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B_c excitations at LHC: first observations and further research prospects

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Abstract. Status of the B_c meson excitatons search is presented in our work. We discuss the $2S$ B_c states first discovered by ATLAS and newly confirmed by CMS and LHCb collaborations. We review the observation prospects for the rest predicted states (B_c^*, $2P$ wave, $3P$ wave and D wave) at LHC experiments. Cascade decays to the ground state as well as direct decays to the lepton pair are regarded for these states.

1 Introduction

B_c-meson is a nonrelativistic two heavy quark system. The potential model framework in analogy with $(b\bar{b})$ and $(c\bar{c})$ states predicts 19 bounded states for $(c\bar{b})$ below the decay threshold to $B\bar{D}$ (see [1–4]). However unlike $(b\bar{b})$- or $(c\bar{c})$-quarkonium, there are no annihilation decay modes for these states, which makes the $(b\bar{c})$ system similar to usual $b\bar{q}$ or $c\bar{q}$-meson. For the first time the ground state of B_c-meson was observed at Tevatron (CDF and D0 experiments) in two decay modes: $B_c \to J\psi l\nu$ $(l = e, \mu)$ and $B_c \to J\psi\pi$ [5–8].

Now this $c\bar{b}$ state is well studied by the LHC Collaborations LHCb, CMS, ATLAS in numeral decay modes [9]. Most of the decays occur due to transformation $b \to cW$. The only observed mode with c quark decay is $B_c \to B_s\pi^+$. The annihilation channel is not observed yet. It is interesting to note that according to theoretical predictions the dominant decay channel is $c \to sW$, the contribution of $b \to cW$ is smaller, and the smallest contribution comes from weak annihilation channel $bc \to W$.

The mass and life time are known with excellent accuracy [9–11] as well:

$$M_{B_c} = 6274.9 \pm 0.8 \text{ MeV},$$

$$\tau_{B_c} = 0.507 \pm 0.009 \text{ ps}.$$

The production mechanism of B_c-meson and its excitations is thoroughly studied theoretically in [12–26]. The ratio $R_{B_c} = \sigma(B_c^*)/\sigma(B_c)$ is predicted to be about ~ 2.6 under assumption that the values of wave functions at origin for B_c^* and B_c are the same. According this study the B_c-meson production at fixed gluon interaction energy resolves itself to just a b-quark fragmentation (like fragmentation $b \to B$) only at high transverse momenta, where the cross section ratio between the production of vector state B_c^* and pseudoscalar state is predicted to be $R_{B_c} \sim 1.4$ [1]. However the transverse momentum value where the fragmenta-

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[1]It is worth to mention the study [27], where the fragmentation functions $b \to B_c$ and $b \to B_c^*$ are estimated within one loop approximation and it is shown that value of R_{B_c} does not change dramatically.

Table 1. Mass values and row relative yields of $B_c(2S)$ states estimated from the experimental data.

	experiment	ATLAS [29]	CMS [30]	LHCb [31]
	luminosity	24.1 fb^{-1}	140 fb^{-1}	8.7 fb^{-1}
	energy	7, 8 TeV	13 TeV	7, 8, 13 TeV
mass, MeV	2^3S_1, shifted	6842 ± 6	6842 ± 2	6841 ± 1
	2^1S_0		6871.0 ± 1.6	6872.1 ± 1.6
row relative yield	2^3S_1		0.0088 ± 0.0014	0.0136 ± 0.0027
	2^1S_0		0.0068 ± 0.0014	0.0063 ± 0.0024
	total	0.18 ± 0.05	0.0156 ± 0.0019	0.0198 ± 0.0036
$N(2^3S_1)/N(2^1S_0)$			1.31 ± 0.32	2.1 ± 0.9

tion becomes dominant depends on the gluon energy: the higher gluon energy, the higher this transverse momentum value. Therefore the non-fragmentation mechanisms contribute to all transverse momenta.

Unfortunately the observation of B_c^* is extremely difficult due to the low energy of the photon in the decay $B_c^* \to B_c\gamma$. That is why in study [28] we discussed the other B_c excitations, namely, $2S$-wave states and P-wave states, and these states are more preferable for observation. The recent observation of $2S$ states in ATLAS [29], CMS [30] and LHCb [31] made us return to this discussion (see also [32] and [33]).

2 $B_c(2S) \to B_c(1S) + \pi\pi$ decays

First observation of $B_c(2S)$ excitations belongs to ATLAS collaboration [29]. The result of ATLAS has been recently confirmed by CMS [30] and LHCb [31] collaborations. We show these measurements in Table 1. Before discussing the experimental results, we would like to remind the theoretical predictions on the subject.

According to [28] about one half of such excitations decay to B_c (B_c^*) and $\pi^+\pi^-$ pair:

$$2^1S_0(B_c) \xrightarrow[\sim 50\%]{\pi^+\pi^-} 1^1S_0(B_c),$$
$$2^3S_1(B_c) \xrightarrow[\sim 40\%]{\pi^+\pi^-} 1^3S_1(B_c),$$
$$\sigma(B_c(2S))/\sigma^{\text{total}}(B_c) \sim 25\ \%.$$

Therefore, the predicted relative yield of $2S$-excitations with $B_c(2S) \to B_c(B_c^*) + \pi^+\pi^-$ decay is up to 10% of total B_c-meson yield. Under assumption that the radial wave functions at origin for 2^3S_1 and for 2^1S_0 states are approximately equal to each other, we predicted that for the total phase space

$$\sigma(2^3S_1)/\sigma(2^1S_0) \sim 2.6.$$

However, some models [34–36] predict that the wave function value is essentially larger for pseudoscalar $2S$ state, than for vector one. According [34] the ratio of radial wave functions at zero $R(B_c^*(2S))\ /\ R(B_c(2S)) = 0.87$ that leads to decreasing of $\sigma(2^3S_1)/\sigma(2^1S_0)$ from 2.6 to 2. Within the approach [35, 36] the $\sigma(2^3S_1)/\sigma(2^1S_0)$ decreases even more dramatically: $R(B_c^*(2S))\ /\ R(B_c(2S)) = 0.567$ and therefore the ratio $\sigma(2^3S_1)/\sigma(2^1S_0)$ becomes close to 0.9.

It was also predicted in [28] that the loss of "soft" photon from B_c^* shifts the vector $2S$-state approximately by 65 MeV and insignificantly broadens the peak:

$$\Delta\tilde{M}_{2S} < 2\frac{\Delta M^* \sqrt{\Delta M^2 - 4m_\pi^2}}{M} \approx 10\,\text{MeV}, \tag{1}$$

where M is a ground state mass, $\Delta M^* = M(B_c^*) - M(B_c)$ is a difference between masses of lowest vector and pseudoscalar states and $\Delta M = M(B_c(2^3S_1)) - M(B_c^*)$ is a difference between the masses of 2^3S_1-wave excitation and lowest vector state. As a result, the mass peak on $B_c + \pi\pi$ spectrum for more massive vector state 2^3S_1 will appear about 30 MeV lower, than for less massive pseudoscalar state 2^1S_0. It is worth noting that in our previous study [28] we have skipped the fact that the $\Delta\tilde{M}_{2S}$ is not a value of additional width itself, but an upper value of additional width. The real value of additional width depends on shape of $\pi\pi$ distribution: the smaller the average value of $\pi\pi$ invariant mass $m_{\pi\pi}$, the narrower the peak. The width can be estimated as follows:

$$\Delta\tilde{M}_{2S} \sim 2\frac{\Delta M^* \langle \sqrt{\Delta M^2 - m_{\pi\pi}^2} \rangle}{M}. \tag{2}$$

Now let us return to the experimental results (see Table 1). At first sight the measured $B_c(2S)$ yields are essentially smaller than the predicted one, which is about 10%. However the registration efficiencies in these experimental studies were not taken into account and therefore at the moment this theoretical prediction does not contradict CMS and LHCb results.

Also it is worth to note that the ATLAS measurement [29] is significantly out of the range of others, and we think that such a huge difference can not be explained by the difference in the registration efficiencies. However the mass value measured by ATLAS is in consistence with shifted value of the vector $B_c(2S)$ excitation. The resolution in ATLAS is not enough to separate peaks and it makes awkward the detailed comparison with the results of other experiments, where the two peaks are clearly seen.

It is very interesting to compare the ratio of yields of $B_c^*(2S)$ and $B_c(2S)$ in CMS and LHCb experiments. If LHCb measured the central value which is close to the predicted one in [28]: ~ 2.1, the CMS Collaboration measured the central value ~ 1.3. However, the errors are quite large, and within uncertainties these results do not contradict each other.

We hope that new data will allow to compare the ratio of yields for different experiments. The situation would be most interesting if the ratio essentially depended on kinematic region, which would indicate a crucial change of production mechanism. It is worth noting that the sharp change of this value is not expected within the conventional production model.

3 Radiative B_c meson decays

According to predictions of the potential model, the mass difference between lowest vector and pseudoscalar states of $c\bar{b}$-quarkonium (B_c^* and B_c) is fairly small:

$$M(B_c^*) - M(B_c) \approx 65\text{ MeV}.$$

LHCb detector is not able to detect photon with transverse momentum about 65 MeV, which means that decaying B_c^* shoud have fairly large transverse momentum. It is known that production cross section is greatly reduced with increasing of the transverse momentum, so that it leads to significant decreasing of amount of events, where such photon could be detected.

The maximum photon transverse energy in the laboratory system can be calculated using the expression

$$\omega_T^{max} \approx \left(\sqrt{M_{B_c^*}^2 + p_T^2} + p_T\right)\frac{\Delta M^*}{M_{B_c^*}} \approx 0.01\left(\sqrt{M_{B_c^*}^2 + p_T^2} + p_T\right) \tag{3}$$

It means for example, that photons with transverse energy $\omega_T > 0.5$ GeV may be produced only if transverse momentum of B_c^*-meson $p_T(B_c^*) > 24$ GeV. Such a cut on transverse momentum decreases the yield of B_c^*-mesons approximately by two orders (see [28]). It is not so for radiation transitions of $2P$-wave states. In the latter case the transverse energy could be sufficiently large, even if initial state of $B_c(2P)$-meson would have small momentum. Therefore, despite the fact that the yield of $2P$-excitations is about 6-20% of total yield of B_c-meson [16, 24], it is much easier to register decays of $2P$-excitations. As it was estimated in [28], the yield of $2P$-excitations emitting photon with $\omega_T > 0.5$ GeV is $25 \div 50$ times more, than yield of vector B_c^*, emitting photon with the same transverse energy.

It should be stressed that only 20% of all $2P$-excitations emit only one photon, immediately transforming to lowest pseudoscalar state:

$$2P1^+(B_c) \xrightarrow[\sim 13\%]{\gamma} 1^1S_0(B_c),$$
$$2P1'^+(B_c) \xrightarrow[\sim 94\%]{\gamma} 1^1S_0(B_c).$$

In all other cases $2P$-states first decay to B_c^* while emitting "hard" photon and then decay to B_c emitting "soft" photon:

$$2P(B_c) \xrightarrow{\gamma^{\rm hard}} 1^3S_1(B_c^*) \xrightarrow{\gamma^{\rm soft}} 1^1S_0(B_c).$$

The second photon ("soft") is considered to be lost in almost all the events. However this fact does not preclude the experimental observation of $2P$-wave states of B_c-meson.

Table 2. Radiative decays of B_c meson P-wave excitations (see [4, 45, 46])

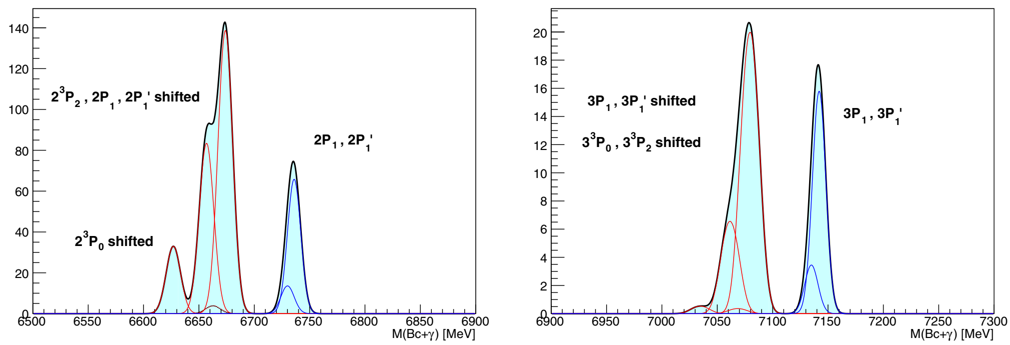
initial state	final state	Br, %	ΔM, MeV
2^3P_0	$1^3S_1+\gamma$	100	363-366
$2P1^+$	$1^3S_1+\gamma$	87	393-400
	$1^1S_0+\gamma$	13	393-400
$2P1'^+$	$1^1S_0+\gamma$	94	472-476
	$1^3S_1+\gamma$	6	472-476
2^3P_2	$1^3S_1+\gamma$	100	410-426
3^3P_0	$1^3S_1+\gamma$	2	741
$3P1^+$	$1^3S_1+\gamma$	8.5	761
	$1^1S_0+\gamma$	3.3	820
$3P1'^+$	$1^1S_0+\gamma$	22.6	825
	$1^3S_1+\gamma$	0.7	769
3^3P_2	$1^3S_1+\gamma$	18	778

Actually, it can be shown that this loss leads to the left shifting of the peak by ΔM^* and to its broadening by the value

$$\Delta\tilde{M} = \tilde{M}_{max} - \tilde{M}_{min} \approx 2\frac{\Delta M^* \Delta M}{M}. \tag{4}$$

Taking $\Delta M^* \approx 65$ MeV and $\Delta M \approx 400$ MeV we obtain the value of broadening for $2P$-wave states: $\Delta\tilde{M}_{2P} \approx 10$ MeV. It is clear that this width is smaller than hardware width of the resonance and doesn't affect its detection quality at all. It should be noted that the value of broadening from "soft" photon loss for $3P$-wave states is also fairly small: $\Delta\tilde{M}_{3P} \approx 20$ MeV.

Predicted distributions over invariant mass for the final pseudoscalar B_c-meson and the "hard" photon in cascade decays of P-wave states of the B_c-meson are shown in Figure 1.



(a) (b)

Figure 1. The mass spectrum of $B_c + \gamma^{\rm hard}$ system for the process: (a) $B_c(2P) \to B_c + \gamma^{\rm hard}[+\gamma^{\rm soft}]$; (b) $B_c(3P) \to B_c + \gamma^{\rm hard}[+\gamma^{\rm soft}]$.

In the picture (a) unshifted peaks from $2P\ 1^+$ and $2P\ 1^{'+}$ states are depicted with blue lines, red lines depict shifted and broadened peaks from 2^3P_0, $2P\ 1^+$, $2P\ 1'^+$ and 2^3P_2 states. At last black line shows sum of the peaks. Picture (b) shows similar distributions for $3P$-wave states. The pictures are plotted by the data from the table 2 and B_c-meson mass spectrum from the work [3]. The peaks are plotted with respect to rough model of detector resolution: broadening the source histograms with Gaussian function with dispersion of 6 MeV. It should be stressed that such simulation is just an estimation and doesn't reflect the actual properties of detector (see also [32] and [33]).

It is important to note that in spite of the fact that the yields of $2P$- and $3P$-states in the proton-proton interaction are nearly the same, $3P$-excitations are harder to detect in the mass spectrum of $B_c + \gamma$. $2P$-excitations always decay via electromagnetic transitions, while $3P$-excitations — only in 20% of the cases. Moreover, it is clear that the shapes of peaks broadened due to the loss of photon will repeat the shapes of distributions over the cosine of the angular between the directions of motion of "soft" and "hard" photons in rest frame of the decaying B_c excitation. Also is is shown that the more minimal transverse energy of photon, the less probability it is radiated by B_c^*-meson.

4 *D*-wave excitations at LHC

The production of D-wave states of B_c quarkonium is not broadly discussed due toupposedly small relative yield $\sim 1\%$ (see for example [37], where the production of D-waves in e^+e^- annihilation was studied), as well as due to technical difficulties in the cross section estimation. However, at the moment, when $2S$ states are already experimentally observed, the theoretical study of hadronic production of D-wave states looks quite reasonable. Indeed, despite that the dominant decay mode for D-wave states is electromagnetic [3, 4, 38, 39], it is shown in [40] that about 20% of such states decay to $1S$ state radiating two π mesons, as well as $2S$ excitations. Thus it provides a chance to extract the D-wave states in the $B_c\pi^+\pi^-$ mass spectrum with large statistics.

Similar to registered $B_c(2S) \xrightarrow{\pi\pi} B_c(1S)$ decay there should be several peaks: corresponding to direct decay to B_c ground state and corresponding to decay with intermediate B_c^*, i.e. lowest vector state. The predicted B_c spectroscopy comprises four D-wave states (we indicate their masses gained by different groups in table (3)). If $B_c(3D) \xrightarrow{\pi\pi} B_c(1S)$ decay goes with conservation of spin (as supposed in [40]), than we should obtain one peak for 3^1D_2 state and three peaks for $3^3D_1, 3^3D_2, 3^3D_3$ states shifted by the value close to $\Delta M^* = M_{B_c^*} - M_{B_c}$. Most likely the latter three ones will overlap each other and will appear earlier than 3^1D_2 peak at the $B_c\pi^+\pi^-$ invariant mass scale. Therefore we could expect, that one narrow peak from D wave states can be observed at $\sim$ 7000 MeV and one broad peak can be observed at $\sim$ 6930 MeV.

Table 3. Masses of D-wave B_c meson states in MeV.

State	GKLT [1]	EFG [38]	GI [4]	MBV [41]	LLLGZ [39]
3^3D_1	7008	7072	7028	6973	7020
$3D_2'$	7016	7079	7036	7003	7032
$3D_2$	7001	7077	7041	6974	7024
3^3D_3	7007	7081	7045	7004	7030

State	EQ [40]	ZVR [42]	FUI [43]	SJSCP [44]
3^3D_1	7012	7010	7024	6998
3^1D_2	7009	7020	7023	6994
3^3D_2	7012	7030	7025	6997
3^3D_3	7005	7040	7022	6990

5 Lepton pair production in radiative B_c meson decays

The branching fraction of lepton pair production in radiative decays of the excited B_c meson $B_1 \to B_2\ell\ell$ can be written in the form [47, 48]

$$\frac{d\mathrm{Br}_{\ell\ell}}{dq^2} = \frac{\alpha}{3\pi}\frac{1}{q^2}\frac{\lambda(M_1; M_2, \sqrt{q^2})}{\lambda(M_1; M_2, 0)}\left(1 - \frac{2m_\ell^2}{q^2}\right)\sqrt{1 - \frac{4m_\ell^2}{q^2}}\mathrm{Br}_\gamma = \frac{dI^{\ell\ell}(q^2)}{dq^2}\mathrm{Br}_\gamma \tag{5}$$

where Br_γ is the branching fraction of the original radiative decay, q^2 is the squared invariant mass of the lepton pair, $\alpha = e^2/4\pi$ is the fine structure coupling constant, $M_{1,2}$ are the masses of initial and final mesons respectively and λ is the velocity of the final particles in $M \to m_1m_2$ decay: $\lambda(M; m_1, m_2) = \sqrt{1 - (m_1 + m_2)^2/M}\sqrt{1 - (m_1 - m_2)^2/M}$.

It should be noted that the relation (5) is universal and does not depend on the physics of the process. The only assumption is that we neglect the q^2 dependence of the $B_1 \to B_2\gamma^*$ decay vertex. This assumption looks quit reasonable since typical energy deposit in the radiative decays of the doubly heavy mesons is small in comparison with quarks' masses. As a result, the conversion factor $I^{\ell\ell}$ depends only on the masses of the initial and final particles. Masses of the leptons and ground state B_c meson can be found easily [9], while for the initial excited particles some theoretical models are required.

The mass of B_c^* meson is not high enough for muon pair production, so only the *ee* channel is opened. The corresponding results are shown in Table 4. As for P wave excitations, both electronic and muonic decays are allowed. One can see in Table 4, that in all cases electron pair emission leads to suppression of the branching fraction by a factor $\sim 10^{-2}$, while in the case of $\mu\mu$ channel the suppression is about an order of magnitude stronger. However, unlike soft photon, the lepton-antilepton pair can be easily detected by the modern detectors, we think that the excited B_c mesons could be observed in the discussed modes.

Table 4. Conversion factors for $B_c^*, B_c(1P_1), B_c(1P_1)' \to B_c\gamma$ decays.

	GI [49]	EFG [38]	FUII [43]	GKLT [46]	EQ [40]	GJ [45]	ZVR [42]
$M(B_c^*)$, MeV	6338	6332	6341	6317	6337	6308	6340
$10^3 \cdot I^{ee}$	6.105	5.616	5.431	5.665	5.869	5.591	6.011
$M(1P_1)$, MeV	6741	6734	6737	6717	6730	6738	6730
$10^3 \cdot I^{ee}$	8.811	8.733	8.689	8.733	8.74	8.821	8.753
$10^3 \cdot I^{\mu\mu}$	0.7192	0.6538	0.6176	0.6538	0.6593	0.7272	0.6703
$M(1P_1')$, MeV	6750	6749	6760	6729	6736	6757	6740
$10^3 \cdot I^{ee}$	8.84	8.782	8.766	8.773	8.76	8.88	8.786
$10^3 \cdot I^{\mu\mu}$	0.7432	0.6949	0.6813	0.6867	0.6758	0.7774	0.6976

6 Conclusion

We discuss the latest results of LHC Collaborations CMS, ATLAS and LHCb, independently observed $2S$ excitations of B_c meson. Certainly these results should trigger for new advances in the double heavy spectroscopy study. We stress that B_c meson production mechanism is largely determined by the dependence of the ratio between 2^3S_1 and 2^1S_0 states on kinematical conditions. We estimate the perspectives for observing the rest excited states at LHC as well. Among B_c^*, P wave and D wave states at least for P excitations we find such perspectives to be rather optimistic. Also we suggest to study P wave states in their radiative decays to the lepton pair.

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