$B_c$ excitations at LHC: first observations and further research prospects

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Abstract. Status of the $B_c$ meson excitations search is presented in our work. We discuss the $2S B_c$ states first discovered by ATLAS and newly confirmed by CMS and LHCb collaborations. We review the observation prospects for the rest predicted states ($B_c^*$, $2P$ wave, $3P$ wave and $D$ wave) at LHC experiments. Cascade decays to the ground state as well as direct decays to the lepton pair are regarded for these states.

1 Introduction

$B_c$-meson is a nonrelativistic two heavy quark system. The potential model framework in analogy with ($b\bar{b}$) and ($c\bar{c}$) states predicts 19 bounded states for ($c\bar{b}$) below the decay threshold to $BD$ (see [1–4]). However unlike ($b\bar{b}$)- or ($c\bar{c}$)-quarkonium, there are no annihilation decay modes for these states, which makes the ($bc$) system similar to usual $b\bar{q}$ or $c\bar{q}$-meson. For the first time the ground state of $B_c$-meson was observed at Tevatron (CDF and D0 experiments) in two decay modes: $B_c \to J/\psi l\nu$ ($l = e, \mu$) and $B_c \to J/\psi \pi$ [5–8].

Now this $c\bar{b}$ state is well studied by the LHC Collaborations LHCb, CMS, ATLAS in numeral decay modes [9]. Most of the decays occur due to transformation $b \to cW$. The only observed mode with $c$ quark decay is $B_c \to B_s \pi^+$. The annihilation channel is not observed yet. It is interesting to note that according to theoretical predictions the dominant decay channel is $c \to sW$, the contribution of $b \to cW$ is smaller, and the smallest contribution comes from weak annihilation channel $bc \to W$.

The mass and life time are known with excellent accuracy [9–11] as well:

$$M_{B_c} = 6274.9 \pm 0.8 \text{ MeV},$$
$$\tau_{B_c} = 0.507 \pm 0.009 \text{ ps}.$$ 

The production mechanism of $B_c$-meson and its excitations is thoroughly studied theoretically in [12–26]. The ratio $R_{B_c} = \sigma(B_c^*)/\sigma(B_c)$ is predicted to be about $\sim 2.6$ under assumption that the values of wave functions at origin for $B_c^*$ and $B_c$ are the same. According this study the $B_c$-meson production at fixed gluon interaction energy resolves itself to just a $b$-quark fragmentation (like fragmentation $b \to B_c$) only at high transverse momenta, where the cross section ratio between the production of vector state $B_c^*$ and pseudoscalar state is predicted to be $R_{B_c} \sim 1.4^4$. However the transverse momentum value where the fragmenta-
made us return to this discussion (see also [32] and [33]).

Table 1. Mass values and row relative yields of $B_c(2S)$ states estimated from the experimental data.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Luminosity</td>
<td>24.1 fb$^{-1}$</td>
<td>140 fb$^{-1}$</td>
<td>8.7 fb$^{-1}$</td>
</tr>
<tr>
<td>Energy</td>
<td>7, 8 TeV</td>
<td>13 TeV</td>
<td>7, 8, 13 TeV</td>
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</tbody>
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<table>
<thead>
<tr>
<th>State</th>
<th>Mass, MeV</th>
<th>Row relative yield</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^1S_1$, shifted</td>
<td>6842 ± 6</td>
<td>0.18 ± 0.05</td>
<td>1.31 ± 0.32</td>
</tr>
<tr>
<td>$2^1S_0$</td>
<td>6842 ± 2</td>
<td>0.0156 ± 0.0019</td>
<td>0.0198 ± 0.0036</td>
</tr>
<tr>
<td>$2^3S_1$</td>
<td>0.0088 ± 0.0014</td>
<td>0.00136 ± 0.00027</td>
<td></td>
</tr>
<tr>
<td>$2^3S_0$</td>
<td>0.0068 ± 0.0014</td>
<td>0.00063 ± 0.00024</td>
<td></td>
</tr>
</tbody>
</table>

The recent observation of $2S$ states in ATLAS [29], CMS [30] and LHCb [31] made us return to this discussion (see also [32] and [33]).

2 $B_c(2S) \rightarrow B_c(1S) + \pi\pi$ decays

First observation of $B_c(2S)$ excitations belongs to ATLAS collaboration [29]. The result of ATLAS has been recently confirmed by CMS [30] and LHCb [31] collaborations. We show these measurements in Table 1. Before discussing the experimental results, we would like to remind the theoretical predictions on the subject.

According to [28] about one half of such excitations decay to $B_c (B_c^*)$ and $\pi^+\pi^-$ pair:

$$2^1S_0(B_c^*) \xrightarrow{\pi^+\pi^- \sim 50\%} 1^1S_0(B_c),$$

$$2^3S_1(B_c^*) \xrightarrow{\pi^+\pi^- \sim 40\%} 1^3S_1(B_c),$$

$$\sigma(B_c(2S))/\sigma^{total}(B_c) \sim 25 \%.$$

Therefore, the predicted relative yield of $2S$-excitations with $B_c(2S) \rightarrow B_c(B_c^*) + \pi^+\pi^-$ decay is up to 10% of total $B_c$-meson yield. Under assumption that the radial wave functions at origin for $2^3S_1$ and for $2^1S_0$ states are approximately equal to each other, we predicted that for the total phase space

$$\sigma(2^3S_1)/\sigma(2^1S_0) \sim 2.6.$$

However, some models [34–36] predict that the wave function value is essentially larger for pseudoscalar $2S$ state, than for vector one. According [34] the ratio of radial wave functions at zero $R(B_c^*(2S))/R(B_c(2S)) = 0.87$ that leads to decreasing of $\sigma(2^3S_1)/\sigma(2^1S_0)$ from 2.6 to 2. Within the approach [35, 36] the $\sigma(2^3S_1)/\sigma(2^1S_0)$ decreases even more dramatically: $R(B_c^*(2S))/R(B_c(2S)) = 0.567$ and therefore the ratio $\sigma(2^3S_1)/\sigma(2^1S_0)$ becomes close to 0.9.

It was also predicted in [28] that the loss of “soft” photon from $B_c^*$ shifts the vector $2S$-state approximately by 65 MeV and insignificantly broadens the peak:
\[ \Delta \tilde{M}_{2S} < 2 \frac{\Delta M' \sqrt{\Delta M^2 - 4m_{\pi}^2}}{M} \approx 10 \text{ MeV}, \]  

(1)

where \( M \) is a ground state mass, \( \Delta M' = M(B_c^e) - M(B_c) \) is a difference between masses of lowest vector and pseudoscalar states and \( \Delta M = M(B_c(2^3S_1)) - M(B_c^e) \) is a difference between the masses of \( 2^3S_1 \)-wave excitation and lowest vector state. As a result, the mass peak on \( B_c + \pi \pi \) spectrum for more massive vector state \( 2^3S_1 \) will appear about 30 MeV lower, than for less massive pseudoscalar state \( 2^1S_0 \). It is worth noting that in our previous study [28] we have skipped the fact that the \( \Delta \tilde{M}_{2S} \) is not a value of additional width itself, but an upper value of additional width. The real value of additional width depends on shape of \( \pi \pi \) distribution: the smaller the average value of \( \pi \pi \) invariant mass \( m_{\pi\pi} \), the narrower the peak. The width can be estimated as follows:

\[ \Delta \tilde{M}_{2S} \sim 2 \frac{\Delta M'(\sqrt{\Delta M^2 - m_{\pi\pi}^2})}{M}. \]  

(2)

Now let us return to the experimental results (see Table 1). At first sight the measured \( B_c(2S) \) yields are essentially smaller than the predicted one, which is about 10%. However the registration efficiencies in these experimental studies were not taken into account and therefore at the moment this theoretical prediction does not contradict CMS and LHCb results.

Also it is worth to note that the ATLAS measurement [29] is significantly out of the range of others, and we think that such a huge difference can not be explained by the difference in the registration efficiencies. However the mass value measured by ATLAS is in consistence with shifted value of the vector \( B_c(2S) \) excitation. The resolution in ATLAS is not enough to separate peaks and it makes awkward the detailed comparison with the results of other experiments, where the two peaks are clearly seen.

It is very interesting to compare the ratio of yields of \( B_c^e(2S) \) and \( B_c(2S) \) in CMS and LHCb experiments. If LHCb measured the central value which is close to the predicted one in [28]: \( \sim 2.1 \), the CMS Collaboration measured the central value \( \sim 1.3 \). However, the errors are quite large, and within uncertainties these results do not contradict each other.

We hope that new data will allow to compare the ratio of yields for different experiments. The situation would be most interesting if the ratio essentially depended on kinematic region, which would indicate a crucial change of production mechanism. It is worth noting that the sharp change of this value is not expected within the conventional production model.

### 3 Radiative \( B_c \) meson decays

According to predictions of the potential model, the mass difference between lowest vector and pseudoscalar states of \( c\bar{b} \)-quarkonium (\( B_c^e \) and \( B_c \)) is fairly small:

\[ M(B_c^e) - M(B_c) \approx 65 \text{ MeV}. \]

LHCb detector is not able to detect photon with transverse momentum about 65 MeV, which means that decaying \( B_c^e \) should have fairly large transverse momentum. It is known that production cross section is greatly reduced with increasing of the transverse momentum, so that it leads to significant decreasing of amount of events, where such photon could be detected.

The maximum photon transverse energy in the laboratory system can be calculated using the expression

\[ \omega_{T}^{\text{max}} \approx \left( \sqrt{M_{B_c}^2 + p_T^2} + p_T \right) \frac{\Delta M'}{M_{B_c}} \approx 0.01 \left( \sqrt{M_{B_c}^2 + p_T^2} + p_T \right) \]  

(3)
It means for example, that photons with transverse energy $\omega_T > 0.5$ GeV may be produced only if transverse momentum of $B_c^*$-meson $p_T(B_c^*) > 24$ GeV. Such a cut on transverse momentum decreases the yield of $B_c^*$-mesons approximately by two orders (see [28]). It is not so for radiation transitions of $2P$-wave states. In the latter case the transverse energy could be sufficiently large, even if initial state of $B_c(2P)$-meson would have small momentum. Therefore, despite the fact that the yield of $2P$-excitations is about 6-20% of total yield of $B_c$-meson [16, 24], it is much easier to register decays of $2P$-excitations. As it was estimated in [28], the yield of $2P$-excitations emitting photon with $\omega_T > 0.5$ GeV is $25 \div 50$ times more, than yield of vector $B_c^*$, emitting photon with the same transverse energy.

It should be stressed that only 20% of all $2P$-excitations emit only one photon, immediately transforming to lowest pseudoscalar state:

$$2P1^+(B_c) \stackrel{\gamma}{\rightarrow} 1^1S_0(B_c^*),$$
$$2P1'+(B_c) \stackrel{\gamma}{\rightarrow} 1^1S_0(B_c^*).$$

In all other cases $2P$-states first decay to $B_c^*$ while emitting “hard” photon and then decay to $B_c$ emitting “soft” photon:

$$2P(B_c) \stackrel{\gamma_{hard}}{\rightarrow} 1^3S_1(B_c^*) \stackrel{\gamma_{soft}}{\rightarrow} 1^1S_0(B_c).$$

The second photon (“soft”) is considered to be lost in almost all the events. However this fact does not preclude the experimental observation of $2P$-wave states of $B_c$-meson.

### Table 2. Radiative decays of $B_c$ meson $P$-wave excitations (see [4, 45, 46])

<table>
<thead>
<tr>
<th>initial state</th>
<th>final state</th>
<th>Br, %</th>
<th>$\Delta M$, MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^3P_0$</td>
<td>$1^3S_1 + \gamma$</td>
<td>100</td>
<td>363-366</td>
</tr>
<tr>
<td>$2P1^+$</td>
<td>$1^3S_1 + \gamma$</td>
<td>87</td>
<td>393-400</td>
</tr>
<tr>
<td></td>
<td>$1^1S_0 + \gamma$</td>
<td>13</td>
<td>393-400</td>
</tr>
<tr>
<td>$2P1'^+$</td>
<td>$1^3S_0 + \gamma$</td>
<td>94</td>
<td>472-476</td>
</tr>
<tr>
<td></td>
<td>$1^3S_1 + \gamma$</td>
<td>6</td>
<td>472-476</td>
</tr>
<tr>
<td>$2^3P_2$</td>
<td>$1^3S_1 + \gamma$</td>
<td>100</td>
<td>410-426</td>
</tr>
<tr>
<td>$3^3P_0$</td>
<td>$1^3S_1 + \gamma$</td>
<td>2</td>
<td>741</td>
</tr>
<tr>
<td>$3P1^+$</td>
<td>$1^3S_1 + \gamma$</td>
<td>8.5</td>
<td>761</td>
</tr>
<tr>
<td></td>
<td>$1^1S_0 + \gamma$</td>
<td>3.3</td>
<td>820</td>
</tr>
<tr>
<td>$3P1'^+$</td>
<td>$1^3S_0 + \gamma$</td>
<td>22.6</td>
<td>825</td>
</tr>
<tr>
<td></td>
<td>$1^3S_1 + \gamma$</td>
<td>0.7</td>
<td>769</td>
</tr>
<tr>
<td>$3^3P_2$</td>
<td>$1^3S_1 + \gamma$</td>
<td>18</td>
<td>778</td>
</tr>
</tbody>
</table>

Actually, it can be shown that this loss leads to the left shifting of the peak by $\Delta M^*$ and to its broadening by the value

$$\Delta \tilde{M} = \tilde{M}_{max} - \tilde{M}_{min} \approx 2 \frac{\Delta M^* \Delta M}{M}.$$

Taking $\Delta M^* \approx 65$ MeV and $\Delta M \approx 400$ MeV we obtain the value of broadening for $2P$-wave states: $\Delta \tilde{M}_{2P} \approx 10$ MeV. It is clear that this width is smaller than hardware width of the resonance and doesn’t affect its detection quality at all. It should be noted that the value of broadening from “soft” photon loss for $3P$-wave states is also fairly small: $\Delta \tilde{M}_{3P} \approx 20$ MeV.

Predicted distributions over invariant mass for the final pseudoscalar $B_c$-meson and the “hard” photon in cascade decays of $P$-wave states of the $B_c$-meson are shown in Figure 1.
In all other cases $P$ does not preclude the experimental observation of $2P$-wave states. In the latter case the transverse energy could be $\gamma c^2$, emitting photon with the same transverse energy.

$\Delta$-states first decay to $c^*\bar{c}$, emitting photon with the same transverse energy. The pictures are plotted by the data from the table 2 and $B_c$-meson mass spectrum from the work [3]. The peaks are plotted with respect to rough model of detector resolution: broadening the source histograms with Gaussian function with dispersion of 6 MeV. It should be stressed that such simulation is just an estimation and doesn’t reflect the actual properties of detector (see also [32] and [33]).

It is important to note that in spite of the fact that the yields of $2P$- and $3P$-states in the proton-proton interaction are nearly the same, $3P$-excitations are harder to detect in the mass spectrum of $B_c + \gamma$. $2P$-excitations always decay via electromagnetic transitions, while $3P$-excitations — only in 20% of the cases. Moreover, it is clear that the shapes of peaks broadened due to the loss of photon will repeat the shapes of distributions over the cosine of the angular between the directions of motion of “soft” and “hard” photons in rest frame of the decaying $B_c$ excitation. Also is is shown that the more minimal transverse energy of photon, the less probability it is radiated by $B_c^*$-meson.

4 $D$-wave excitations at LHC

The production of $D$-wave states of $B_c$ quarkonium is not broadly discussed due toupposedly small relative yield $\sim 1\%$ (see for example [37], where the production of $D$-waves in $e^+e^-$ annihilation was studied), as well as due to technical difficulties in the cross section estimation. However, at the moment, when $2S$ states are already experimentally observed, the theoretical study of hadronic production of $D$-wave states looks quite reasonable. Indeed, despite that the dominant decay mode for $D$-wave states is electromagnetic [3, 4, 38, 39], it is shown in [40] that about 20% of such states decay to $1S$ state radiating two $\pi$ mesons, as well as $2S$ excitations. Thus it provides a chance to extract the $D$-wave states in the $B_c\pi^+\pi^-$ mass spectrum with large statistics.

Similar to registered $B_c(2S) \rightarrow B_c(1S)$ decay there should be several peaks: corresponding to direct decay to $B_c$ ground state and corresponding to decay with intermediate $B_c^*$, i.e. lowest vector state. The predicted $B_c$ spectroscopy comprises four $D$-wave states (we indicate their masses gained by different groups in table (3)). If $B_c(3D) \rightarrow B_c(1S)$ decay goes with conservation of spin (as supposed in [40]), then we should obtain one peak for $3^1D_2$ state and
three peaks for $3^3D_1, 3^3D_2, 3^3D_3$ states shifted by the value close to $\Delta M^* = M_{B^*} - M_{B_c}$. Most likely the latter three ones will overlap each other and will appear earlier than $3^1D_2$ peak at the $B_c \pi^+\pi^-$ invariant mass scale. Therefore we could expect, that one narrow peak from $D$ wave states can be observed at $\sim 7000$ MeV and one broad peak can be observed at $\sim 6930$ MeV.

Table 3. Masses of $D$-wave $B_c$ meson states in MeV.

<table>
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<tr>
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<tr>
<td>$3^3D_1$</td>
<td>7008</td>
<td>7072</td>
<td>7028</td>
<td>6973</td>
<td>7020</td>
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<tr>
<td>$3^3D'_2$</td>
<td>7016</td>
<td>7079</td>
<td>7036</td>
<td>7003</td>
<td>7032</td>
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<tr>
<td>$3^3D_2$</td>
<td>7001</td>
<td>7077</td>
<td>7041</td>
<td>6974</td>
<td>7024</td>
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<tr>
<td>$3^3D_3$</td>
<td>7007</td>
<td>7081</td>
<td>7045</td>
<td>7004</td>
<td>7030</td>
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<tbody>
<tr>
<td>$3^3D_1$</td>
<td>7012</td>
<td>7010</td>
<td>7024</td>
<td>6998</td>
</tr>
<tr>
<td>$3^1D_2$</td>
<td>7009</td>
<td>7020</td>
<td>7023</td>
<td>6994</td>
</tr>
<tr>
<td>$3^3D_2$</td>
<td>7012</td>
<td>7030</td>
<td>7025</td>
<td>6997</td>
</tr>
<tr>
<td>$3^3D_3$</td>
<td>7005</td>
<td>7040</td>
<td>7022</td>
<td>6990</td>
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</table>

5 Lepton pair production in radiative $B_c$ meson decays

The branching fraction of lepton pair production in radiative decays of the excited $B_c$ meson $B_1 \to B_2\ell\ell$ can be written in the form [47, 48]

$$
\frac{d\text{Br}_{\ell\ell}}{dq^2} = \frac{\alpha}{3\pi} \frac{1}{q^2} \frac{\lambda(M_1; M_2, \sqrt{q^2})}{\lambda(M_1; M_2, 0)} \left( 1 - \frac{2m_\ell^2}{q^2} \right) \sqrt{1 - \frac{4m_\ell^2}{q^2}} \text{Br}_\gamma = \frac{dI_{\ell\ell}(q^2)}{dq^2} \text{Br}_\gamma
$$

(5)

where $\text{Br}_\gamma$ is the branching fraction of the original radiative decay, $q^2$ is the squared invariant mass of the lepton pair, $\alpha = e^2/4\pi$ is the fine structure coupling constant, $M_{1,2}$ are the masses of initial and final mesons respectively and $\lambda$ is the velocity of the final particles in $M \to m_1m_2$ decay: $\lambda(M; m_1, m_2) = \sqrt{1 - (m_1 + m_2)^2/M} \sqrt{1 - (m_1 - m_2)^2/M}$.

It should be noted that the relation (5) is universal and does not depend on the physics of the process. The only assumption is that we neglect the $q^2$ dependence of the $B_1 \to B_2\gamma^*$ decay vertex. This assumption looks quite reasonable since typical energy deposit in the radiative decays of the doubly heavy mesons is small in comparison with quarks’ masses. As a result, the conversion factor $I_{\ell\ell}$ depends only on the masses of the initial and final particles. Masses of the leptons and ground state $B_c$ meson can be found easily [9], while for the initial excited particles some theoretical models are required.

The mass of $B_c^*$ meson is not high enough for muon pair production, so only the $ee$ channel is opened. The corresponding results are shown in Table 4. As for $P$ wave excitations, both electronic and muonic decays are allowed. One can see in Table 4, that in all cases electron pair emission leads to suppression of the branching fraction by a factor $\sim 10^{-2}$, while in the case of $\mu\mu$ channel the suppression is about an order of magnitude stronger. However, unlike soft photon, the lepton-antilepton pair can be easily detected by the modern detectors, we think that the excited $B_c$ mesons could be observed in the discussed modes.
Table 4. Conversion factors for $B^+_c, B_c(1P_1), B_c(1P_1)' \rightarrow B_c\gamma$ decays.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>$10^3 \cdot f_{ee}$</td>
<td>6.105</td>
<td>5.616</td>
<td>5.431</td>
<td>5.665</td>
<td>5.869</td>
<td>5.591</td>
<td>6.011</td>
</tr>
<tr>
<td>$M(1P_1)$, MeV</td>
<td>6741</td>
<td>6734</td>
<td>6737</td>
<td>6717</td>
<td>6730</td>
<td>6738</td>
<td>6730</td>
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<tr>
<td>$10^3 \cdot f_{ee}$</td>
<td>8.811</td>
<td>8.733</td>
<td>8.689</td>
<td>8.733</td>
<td>8.74</td>
<td>8.821</td>
<td>8.753</td>
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<tr>
<td>$10^3 \cdot f_{po}$</td>
<td>0.7192</td>
<td>0.6538</td>
<td>0.6176</td>
<td>0.6538</td>
<td>0.6593</td>
<td>0.7272</td>
<td>0.6703</td>
</tr>
</tbody>
</table>

6 Conclusion

We discuss the latest results of LHC Collaborations CMS, ATLAS and LHCb, independently observed $2S$ excitations of $B_c$ meson. Certainly these results should trigger for new advances in the double heavy spectroscopy study. We stress that $B_c$ meson production mechanism is largely determined by the dependence of the ratio between $2^3S_1$ and $2^1S_0$ states on kinematical conditions. We estimate the perspectives for observing the rest excited states at LHC as well. Among $B^+_c$, $P$ wave and $D$ wave states at least for $P$ excitations we find such perspectives to be rather optimistic. Also we suggest to study $P$ wave states in their radiative decays to the lepton pair.

A. Berezhnoy and I. Belov acknowledge the support from “Basis” Foundation (grants 17-12-244-1 and 17-12-244-41). Authors thank V. Galkin and A. Martynenko for help and fruitful discussion.

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