
 &\quad + \frac{1}{3} (-3h_1^2 + 3h_2^2) b_{02} a_2 + \frac{1}{3} \left( (h_{03} + 2h_2) (h_{03} - h_2) b_{02} - \right. \\
 &\quad \left. - b_{03} (h_{03}^2 + h_{03} h_2 + h_2^2 - 3h_3^2) \right) a_3, \\
 f_4 &= \left( \frac{1}{12} b_{02}^2 (h_{01} + 3h_1) (h_{01} - h_1) - \frac{1}{6} b_{02} b_{01} (-h_{01}^2 + h_1^2) - \right. \\
 &\quad \left. - \frac{1}{4} b_{01}^2 \left( h_{01}^2 + \frac{2}{3} h_{01} h_1 + \frac{1}{3} h_1^2 \right) \right) a_1 + \frac{1}{2} (h_1^2 - h_2^2) b_{02}^2 a_2 - \left( \frac{1}{12} (h_{03} + 3h_2) (h_{03} - h_2) b_{02}^2 + \right. \\
 &\quad \left. \frac{1}{12} (2h_{03}^2 - 2h_2^2) b_{03} b_{02} - \frac{1}{4} b_{03}^2 \left( h_{03}^2 + \frac{2}{3} h_{03} h_2 + \frac{1}{3} h_2^2 - 2h_3^2 \right) \right) a_3, \\
 f_5 &= \frac{1}{6} \left( b_{02} (-h_{01}^3 - h_1 h_{01}^2 - h_{01} h_1^2 + 3h_1^3) + b_{01} (h_{01} + h_1) (h_{01}^2 + h_1^2) \right) a_1 + \\
 &\quad + \frac{2}{3} b_{02} (-h_1^3 + h_2^3) a_2 + \frac{1}{6} \left( (h_{03} - h_2) (h_{03}^2 + 2h_{03} h_2 + 3h_2^2) b_{02} - \right. \\
 &\quad \left. - (h_{03}^3 + h_{03}^2 h_2 + h_{03} h_2^2 + h_2^3 - 4h_3^3) b_{03} \right) a_3, \\
 f_6 &= \left( \frac{1}{6} (h_1 - h_{01}) b_{02}^3 - \frac{1}{6} b_{01} (h_{01} - h_1) b_{02}^2 - \frac{1}{6} b_{01}^2 (h_{01} - h_1) b_{02} + \frac{1}{2} (h_{01} + \frac{1}{3} h_1) b_{01}^3 \right) a_1 + \\
 &\quad + \frac{2}{3} (h_2 - h_1) b_{02}^3 a_2 + \left( \frac{1}{6} (h_{03} - h_2) b_{02}^3 + \frac{1}{6} b_{03} (h_{03} - h_2) b_{02}^2 + \right. \\
 &\quad \left. + \frac{1}{6} b_{03}^2 (h_{03} - h_2) b_{02} - \frac{1}{6} (3h_{03} + h_2 - 4h_3) b_{03}^3 \right) a_3, \\
 r_1 &= ((h_1 - h_{01}) b_{02} + (h_{01} + h_1) b_{01}) c_1 + (-2h_1 + 2h_2) b_{02} c_2 + \\
 &\quad + ((h_{03} - h_2) b_{02} - b_{03} (h_{03} + h_2 - 2h_3)) c_3, \\
 r_2 &= \left( \frac{1}{3} (-h_{01} + h_1) b_{02}^2 - \frac{1}{3} b_{01} (h_{01} - h_1) b_{02} + \frac{2}{3} b_{01}^2 (h_{01} + \frac{1}{2} h_1) \right) c_1 + \\
 &\quad + (-h_1 + h_2) b_{02}^2 c_2 + \frac{1}{3} \left( (h_{03} - h_2) b_{02}^2 - \frac{1}{3} b_{03} (h_{03} - h_2) b_{02} - \right. \\
 &\quad \left. - \frac{2}{3} \left( h_{03} + \frac{1}{2} h_2 - \frac{3}{2} h_3 \right) b_{03}^2 \right) c_3, \\
 r_3 &= \left( \frac{1}{3} (h_{01} + 2h_1) (h_{01} - h_1) b_{02} - \frac{1}{3} b_{01} (h_{01}^2 + h_{01} h_1 + h_1^2) \right) c_1 -
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{3}(-3h_1^2 + 3h_2^2)b_{02}c_2 - \frac{1}{3}((h_{03} + 2h_2)(h_{03} - h_2)b_{02} + \\
 & \quad + b_{03}(h_{03}^2 + h_{03}h_2 + h_2^2 - 3h_3^2))c_3, \\
 r_4 = & \left(\frac{1}{6}(h_1 - h_{01})b_{02}^3 - \frac{1}{6}b_{01}(h_{01} - h_1)b_{02}^2 - \frac{1}{6}b_{01}^2(h_{01} - h_1)b_{02} + \right. \\
 & \left. + \frac{1}{2}(h_{01} + \frac{1}{3}h_1)b_{01}^3\right)c_1 + \frac{2}{3}(h_2 - h_1)b_{02}^3c_2 + \left(\frac{1}{6}(h_{03} - h_2)b_{02}^3 + \frac{1}{6}b_{03}(h_{03} - h_2)b_{02}^2 + \right. \\
 & \quad \left. + \frac{1}{6}b_{03}^2(h_{03} - h_2)b_{02} - \frac{1}{6}(3h_{03} + h_2 - 4h_3)b_{03}^3\right)c_3, \\
 r_5 = & \left(\frac{1}{12}b_{02}^2(h_{01} + 3h_1)\right)(h_{01} - h_1) - \frac{1}{6}b_{02}b_{01}(-h_{01}^2 + h_1^2) - \\
 & -\frac{1}{4}b_{01}^2\left(h_{01}^2 + \frac{2}{3}h_{01}h_1 + \frac{1}{3}h_1^2\right)c_1 - \frac{1}{2}(-h_1^2 + h_2^2)b_{02}^2c_2 - \left(\frac{1}{12}(h_{03} + 3h_2)(h_{03} - h_2)b_{02}^2 + \right. \\
 & \quad \left. + \frac{1}{12}(2h_{03}^2 - 2h_2^2)b_{03}b_{02} - \frac{1}{4}b_{03}^2\left(h_{03}^2 + \frac{2}{3}h_{03}h_2 + \frac{1}{3}h_2^2 - 2h_3^2\right)\right)c_3, \\
 r_6 = & \frac{1}{6}\left(b_{02}(-h_{01}^3 - h_1h_{01}^2 - h_{01}h_1^2 + 3h_1^3) + b_{01}(h_{01} + h_1)(h_{01}^2 + h_1^2)\right)c_1 + \\
 & \quad + \frac{2}{3}b_{02}(-h_1^3 + h_2^3)c_2 + \frac{1}{6}\left((h_{03} - h_2)(h_{03}^2 + 2h_{03}h_2 + 3h_2^2)b_{02} - \right. \\
 & \quad \left. -(h_{03}^3 + h_{03}^2h_2 + h_{03}h_2^2 + h_2^3 - 4h_3^3)b_{03}\right)c_3, \\
 r_7 = & \left(-\frac{1}{30}(h_{01} + 4h_1)(h_{01} - h_1)b_{02}^3 - \frac{1}{15}\left(h_{01} + \frac{3}{2}h_1\right)(h_{01} - h_1)b_{01}b_{02}^2 - \right. \\
 & \quad \left. - \frac{1}{10}(h_{01} - h_1)b_{01}^2\left(h_{01} + \frac{2}{3}h_1\right)b_{02} + \frac{1}{5}b_{01}^3\left(h_{01}^2 + \frac{1}{2}h_{01}h_1 + \frac{1}{6}h_1^2\right)\right)c_1 + \\
 & \quad + \frac{1}{3}(h_2^2 - h_1^2)b_{02}^3c_2 + \left(\frac{1}{30}(h_{03} + 4h_2)(h_{03} - h_2)b_{02}^3 + \frac{1}{15}\left(h_{03} + \frac{3}{2}h_2\right)(h_{03} - h_2)b_{02}^2b_{03} + \right. \\
 & \quad \left. + \frac{1}{10}b_{03}^2b_{02}(h_{03} - h_2)\left(h_{03} + \frac{2}{3}h_2\right) - \frac{1}{5}\left(h_{03}^2 + \frac{1}{2}h_{03}h_2 + \frac{1}{6}h_2^2 - \frac{5}{3}h_3^2\right)b_{03}^3\right)c_3, \\
 r_8 = & \frac{1}{10}\left((-h_{01}^4 - h_{01}^3h_1 - h_{01}^2h_1^2 - h_{01}h_1^3 + 4h_1^4)b_{02} + \right. \\
 & \quad \left. + (h_{01}^4 + h_{01}^3h_1 + h_{01}^2h_1^2 + h_{01}h_1^3 + h_1^4)b_{01}\right)c_1 + \\
 & \quad + \frac{1}{2}(h_2^4 - h_1^4)b_{02}c_2 + \left(\frac{1}{10}(h_{03} - h_2)(h_{03}^3 + 2h_{03}^2h_2 + 3h_{03}h_2^2 + 4h_2^3)b_{02} - \right. \\
 & \quad \left. - \frac{1}{10}b_{03}\left(h_{03}^4 + h_{03}^3h_2 + h_{03}^2h_2^2 + h_{03}h_2^3 + h_2^4 - 5h_3^4\right)\right)c_3, \\
 r_9 = & \left(-\frac{1}{15}(-h_{01}^3 - 2h_{01}^2h_1 - 3h_{01}h_1^2 + 6h_1^3)b_{02}^2 + \frac{1}{5}b_{01}\left(h_{01}^2 + \frac{4}{3}h_{01}h_1 + h_1^2\right)(h_{01} - h_1)b_{02} - \right. \\
 & \quad \left. - \frac{4}{15}b_{01}^2\left(h_{01}^3 + \frac{3}{4}h_{01}^2h_1 + \frac{1}{2}h_{01}h_1^2 + \frac{1}{4}h_1^3\right)\right)c_1 - \frac{2}{3}(h_2^3 - h_1^3)b_{02}^2c_2 - \\
 & \quad - \left(\frac{1}{15}(h_{03} - h_2)(h_{03}^2 + 3h_{03}h_2 + 6h_2^2)b_{02}^2 + \frac{1}{5}(h_{03} - h_2)b_{02}b_{03}\left(h_{03}^2 + \frac{4}{3}h_{03}h_2 + h_2^2\right) - \right. \\
 & \quad \left. - \frac{4}{15}b_{03}^2\left(h_{03}^3 + \frac{3}{4}h_{03}^2h_2 + \frac{1}{2}h_{03}h_2^2 + \frac{1}{4}h_2^3 - \frac{5}{2}h_3^3\right)\right)c_3,
 \end{aligned}$$

$$\begin{aligned}
 r_{10} = & \left( \frac{1}{10} (h_1 - h_{01}) b_{02}^4 - \frac{1}{10} (h_{01} - h_1) b_{01} b_{02}^3 - \frac{1}{10} (h_{01} - h_1) b_{01}^2 b_{02}^2 - \right. \\
 & - \frac{1}{10} (h_{01} - h_1) b_{02} b_{01}^3 + \frac{2}{5} \left( h_{01} + \frac{1}{4} h_1 \right) b_{01}^4 \Big) c_1 + \frac{1}{2} (h_2 - h_1) b_{02}^4 c_2 + \\
 & + \left( \frac{1}{10} (h_{03} - h_2) b_{02}^4 + \frac{1}{10} b_{03} (h_{03} - h_2) b_{02}^3 + \right. \\
 & \left. + \frac{1}{10} (h_{03} - h_2) b_{02}^2 b_{03}^2 + \frac{1}{10} (h_{03} - h_2) b_{02} b_{03}^3 - \frac{2}{5} \left( h_{03} + \frac{1}{4} h_2 - \frac{5}{4} h_3 \right) b_{03}^4 \right) c_3.
 \end{aligned}$$

From fastening conditions of the rod, we have

$$w_0^{(0)}(0) = w_0^{(0)}(l) = v_0^{(0)}(0) = v_0^{(0)}(l) = u_0^{(0)}(0) = u_0^{(0)}(l) = 0, \tag{40}$$

$$\left. \frac{dw_0^{(0)}}{dx} \right|_{x=0} = \left. \frac{dv_0^{(0)}}{dx} \right|_{x=l} = \left. \frac{du_0^{(0)}}{dx} \right|_{x=0} = \left. \frac{dw_0^{(0)}}{dx} \right|_{x=l} = 0. \tag{41}$$

Then the ratios (24), (25) have the form

$$\frac{d^4 w_0^{(0)}}{dx^4} = \alpha_3 + \alpha_4 x, \quad \frac{d^4 v_0^{(0)}}{dx^4} = \alpha_5 + \alpha_6 x, \tag{42}$$

where

$$\alpha_3 = \frac{1}{l} (g_4 q_{0x} + g_5 m_{0z} + g_6 m_{0y}) - g_5 q_{0y} \alpha_1 - g_6 q_{0z} \beta_1, \quad \alpha_4 = -g_5 \alpha_2 q_{0y} - g_6 \beta_2 q_{0z},$$

$$\alpha_5 = \frac{1}{l} (g_7 q_{0x} + g_8 m_{0z} + g_9 m_{0y}) - g_8 q_{0y} \alpha_1 - g_9 q_{0z} \beta_1, \quad \alpha_6 = -g_8 \alpha_2 q_{0y} - g_9 \beta_2 q_{0z}.$$

Integrating the ratios (42), we obtain

$$w_0^{(0)} = \alpha_4 \frac{x^5}{120} + \alpha_3 \frac{x^4}{24} + C_1 x^3 + C_2 x^2 + C_3 x + C_4, \tag{43}$$

$$v_0^{(0)} = \alpha_6 \frac{x^5}{120} + \alpha_5 \frac{x^4}{24} + C_5 x^3 + C_6 x^2 + C_7 x + C_8. \tag{44}$$

From the ratios (26), (43), (44) we obtain

$$u_0^{(0)} = \alpha_7 \frac{x^4}{12} + \alpha_8 \frac{x^3}{6} - \left( \frac{f_3}{f_1} C_1 + \frac{f_2}{f_1} C_5 \right) \frac{x^2}{2} + C_9 x + C_{10}, \tag{45}$$

where

$$\alpha_7 = -\frac{f_3}{f_1} \frac{\alpha_4}{2} - \frac{f_2}{f_1} \frac{\alpha_6}{2}, \quad \alpha_8 = -\frac{q_0}{lf_1} - \frac{f_3}{f_1} \alpha_3 - \frac{f_2}{f_1} \alpha_5.$$

From (43), (44), (45) and expression (21) we obtain

$$\begin{aligned}
 N^{(0)} = & \left( \frac{f_1 \alpha_7}{3} + \frac{f_2 \alpha_6}{6} + \frac{f_3 \alpha_4}{6} \right) x^3 + \left( \frac{f_1 \alpha_8}{2} + \frac{f_2 \alpha_5}{2} + \frac{f_3 \alpha_3}{2} \right) x^2 + \\
 & + C_9 f_1 + C_6 f_2 + C_2 f_3, \\
 M_y^{(0)} = & \left( \frac{f_3 \alpha_7}{3} + \frac{f_4 \alpha_6}{6} + \frac{f_5 \alpha_4}{6} \right) x^3 + \left( \frac{f_3 \alpha_8}{2} + \frac{f_4 \alpha_5}{2} + \frac{f_5 \alpha_3}{2} \right) x^2 + \\
 & + \left( C_1 (f_5 - f_3^2 f_1) + C_5 \left( f_4 - \frac{f_2 f_3}{f_1} \right) \right) x + C_9 f_3 + C_6 f_4 + C_2 f_5,
 \end{aligned}$$

$$M_z^{(0)} = \left( \frac{f_2 \alpha_7}{3} + \frac{f_5 \alpha_6}{6} + \frac{f_6 \alpha_4}{6} \right) x^3 + \left( \frac{f_2 \alpha_8}{2} + \frac{f_5 \alpha_5}{2} + \frac{f_6 \alpha_3}{2} \right) x^2 + \left( C_1 \left( f_6 - \frac{f_3 f_2}{f_1} \right) + C_5 \left( f_5 - \frac{f_2^2}{f_1} \right) \right) x + C_9 f_2 + C_6 f_5 + C_2 f_6,$$

From fastening conditions (40) we have

$$C_3 = C_4 = C_7 = C_8 = C_{10} = 0,$$

$$C_1 = \frac{\alpha_8}{l^2} - 2 \frac{\alpha_7}{l^3}, \quad C_2 = 3 \frac{\alpha_7}{l^2} - \frac{\alpha_8}{l}, \quad C_5 = \frac{\alpha_{10}}{l^2} - 2 \frac{\alpha_9}{l^3}, \quad C_6 = 3 \frac{\alpha_9}{l^2} - \frac{\alpha_{10}}{l},$$

$$C_9 = -\alpha_7 \frac{l^3}{12} - \alpha_8 \frac{l^2}{6} + \left( \frac{f_3}{f_1} C_1 + \frac{f_2}{f_1} C_5 \right) \frac{l}{2}.$$

where

$$\alpha_7 = -\alpha_4 \frac{l^5}{120} - \alpha_3 \frac{l^4}{24}, \quad \alpha_8 = -\alpha_4 \frac{l^4}{24} - \alpha_3 \frac{l^3}{6}, \quad \alpha_9 = -\alpha_6 \frac{l^5}{120} - \alpha_5 \frac{l^4}{24}, \quad \alpha_{10} = -\alpha_6 \frac{l^4}{24} - \alpha_5 \frac{l^3}{6},$$

Thus, the movements in the zero approximation are completely determined.

$$u_0^{(0)} = \alpha_7 \frac{x^4}{12} + \alpha_8 \frac{x^3}{6} - \left( \frac{f_3}{f_1} C_1 + \frac{f_2}{f_1} C_5 \right) \frac{x^2}{2} + C_9 x, \tag{46}$$

$$v_0^{(0)} = \alpha_6 \frac{x^5}{120} + \alpha_5 \frac{x^4}{24} + C_5 x^3 + C_6 x^2, \tag{47}$$

$$w_0^{(0)} = \alpha_4 \frac{x^5}{120} + \alpha_3 \frac{x^4}{24} + C_1 x^3 + C_2 x^2. \tag{48}$$

From (29), (30), (31), (46), (47), (48) we obtain

$$j_1(x) = \alpha_{11} x^6 + \alpha_{12} x^5 + \alpha_{13} x^4 + \alpha_{14} x^3 + \alpha_{15} x^2 + \alpha_{16} x + \alpha_{17}, \tag{49}$$

where

$$\alpha_{11} = \frac{1}{36} \alpha_4^2 r_6 + \frac{1}{18} (\alpha_6 r_5 + 2 \alpha_7 r_3) \alpha_4 + \frac{1}{9} \alpha_7^2 r_1 + \frac{1}{9} \alpha_6 \alpha_7 r_2 + \frac{1}{36} \alpha_6^2 r_4,$$

$$\alpha_{12} = \frac{1}{6} (\alpha_4 r_6 + \alpha_6 r_5 + 2 \alpha_7 r_3) \alpha_3 + \frac{1}{6} (\alpha_5 r_5 + \alpha_8 r_3) \alpha_4 + \frac{1}{6} (\alpha_6 r_4 + 2 \alpha_7 r_2) \alpha_5 + \frac{1}{3} r_1 \alpha_7 \alpha_8 + \frac{1}{6} \alpha_8 r_2 \alpha_6,$$

$$\alpha_{13} = \frac{1}{12} \frac{1}{f_1} \left( (24 \alpha_4 r_6 + 24 \alpha_6 r_5 + 48 \alpha_7 r_3) C_1 + (24 \alpha_4 r_5 + 24 \alpha_6 r_4 + 48 \alpha_7 r_2) C_5 + 6 r_5 \alpha_5 \alpha_3 + 3 r_6 \alpha_3^2 + 6 \alpha_3 \alpha_8 r_3 + 3 \alpha_5^2 r_4 + 6 \alpha_5 \alpha_8 r_2 + 3 \alpha_8^2 r_1 \right) f_1 - 4 (\alpha_4 r_3 + \alpha_6 r_2 + 2 \alpha_7 r_1) (C_1 f_3 + C_5 f_2),$$

$$\alpha_{14} = \frac{1}{3 f_1} \left( (18 \alpha_3 r_6 + 18 \alpha_5 r_5 + 18 \alpha_8 r_3) C_1 + (18 \alpha_3 r_5 + 18 \alpha_5 r_4 + 18 \alpha_8 r_2) C_5 + (\alpha_6 C_9 + 4 \alpha_7 C_6) r_2 + (\alpha_4 C_9 + 4 \alpha_7 C_2) r_3 + (2 \alpha_4 C_6 + 2 \alpha_6 C_2) r_5 + 2 r_4 \alpha_6 C_6 + 2 r_6 \alpha_4 C_2 + 2 \alpha_7 C_9 r_1 \right) f_1 - 3 (\alpha_3 r_3 + \alpha_5 r_2 + \alpha_8 r_1) (C_1 f_3 + C_5 f_2),$$

$$\alpha_{15} = \frac{1}{f_1^2} \left( (36 C_1^2 r_6 + 72 r_5 C_5 C_1 + 36 r_4 C_5^2 + \alpha_8 C_9 r_1 + (\alpha_5 C_9 + 2 \alpha_8 C_6) r_2 + (\alpha_3 C_9 + 2 \alpha_8 C_2) r_3 + (2 C_2 r_6 + 2 C_6 r_5) \alpha_3 + 2 \alpha_5 (C_2 r_5 + C_6 r_4)) f_1^2 - 12 (C_1 f_3 + C_5 f_2) (C_1 r_3 + C_5 r_2) f_1 + r_1 (C_1 f_3 + C_5 f_2)^2 \right),$$

$$\begin{aligned} \alpha_{16} &= \frac{1}{f_1} (((24C_2r_6 + 24C_6r_5 + 12C_9r_3) C_1 + 12C_5 (2C_2r_5 + 2C_6r_4 + C_9r_2)) f_1 - \\ &\quad - (C_1f_3 + C_5f_2) (4r_2C_6 + 4r_3C_2 + 2r_1C_9)), \\ \alpha_{17} &= C_9^2r_1 + (4C_2r_3 + 4C_6r_2) C_9 + 4C_2^2r_6 + 8C_2C_6r_5 + 4C_6^2r_4, \\ j_2(x) &= \alpha_{18}x^6 + \alpha_{19}x^5 + \alpha_{20}x^4 + \alpha_{21}x^3 + \alpha_{22}x^2 + \alpha_{23}x + \alpha_{24}, \end{aligned} \quad (50)$$

where

$$\begin{aligned} \alpha_{18} &= \frac{1}{36}\alpha_4^2r_8 + \frac{1}{18}(\alpha_6r_9 + 2\alpha_7r_5)\alpha_4 + \frac{1}{9}\alpha_7^2r_2 + \frac{1}{9}\alpha_6\alpha_7r_4 + \frac{1}{36}\alpha_6^2r_7, \\ \alpha_{19} &= \frac{1}{6}(\alpha_4r_8 + \alpha_6r_9 + 2\alpha_7r_5)\alpha_3 + \frac{1}{6}(\alpha_5r_9 + \alpha_8r_5)\alpha_4 + \frac{1}{6}(\alpha_6r_7 + 2\alpha_7r_4)\alpha_5 + \\ &\quad + \frac{1}{3}r_2\alpha_7\alpha_8 + \frac{1}{6}\alpha_6\alpha_8r_4, \\ \alpha_{20} &= \frac{1}{12f_1} (((24\alpha_4r_8 + 24\alpha_6r_9 + 48\alpha_7r_5) C_1 + (24\alpha_4r_9 + 24\alpha_6r_7 + 48\alpha_7r_4) C_5 + \\ &\quad + 3r_8\alpha_3^2 + 6r_9\alpha_5\alpha_3 + 6\alpha_3\alpha_8r_5 + 3r_7\alpha_5^2 + 6\alpha_5\alpha_8r_4 + 3r_2\alpha_8^2) f_1 - \\ &\quad - 4(\alpha_4r_5 + \alpha_6r_4 + 2\alpha_7r_2)(C_1f_3 + C_5f_2)), \\ \alpha_{21} &= \frac{1}{3f_1} (((18\alpha_3r_8 + 18\alpha_5r_9 + 18\alpha_8r_5) C_1 + (18\alpha_3r_9 + 18\alpha_5r_7 + 18\alpha_8r_4) C_5 + \\ &\quad + (\alpha_6C_9 + 4\alpha_7C_6) r_4 + (\alpha_4C_9 + 4\alpha_7C_2) r_5 + (2\alpha_4C_6 + 2\alpha_6C_2) r_9 + 2r_7C_6\alpha_6 + \\ &\quad + 2r_8C_2\alpha_4 + 2r_2\alpha_7C_9) f_1 - 3(\alpha_3r_5 + \alpha_5r_4 + \alpha_8r_2)(C_1f_3 + C_5f_2)), \\ \alpha_{22} &= \frac{1}{f_1^2} ((36r_8C_1^2 + 72r_9C_5C_1 + 36r_7C_5^2 + r_2\alpha_8C_9 + (\alpha_5C_9 + 2\alpha_8C_6) r_4 + \\ &\quad + (\alpha_3C_9 + 2\alpha_8C_2) r_5 + ((2C_2r_8 + 2C_6r_9) \alpha_3 + 2\alpha_5 (C_2r_9 + C_6r_7)) f_1^2 - \\ &\quad - 12(C_1f_3 + C_5f_2)(C_1r_5 + C_5r_4) f_1 + r_2(C_1f_3 + C_5f_2)^2), \\ \alpha_{23} &= \frac{1}{f_1} (((864C_2r_8 + 864C_6r_9 + 432C_9r_5) C_1 + 432C_5 (2C_2r_9 + 2C_6r_7 + C_9r_4)) f_1^3 - \\ &\quad - (C_1f_3 + C_5f_2) (4C_2r_5 + C_6r_4 + 2C_9r_2)), \\ \alpha_{24} &= 4C_2^2r_8 + 8C_2C_6r_9 + 4C_2C_9r_5 + 4C_6^2r_7 + 4C_6C_9r_4 + C_9^2r_2, \\ j_3(x) &= \alpha_{25}x^6 + \alpha_{26}x^5 + \alpha_{27}x^4 + \alpha_{28}x^3 + \alpha_{29}x^2 + \alpha_{30}x + \alpha_{31}, \end{aligned} \quad (51)$$

where

$$\begin{aligned} \alpha_{25} &= \frac{1}{36}\alpha_4^2r_{10} + \frac{1}{72}(4\alpha_6r_8 + 8\alpha_7r_6)\alpha_4 + \frac{1}{9}\alpha_7^2r_3 + \frac{1}{9}\alpha_6\alpha_7r_5 + \frac{1}{72}\alpha_6^2r_9, \\ \alpha_{26} &= \frac{1}{6}(\alpha_4r_{10} + \alpha_6r_8 + 2\alpha_7r_6)\alpha_3 + \frac{1}{6}(\alpha_5r_8 + \alpha_8r_6)\alpha_4 + \frac{1}{12}(\alpha_6r_9 + 4\alpha_7r_5)\alpha_5 + \\ &\quad + \frac{1}{3}r_3\alpha_7\alpha_8 + \frac{1}{6}\alpha_6\alpha_8r_5, \\ \alpha_{27} &= \frac{1}{24f_1} (((48\alpha_4r_{10} + 48\alpha_6r_8 + 96\alpha_7r_6) C_1 + (48\alpha_4r_8 + 24\alpha_6r_9 + 96\alpha_7r_5) C_5 + 3r_9\alpha_5^2 + \\ &\quad + 6r_{10}\alpha_3^2 + 12r_8\alpha_5\alpha_3 + 12r_6\alpha_3\alpha_8 + 12r_5\alpha_5\alpha_8 + 6r_3\alpha_8^2) f_1 - \\ &\quad - (\alpha_4r_6 + \alpha_6r_5 + 2\alpha_7r_3)(C_1f_3 + C_5f_2)), \end{aligned}$$

$$\begin{aligned} \alpha_{28} &= \frac{1}{3f_1} (((18\alpha_3r_{10} + 18\alpha_5r_8 + 18\alpha_8r_6) C_1 + (18\alpha_3r_8 + 9\alpha_5r_9 + 18\alpha_8r_5) C_5 + \\ &+ (\alpha_6C_9 + 4\alpha_7C_6) r_5 + (\alpha_4C_9 + 4\alpha_7C_2) r_6 + (2\alpha_4C_6 + 2\alpha_6C_2) r_8 + r_9C_6\alpha_6 + 2r_{10}C_2\alpha_4 + \\ &+ 2r_3\alpha_7C_9) f_1 - 3(\alpha_3r_6 + \alpha_5r_5 + \alpha_8r_3)(C_1f_3 + C_5f_2)), \\ \alpha_{29} &= \frac{1}{f_1^2} ((36r_{10}C_1^2 + 72r_8C_5C_1 + 18r_9C_5^2 + r_3\alpha_8C_9 + (\alpha_5C_9 + 2\alpha_8C_6) r_5 + \\ &+ (\alpha_3C_9 + 2\alpha_8C_2) r_6 + (2C_2r_{10} + 2C_6r_8) \alpha_3 + \alpha_5(2C_2r_8 + C_6r_9)) f_1^2 - \\ &- 12(C_1f_3 + C_5f_2)(C_1r_6 + C_5r_5) f_1 + r_3(C_1f_3 + C_5f_2)^2), \\ \alpha_{30} &= \frac{1}{f_1} (((24C_2r_{10} + 24C_6r_8 + 12C_9r_6) C_1 + 12C_5(2C_2r_8 + C_6r_9 + C_9r_5)) f_1 - \\ &- (C_1f_3 + C_5f_2)(4C_6r_5 + 4C_2r_6 + 2C_9r_3)), \\ \alpha_{31} &= C_9^2r_3 + (4C_2r_6 + 4C_6r_5) C_9 + 4C_2^2r_{10} + 8C_2C_6r_8 + 2C_6^2r_9. \end{aligned}$$

Substituting the ratios (49), (50), (51) in expressions (34), (35), (36), we get the differential equations to determine the movements in the first approximation

$$\frac{d^4v_0^{(1)}}{dx^4} = \alpha_{32}x^4 + \alpha_{33}x^3 + \alpha_{34}x^2 + \alpha_{35}x + \alpha_{36}, \tag{52}$$

$$\frac{d^4w_0^{(1)}}{dx^4} = \alpha_{37}x^4 + \alpha_{38}x^3 + \alpha_{39}x^2 + \alpha_{40}x + \alpha_{41}, \tag{53}$$

$$\frac{d^2u_0^{(1)}}{dx^2} = \alpha_{42}x^5 + \alpha_{43}x^4 + \alpha_{44}x^3 + \alpha_{45}x^2 + \alpha_{46}x + \alpha_{47}, \tag{54}$$

where

$$\begin{aligned} \alpha_{32} &= 30(\alpha_{11}r_{12} + \alpha_{18}r_{13} + \alpha_{25}r_{14}), & \alpha_{33} &= 20(\alpha_{12}r_{12} + \alpha_{19}r_{13} + \alpha_{26}r_{14}), \\ \alpha_{34} &= 12(\alpha_{13}r_{12} + \alpha_{20}r_{13} + \alpha_{27}r_{14}), & \alpha_{35} &= 6(\alpha_{14}r_{12} + \alpha_{21}r_{13} + \alpha_{28}r_{14}), \\ \alpha_{36} &= 2(\alpha_{15}r_{12} + \alpha_{22}r_{13} + \alpha_{29}r_{14}), & \alpha_{37} &= 30(\alpha_{11}r_{15} + \alpha_{18}r_{16} + \alpha_{25}r_{17}), \\ \alpha_{38} &= 20(\alpha_{12}r_{15} + \alpha_{19}r_{16} + \alpha_{26}r_{17}), & \alpha_{39} &= 12(\alpha_{13}r_{15} + \alpha_{20}r_{16} + \alpha_{27}r_{17}), \\ \alpha_{40} &= 6(\alpha_{14}r_{15} + \alpha_{21}r_{16} + \alpha_{28}r_{17}), & \alpha_{41} &= 2(\alpha_{15}r_{15} + \alpha_{22}r_{16} + \alpha_{29}r_{17}). \\ \alpha_{42} &= -\frac{6\alpha_{11}}{f_1}, & \alpha_{43} &= -\frac{5\alpha_{12}}{f_1}, & \alpha_{44} &= -\frac{4\alpha_{13}}{f_1}, & \alpha_{45} &= -\frac{3\alpha_{14}}{f_1}, \end{aligned}$$

$$\alpha_{46} = -\frac{1}{f_1} (2\alpha_{15} + 720(f_2(r_{12}\alpha_{11} + r_{13}\alpha_{18} + r_{14}\alpha_{25}) + f_3(r_{15}\alpha_{11} + r_{16}\alpha_{18} + r_{17}\alpha_{25}))),$$

$$\alpha_{47} = -\frac{1}{f_1} (\alpha_{16} + 120(f_2(r_{12}\alpha_{12} + r_{13}\alpha_{19} + r_{14}\alpha_{26}) + f_3(r_{15}\alpha_{12} + r_{16}\alpha_{19} + r_{17}\alpha_{26}))).$$

Integrating the equations (52), (53), (54), taking into account the initial conditions for the first approximation

$$w_0^{(1)}(0) = w_0^{(1)}(l) = v_0^{(1)}(0) = v_0^{(1)}(l) = u_0^{(1)}(0) = u_0^{(1)}(l) = 0, \tag{55}$$

$$\left. \frac{dv_0^{(1)}}{dx} \right|_{x=0} = \left. \frac{dv_0^{(1)}}{dx} \right|_{x=l} = \left. \frac{dw_0^{(1)}}{dx} \right|_{x=0} = \left. \frac{dw_0^{(1)}}{dx} \right|_{x=l} = 0, \tag{56}$$



we get

$$v_0^{(1)} = \alpha_{32} \frac{x^8}{1680} + \alpha_{33} \frac{x^7}{840} + \alpha_{34} \frac{x^6}{360} + \alpha_{35} \frac{x^5}{120} + \alpha_{36} \frac{x^4}{24} + C_{11}x^3 + C_{12}x^2, \quad (57)$$

$$w_0^{(1)} = \alpha_{37} \frac{x^8}{1680} + \alpha_{38} \frac{x^7}{840} + \alpha_{39} \frac{x^6}{360} + \alpha_{40} \frac{x^5}{120} + \alpha_{41} \frac{x^4}{24} + C_{13}x^3 + C_{14}x^2, \quad (58)$$

$$u_0^{(1)} = \alpha_{42} \frac{x^7}{42} + \alpha_{43} \frac{x^6}{30} + \alpha_{44} \frac{x^5}{20} + \alpha_{45} \frac{x^4}{12} + \alpha_{46} \frac{x^3}{6} + \alpha_{47} \frac{x^2}{2} + C_{15}x, \quad (59)$$

where

$$C_{11} = \frac{1}{l^2}\alpha_{48} - \frac{2}{l^3}\alpha_{49}, \quad C_{12} = \frac{3}{l^2}\alpha_{49} - \frac{1}{l}\alpha_{48}, \quad C_{13} = \frac{1}{l^2}\alpha_{50} - \frac{2}{l^3}\alpha_{51}, \quad C_{14} = \frac{3}{l^2}\alpha_{51} - \frac{1}{l}\alpha_{50},$$

$$C_{15} = -\alpha_{42} \frac{l^6}{42} - \alpha_{43} \frac{l^5}{30} - \alpha_{44} \frac{l^4}{20} - \alpha_{45} \frac{l^3}{12} - \alpha_{46} \frac{l^2}{6} - \alpha_{47} \frac{l}{2},$$

$$\alpha_{48} = -\alpha_{32} \frac{l^7}{210} - \alpha_{33} \frac{l^6}{140} - \alpha_{34} \frac{l^5}{60} - \alpha_{35} \frac{l^4}{24} - \alpha_{36} \frac{l^3}{6},$$

$$\alpha_{49} = -\alpha_{32} \frac{l^8}{1680} - \alpha_{33} \frac{l^7}{840} - \alpha_{34} \frac{l^6}{360} - \alpha_{35} \frac{l^5}{120} - \alpha_{36} \frac{l^4}{24},$$

$$\alpha_{50} = -\alpha_{37} \frac{l^7}{210} - \alpha_{38} \frac{l^6}{140} - \alpha_{39} \frac{l^5}{60} - \alpha_{40} \frac{l^4}{24} - \alpha_{41} \frac{l^3}{6},$$

$$\alpha_{51} = -\alpha_{37} \frac{l^8}{1680} - \alpha_{38} \frac{l^7}{840} - \alpha_{39} \frac{l^6}{360} - \alpha_{40} \frac{l^5}{120} - \alpha_{41} \frac{l^4}{24}.$$

The expression for longitudinal force and bending moments in the first approximation can be obtained from (29), (30), (31), (49), (50), (51), (57), (58), (59).

## 4 Discussion

Thus, the problem of complex bend of the rod is solved by the method of small parameter in the zero and first approximations. This method can be used to solve the problem in the second and subsequent approximations.

This work is carried out with the partial financial support of RFBR grant (project number 19-01-00038).

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