Effects of light-by-light scattering in the Lamb shift and hyperfine structure of muonic hydrogen

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Abstract. We calculate the meson exchange contribution to the interaction operator of muon and proton, which is determined by the meson coupling with two photon state. For the construction of transition form factor $Meson \rightarrow \gamma\gamma$ we use the existing parametrizations based on experimental data including the monopole parametrization over photon four-momenta. For an estimate of the form factor value at zero photon four-momenta squared we use experimental data on the decay width $\Gamma_{Meson \rightarrow \gamma\gamma}$. It is shown that scalar, pseudoscalar, axial vector and tensor mesons exchanges give significant contribution to the Lamb shift (LS) and hyperfine splitting (HFS) in muonic hydrogen which should be taken into account for a comparison with precise experimental data.

1 Introduction

The discrepancy between results for the proton charge radius obtained by different methods got the name "proton radius puzzle". In particular, measurements using electronic hydrogen lead to a different proton charge radius compared with those using muonic hydrogen. This problem arose after the CREMA experiments on Lamb shift measurement in muonic hydrogen [1, 2]. Increasingly accurate experiments require corresponding theoretical calculations. To achieve high accuracy it is necessary to take into account the problem of a more accurate construction of the particle interaction operator in muonic atoms and the inclusion of new contributions to this operator. Emerging new experimental data on the Lamb shift in electron hydrogen, as well as a new analysis of experiments already performed on the scattering of leptons by nucleons, show that the values of the proton charge radius obtained from electron and muon systems are approaching [3, 4]. The problem of the proton charge radius is gradually beginning to be solved. However, the analysis of new interactions between the proton and the muon is important for future more accurate experiments. Among the interactions of the proton and the muon there are those in which two virtual photons turn into a meson, which leads to an effective one-meson potential. The calculation of the transition form factor (TFF) of two photons into a meson can be performed within the framework of nonperturbative quantum chromodynamics. In this work we continue our investigations [5–9] (see also [10–14]) of the role of one-meson exchange interactions in muonic hydrogen (μp).

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2 One meson exchange contribution

2.1 Scalar meson exchange

Our approach to the calculation of one-meson exchange contributions to the energy levels of (μp) is based on quasipotential approach in quantum field theory [15–18]. The general parametrization of scalar meson $\rightarrow \gamma^* + \gamma^*$ vertex function takes the form [14]:

$$T_S^{\mu\nu}(t, k_1, k_2) = 4\pi\alpha \left\{ A(t^2, k_1^2, k_2^2)(g^{\mu\nu}(k_1 \cdot k_2) - k_1^\nu k_2^\mu) + \right. \\ \left. B(t^2, k_1^2, k_2^2)(k_2^\mu k_1^\nu - k_1^\mu(k_1 \cdot k_2))(k_1^\nu k_2^\mu - k_2^\nu(k_1 \cdot k_2)) \right\}, \quad (1)$$

where $A(t^2, k_1^2, k_2^2)$, $B(t^2, k_1^2, k_2^2)$ are two scalar functions, $k_{1,2}$ are four momenta of virtual photons, t is the four momentum of scalar meson. Then the muon-proton interaction amplitude via the scalar meson exchange can be written as follows:

$$iM_S = \frac{\alpha^2 g_s}{\pi^2} \int \frac{d^4 k A(t^2, k^2, k^2)(g^{\mu\nu}(k_1 \cdot k_2) - k_1^\nu k_2^\mu)[\bar{u}(q_1)\gamma_\mu(\hat{p}_1 - \hat{k} + m_1)\gamma_\nu u(p_1)][\bar{v}(p_2)v(q_2)]}{k^4(k^2 - 2m_1 k_0)(t^2 + M_s^2)}, \quad (2)$$

where p_1, p_2 are four momenta of particles in initial state, q_1, q_2 are four momenta of particles in final state. We set $t = q_1 - p_1 = 0$ because this momentum is small and we consider further the leading order in fine structure constant α contribution to the interaction operator ($|\mathbf{t}| \sim \mu\alpha$). This leads to the cancelation of the term with the function $B(p^2, k_1^2, k_2^2)$. Using projection operator on muon-proton states with spin $S=0$, $S=1$ [15] we can construct the interaction operator for these states. In the case of triplet state we find:

$$iM_S(^3S_1) = \frac{\alpha^2 g_s}{16m_1^2 m_2^2 \pi^2} \int \frac{d^4 k}{k^4(k^2 - 2k_0 m_1)} A(t^2, k^2, k^2)(g_{\mu\nu}k^2 - k_\mu k_\nu) \quad (3)$$

$$Tr \left[\frac{1 + \gamma_0}{2\sqrt{2}} \hat{\varepsilon}(\hat{p}_2 - m_2)(\hat{q}_2 - m_2) \hat{\varepsilon}^* \frac{1 + \gamma_0}{2\sqrt{2}} (\hat{q}_1 + m_1)\gamma_\mu(\hat{p}_1 - \hat{k} + m_1)\gamma_\nu(\hat{p}_1 + m_1) \right] \frac{1}{t^2 + M_s^2},$$

where m_1, m_2 are the masses of a muon and proton, M_s is the mass of scalar meson. After the trace calculation using the package Form [19] we obtain in the leading order:

$$\mathcal{T}_{1S_0} = \mathcal{T}_{3S_1} = k^2(3k_0 + 2m_1) - 2m_1 k_0^2. \quad (4)$$

So in the leading order the scalar meson exchange doesn't contribute to hyperfine structure. The typical momentum integral contributing to the shift has the following form:

$$\mathcal{I}_1 = \int \frac{d^4 k (k^2 + 2k_0^2)}{k^2(k^4 + a_1^2 k_0^2)} \frac{1}{(k^2 + 1)^2} = \frac{\pi^2}{12} \left[-9 + 36 \ln 2 + 2a_1^2(-7 + 12 \ln 2) - 12(3 + 2a_1^2) \ln a_1 \right], \quad (5)$$

where $a_1 = 2m_1/\Lambda$. An analytical value $\mathcal{I}_1$ is presented after an expansion over $2m_1/\Lambda$ up to terms of the second order. Such integral appears if we suppose that the parametrization of function $A(t^2, k^2, k^2)$ for scalar mesons has monopole form for variables k_1^2 and k_2^2 :

$$A(t^2, k^2, k^2) = A_s \frac{\Lambda^4}{(\Lambda^2 - k_1^2)(\Lambda^2 - k_2^2)}, \quad k_1 = k, \quad k_2 = -k, \quad (6)$$

where $A_s = A(0, 0, 0)$. Then the interaction potential takes the form :

$$\Delta V_{2S}^{LS,S}(t) = \frac{\alpha^2 g_s m_1 A_s}{6} \left[-9 + 36 \ln 2 + 2a_1^2(-7 + 12 \ln 2) - 12(3 + 2a_1^2) \ln a_1 \right] \frac{1}{t^2 + M_s^2}. \quad (7)$$

Averaging (7) over wave functions of 2S-state we obtain the shift of 2S-level in the form:

$$\Delta E_{\sigma(550)}^{Ls}(2S) = \frac{\alpha^5 \mu^3 g_s m_1 A_S}{96\pi M_s^2} \frac{(2 + \frac{W^2}{M_s^2})}{(1 + \frac{W}{M_s})^4} \left[-9 + 36 \ln 2 + 2a_1^2(-7 + 12 \ln 2) - 12(3 + 2a_1^2) \ln a_1 \right] \quad (8)$$

$$= -0.011 \text{ meV}.$$

The contribution to 2P-state is suppressed by additional degrees of α :

$$\Delta E_{\sigma(550)}^{Ls}(2P) = \frac{\alpha^7 \mu^5 g_s A_S}{288\pi m_1 M_s^2 (1 + \frac{W}{M_s})^4} \left\{ \left[\frac{3}{4} + \frac{W}{M_s} + \frac{3}{8} \frac{W^2}{M_s^2} \right] [-9 + a_1^2(-5 + 6 \ln 2) - 6a_1^2 \ln a_1] - \right. \quad (9)$$

$$\left. \frac{3m_1^2}{M_s^2} [-9 + 36 \ln 2 + 2a_1^2(-7 + 12 \ln 2) - 12(3 + 2a_1^2) \ln a_1] \right\} = 0.000014 \mu\text{eV}.$$

2.2 Pseudoscalar meson exchange

The general parametrization of pseudoscalar meson (π^0, η, η') $\rightarrow$ two virtual photon vertex function is expressed through transition form factor in the form [20]:

$$V^{\mu\nu}(k_1, k_2) = i\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \frac{\alpha}{\pi F_\pi} F_{\pi^0 \gamma^* \gamma^*}(t^2, k_1^2, k_2^2), \quad (10)$$

Transition form factor has normalization $F_{\pi^0 \gamma^* \gamma^*}(0, 0, 0) = 1$. At first we consider construction of the hyperfine part of interaction potential in the case of S-states. Using projection operators technique we present interaction amplitude via pseudoscalar meson exchange as follows:

$$i\mathcal{M}_P = 4\pi Z\alpha \frac{1}{16m_1^2 m_2^2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(p_1 - k)^2 - m_1^2} F_{\pi^0 \gamma^* \gamma^*}(t^2, k_1^2, k_2^2) \frac{1}{\mathbf{t}^2 + M^2} \quad (11)$$

$$k_\alpha t_\beta \epsilon^{\mu\nu\alpha\beta} Tr[(\hat{q}_1 + m_1)\gamma^\nu(\hat{p}_1 - \hat{k} + m_1)\gamma^\mu(\hat{p}_1 + m_1)\hat{\Gamma}(\hat{p}_2 - m_2)\gamma_5(\hat{q}_2 - m_2)\hat{\Gamma}^+],$$

Introduction total and relative momenta of particles in the initial and final states $p = (0, \mathbf{p})$, $q = (0, \mathbf{q})$ instead $p_{1,2}$, $q_{1,2}$ and taking into account that relative momenta are small ($|\mathbf{p}| \sim \mu\alpha$, $|\mathbf{q}| \sim \mu\alpha$) we obtain for the numerator of the amplitude the result in the leading order (proportional to t^2):

$$\mathcal{N}^{hfs} = \frac{512}{3} m_1^2 m_2 [t^2 k^2 - (tk)^2] \quad (12)$$

As a result the hyperfine part of muon proton interaction potential takes the form:

$$\Delta V_{\mathcal{P}}^{hfs}(\mathbf{p}, \mathbf{q}) = \frac{\alpha^2}{6\pi^2} \frac{g_p}{m_2 F_\pi} \frac{(\mathbf{p} - \mathbf{q})^2}{(\mathbf{p} - \mathbf{q})^2 - m_\pi^2} \mathcal{A}(t^2), \quad (13)$$

where

$$\mathcal{A}(t^2) = \frac{2i}{\pi^2 t^2} \int d^4 k \frac{t^2 k^2 - (tk)^2}{k^2(k-t)^2(k^2 - 2kp_1)} F_{\pi \gamma^* \gamma^*}(k^2, (k-t)^2). \quad (14)$$

For $\mathcal{A}(t^2)$ there is a dispersion relation with one subtraction, which has the form:

$$\mathcal{A}(t^2) = \mathcal{A}(0) - \frac{t^2}{\pi} \int_0^\infty ds \frac{Im\mathcal{A}(s)}{s(s + t^2)}. \quad (15)$$

Imaginary part of $\mathcal{A}(t^2)$ doesn't depend on specific form of transition form factor parametrization and is well known [21]:

$$\text{Im}\mathcal{A}(t^2) = \frac{\pi}{2\beta(t^2)} \ln \frac{1-\beta(t^2)}{1+\beta(t^2)}, \quad \beta(t^2) = \sqrt{1-4\frac{m_1^2}{t^2}}. \quad (16)$$

Numerical value of the $\mathcal{A}(0)$ for an electron is equal $\mathcal{A}^e(0) = -21.9 \pm 0.3$ [22] and for a muon $\mathcal{A}^\mu(0) = -6.1 \pm 0.3$. Going in (13) to coordinate representation by means of the Fourier transform we obtain:

$$\Delta V_{\mathcal{P}}(r) = \frac{\alpha^2 g_p}{6F_\pi m_2 \pi^2} \{ \mathcal{A}(0) \left[\delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r} e^{-m_\pi r} \right] - \frac{1}{\pi} \int_0^\infty \frac{ds}{s} \text{Im}\mathcal{A}(s) \left[\delta(\mathbf{r}) + \frac{1}{4\pi r(s - m_\pi^2)} (m_\pi^4 e^{-m_\pi r} - s^2 e^{-\sqrt{s}r}) \right] \} \quad (17)$$

For numerical calculations we also use the Goldberg-Treiman relation for the pion-nucleon interaction constant $g_p = g_{\pi NN} = \frac{m_p g_A}{F_\pi}$ with $g_A = 1.27$, $F_\pi = 0.0924$ GeV. Averaging (17) over wave functions of 1S and 2S-state we obtain the hyperfine splitting of 1S and 2S states:

$$\Delta E_{\mathcal{P}}^{hfs}(1S) = -0.0017 \text{ meV}, \quad \Delta E_{\mathcal{P}}^{hfs}(2S) = -0.0002 \text{ meV} \quad (18)$$

In a similar way we obtain the potential of $2P_{1/2}$ -state hyperfine splitting:

$$\Delta V_{2P_{1/2}}^{hfs, \mathcal{P}}(\mathbf{p}, \mathbf{q}) = -\frac{\alpha^2 g_A}{24\pi^3 F_\pi^2} \frac{(\mathbf{p}\mathbf{q}) \left(\frac{p}{q} + \frac{q}{p} \right) - 2pq}{(\mathbf{p} - \mathbf{q})^2 + m_\pi^2} \mathcal{A}(0) \quad (19)$$

Averaging (19) over wave function of 2P-state we obtain the numerical value of the hyperfine splitting of the level $2P_{1/2}$:

$$\Delta E_{2P_{1/2}}^{hfs, \mathcal{P}} = 0.0004 \text{ } \mu\text{eV}. \quad (20)$$

2.3 Axial vector meson exchange

The general parametrization of axial vector meson $\rightarrow \gamma^* \gamma^*$ vertex function has the form [23]:

$$T^{\mu\nu\alpha} = 4\pi i \alpha \varepsilon_{\mu\nu\alpha\tau} (k_1^\tau k_2^\tau - k_2^\tau k_1^\tau) F_{AV\gamma^*\gamma^*}(k^2, k^2). \quad (21)$$

Interaction amplitude via axial vector meson exchange has the following general structure:

$$i\mathcal{M}_{\mathcal{AV}} = 4\pi Z\alpha \int \frac{d^4 k}{(2\pi)^4} \frac{T^{\mu\nu\alpha}}{(p_1 - k)^2 - m_1^2} [\bar{u}_1(q_1) \gamma^\mu (\hat{p}_1 - \hat{k} + m_1) \gamma^\nu u_1(p_1)] \mathcal{D}^{\alpha\beta}(t) [\bar{v}_2(p_2) \Gamma_\beta^{(p)} v_2(q_2)], \quad (22)$$

where the vertex function

$$\Gamma_\beta^{(p)} = \gamma_\mu \gamma_5 G^A(t^2) - i\gamma_5 \frac{t_\mu}{2m_2} G^P(t^2), \quad (23)$$

where $G^A(t^2)$, $G^P(t^2)$ are axial and induced form factors respectively. Axial vector meson propagator is $\mathcal{D}^{\alpha\beta}(t) = \frac{g_{\alpha\beta} - t_\alpha t_\beta / M_A^2}{t^2 + M_A^2}$. Using the Lorentz transformation for wave function of muon and proton and formalism of projection operators we obtain for numerator of the amplitude in the case of state with $F = 1$:

$$\mathcal{N}_{AV} = \pi i \alpha \varepsilon_{\rho\sigma\tau\alpha} g^{\mu\rho} g^{\nu\sigma} (2k - t)^\tau g^{\alpha\beta} F^{(0)}(k_1^2, k_2^2) G^A(0) \times \quad (24)$$

$$Tr \left[(\hat{q}_1 + m_1) \gamma^\nu (\hat{p}_1 - \hat{k} + m_1) \gamma^\mu \hat{p}_1 + m_1 \right] \frac{1 + \gamma_0}{2\sqrt{2}} \hat{\varepsilon}(\hat{p}_2 - m_2) \gamma_\beta \gamma_5 (\hat{q}_2 - m_2) \hat{\varepsilon}^* \frac{1 + \gamma_0}{2\sqrt{2}} \Big],$$

For transition form factor $F_{AV\gamma^*\gamma^*}(k^2, k^2)$ we use the following representation [24]:

$$F_{AV\gamma^*\gamma^*}(k^2, k^2) = F_{AV\gamma^*\gamma^*}(0, 0) \frac{\Lambda^8}{(\Lambda^2 - k^2)^4}, F_{AV\gamma^*\gamma^*}(0, 0) = 24 < e_q^2 > R'(0) \frac{\sqrt{2}}{\sqrt{\pi} M_A^{9/2}}, \quad (25)$$

where $R'(0)$ is the value of derivative of radial wave function at zero, $< e_q^2 >$ is squared effective charge of a light quark in a bound state in units of electron charge. As a result the potential contributing to hyperfine structure has the form of a Yukawa potential:

$$\Delta V_{\mathcal{AV}}^{hfs}(\mathbf{p} - \mathbf{q}) = \frac{32\alpha^2 G^A(0) F_{AV\gamma\gamma}(0, 0)}{3\pi^2 e(\mathbf{t}^2 + M_A^2)} \int id^4k \frac{2k^2 + k_0^2}{k^2(k^2 - 2m_1 k_0)} \frac{\Lambda^8}{(k^2 - \Lambda^2)^4} \quad (26)$$

Equation (26) contains a characteristic loop momentum integral, that can be calculated analytically:

$$\begin{aligned} \mathcal{I}\left(\frac{m_1}{\Lambda}\right) &= \int (-id^4k) \frac{2k^2 + k_0^2}{k^2(k^2 - 2m_1 k_0)} \frac{\Lambda^6}{(k^2 - \Lambda^2)^4} = \\ &= -\frac{\pi^2}{4(a_1^2 - 1)^{5/2}} \left[3\sqrt{a_1^2 - 1} + a_1^2(2a_1^2 - 5)\text{arcsec}(a_1) \right] = -6.78645, \quad a_1 = \frac{2m_1}{\Lambda} \end{aligned} \quad (27)$$

After the Fourier transform of the potential (26) and averaging it over the wave functions, we obtain the contribution to the HFS spectrum:

$$\Delta E_{f_1}^{hfs}(1S) = \frac{32\alpha^5 \mu^3 \Lambda^2 g_{AVNN} F_{AV\gamma^*\gamma^*}(0, 0) \left(2 + \frac{W^2}{M_A^2}\right)}{3M_A^2 \pi^2 e \left(1 + \frac{W}{M_A}\right)^2} \mathcal{I}\left(\frac{m_1}{\Lambda}\right) = -0.0093 \text{ meV}, \quad (28)$$

$$\Delta E_{f_1}^{hfs}(2S) = \frac{2\alpha^5 \mu^3 \Lambda^2 g_{AVNN} F_{AV\gamma^*\gamma^*}(0, 0)}{3M_A^2 \pi^2 e \left(1 + \frac{W}{M_A}\right)^4} \mathcal{I}\left(\frac{m_1}{\Lambda}\right) = -0.0012 \text{ meV}. \quad (29)$$

2.4 Tensor meson exchange

Following [25] we will assume further that hadronic light-by-light amplitude for tensor mesons is dominated by helicity $\Lambda = 2$ exchange. The amplitude of the process $\gamma^* + \gamma^* \rightarrow T$ can be parameterized as follows [26, 27]:

$$T_{\mu\nu\alpha\beta}^T(k_1, k_2) = e^2 \frac{k_1 k_2}{M} \mathcal{M}_{\mu\nu\alpha\beta}(k_1, k_2) \mathcal{F}_{T\gamma^*\gamma^*}(k_1^2, k_2^2), \quad (30)$$

where $\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2, k_2^2)$ is a transition form factor (TFF) of tensor meson,

$$\mathcal{M}_{\mu\nu\alpha\beta}(k_1, k_2) = \left\{ R_{\mu\alpha}(k_1, k_2) R_{\nu\beta}(k_1, k_2) + \frac{1}{8(k_1 + k_2)^2 \left[(k_1 k_2)^2 - k_1^2 k_2^2 \right]} R_{\mu\nu}(k_1, k_2) \times \right. \quad (31)$$

$$\left. \left[(k_1 + k_2)^2 (k_1 - k_2)_\alpha - (k_1^2 - k_2^2)(k_1 + k_2)_\alpha \right] \times \left[(k_1 + k_2)^2 (k_1 - k_2)_\beta - (k_1^2 - k_2^2)(k_1 + k_2)_\beta \right] \right\},$$

$$R_{\mu\nu}(k_1, k_2) = -g_{\mu\nu} + \frac{1}{X} \left[(k_1 k_2)(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu) - k_1^2 k_2^\mu k_2^\nu - k_2^2 k_1^\mu k_1^\nu \right], \quad X = (k_1 k_2)^2 - k_1^2 k_2^2.$$

Then the muon-proton interaction amplitude via tensor meson exchange can be presented as

$$i\mathcal{M}_T = \frac{4\pi Z\alpha}{16m_1^2 m_2^2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(p_1 - k)^2 - m_1^2} \mathcal{D}_{\mu\mu'}(t - k) \mathcal{D}_{\nu\nu'}(k) \mathcal{D}_T^{\alpha'\beta'\alpha\beta}(t) \mathcal{M}_{\mu'\nu'\alpha'\beta'}(k_1, k_2) \quad (32)$$

$$[\bar{u}(0)(\hat{q}_1 + m_1)\gamma_\mu(\hat{p}_1 - \hat{k} + m_1)\gamma_\nu(\hat{p}_1 + m_1)u(0)][\bar{v}(0)(\hat{p}_2 - m_2)\Gamma_{TNN}^{\alpha\beta}(\hat{q}_2 - m_2)v(0)],$$

where the vertex function of tensor meson nucleon interaction is [28]

$$\Gamma_{TNN}^{\alpha\beta}(p_2, q_2) = \frac{G_{TNN}}{m_2} \left[(q_2 + p_2)_\alpha \gamma_\beta + (q_2 + p_2)_\beta \gamma_\alpha \right] + \frac{F_{TNN}}{m_2^2} (q_2 + p_2)_\alpha (q_2 + p_2)_\beta.$$

We will use only first term of this vertex function, because different estimations [28] show, that there take place an inequality $G_{TNN} \gg F_{TNN}$. The tensor meson propagator has the form:

$$\mathcal{D}_T^{\mu\nu\alpha\beta}(t) = \frac{1}{t^2 - M_T^2 + i\varepsilon} \left\{ \frac{1}{2} (g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\beta}) + \right. \quad (33)$$

$$\left. \frac{1}{2} \left(g_{\mu\alpha} \frac{t^\nu t^\beta}{M_T^2} + g_{\nu\beta} \frac{t^\mu t^\alpha}{M_T^2} + g_{\mu\beta} \frac{t^\nu t^\alpha}{M_T^2} + g_{\nu\alpha} \frac{t^\mu t^\beta}{M_T^2} \right) + \frac{2}{3} \left(\frac{1}{2} g_{\mu\nu} + \frac{t^\mu t^\nu}{M_T^2} \right) \left(\frac{1}{2} g_{\alpha\beta} + \frac{t^\alpha t^\beta}{M_T^2} \right) \right\}.$$

To obtain the interaction potential via tensor meson exchange we use the formalism of projection operators. Projecting the interaction amplitude on the muon-proton S-state with $F = 1$ we obtain in the numerator of the amplitude ($v = (1, 0, 0, 0)$ is auxiliary time-like 4-vector):

$$\mathcal{T}(^3S_1) = Tr \left[\gamma_{\varepsilon_1} \frac{1 + \gamma_0}{2\sqrt{2}} (\hat{q}_1 + m_1) \gamma_\mu (\hat{p}_1 - \hat{k} + m_1) \gamma_\nu (\hat{p}_1 + m_1) \right. \quad (34)$$

$$\left. \frac{1 + \gamma_0}{2\sqrt{2}} \gamma_{\varepsilon_2} (\hat{p}_2 - m_2) \Gamma_{TNN}^{\alpha\beta}(\hat{q}_2 - m_2) \right] \frac{1}{3} (-g_{\varepsilon_1 \varepsilon_2} + v_{\varepsilon_1} v_{\varepsilon_2}).$$

Using the package Form for the trace calculation we obtain that in the leading order in t the contributions to the interaction amplitudes in 3S_1 and 1S_0 states are the following:

$$\mathcal{T}(^3S_1) = \mathcal{T}(^1S_0) = \frac{G_{TNN}}{M_T} 4m_1 k^4 \left(1 + \frac{k_0^4 t^4}{[(kt)^2 - k^2 t^2]^2} + 2 \frac{k_0^2 t^2}{[(kt)^2 - k^2 t^2]} \right). \quad (35)$$

The contribution to the ground state hyperfine splitting contains additional degrees t :

$$\mathcal{T}^{hfs} = \frac{4}{3} \frac{G_{TNN}}{M_T m_2} t^2 k^4 \left(1 - \frac{t^4 k_0^4}{[(kt)^2 - k^2 t^2]^2} \right). \quad (36)$$

The Belle Collaboration investigated the reaction $\gamma^* \gamma^* \rightarrow \pi^0 \pi^0$ in a wide range of photon virtualities and invariant mass W of the $\pi^0 \pi^0$ system in the range $0.5 \text{ GeV} < W < 2.1 \text{ GeV}$ [29]. From analysis [29] it was obtained first empirical results for the tensor meson f_2 TFF in the dipole form on one photon four-momentum squared when other four-momentum squared is equal to 0. Because in the interaction amplitude both photons are virtual we use the dipole parametrization for the TFF $\gamma^* + \gamma^* \rightarrow T$:

$$\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2, k_2^2) = \mathcal{F}_{T\gamma^*\gamma^*}(0, 0) \frac{\Lambda^4}{(k^2 - \Lambda^2)^2}, \quad k_1 = k, \quad k_2 = -k. \quad (37)$$

The parametrization (37) is widely used for the description of experimental data for all exchanged mesons (see previous sections). Making the transition to the Euclidean space we present the loop momentum integral in the integral form:

$$\begin{aligned} \mathcal{I} &= \int_0^\infty \frac{d^4 k}{k^4 - 4k_0^2 m_1^2} \mathcal{F}_{T\gamma^* \gamma^*}(0, 0) \frac{\Lambda^4}{(k^2 - \Lambda^2)^2} \left(1 + \frac{k_0^4 t^4}{[(kt)^2 - k^2 t^2]^2} + 2 \frac{k_0^2 t^2}{[(kt)^2 - k^2 t^2]} \right) = \quad (38) \\ &= -\mathcal{F}_{T\gamma^* \gamma^*}(0, 0) \int_0^\pi \frac{\sin \psi d\psi}{4(a_1^2 \cos \psi + a_1^2 - 2)^2} (a_1^2 \cos 2\psi - 2 \ln(a_1^2 \cos^2 \psi) + a_1^2 - 2) \times \\ &\quad \left(\sin^3 \psi - 3 \sin \psi (\cos^2 \psi + 3) + \cos^2 \psi (\cos 2\psi - 7) \ln \left(\frac{2}{\sin \psi + 1} - 1 \right) \right) \end{aligned}$$

where $a_1 = \frac{2m_1}{\Lambda}$, $\mathcal{F}_{T\gamma^* \gamma^*}(0, 0) = \frac{2\sqrt{5}\Gamma_{\gamma\gamma}}{\alpha\sqrt{\pi}M_T}$. The integral (38) is calculated numerically. Then the particle interaction operators for the Lamb shift and hyperfine splitting are:

$$\Delta V_T^{Ls}(r) = -\frac{16\alpha^2 m_1 G_{TNN}}{M_T} \frac{2\sqrt{5}\Gamma_{\gamma\gamma}}{\alpha\sqrt{\pi}M_T} \mathcal{I} \frac{1}{4\pi r} e^{-M_T r}, \quad (39)$$

$$\Delta V_T^{hfs}(r) = \frac{8\alpha^2 G_{TNN}}{3\pi m_2 M_T} \frac{2\sqrt{5}\Gamma_{\gamma\gamma}}{\alpha\sqrt{\pi}M_T} \mathcal{J} \left(\delta(\mathbf{r}) - \frac{M_T^2}{4\pi r} e^{-M_T r} \right), \quad (40)$$

$$\begin{aligned} \mathcal{J} &= \int_0^\pi \frac{\sin \psi d\psi}{(-2 + a_1^2 + a_1^2 \cos 2\psi)^2} (a_1^2 \cos 2\psi - 2 \ln(a_1^2 \cos^2 \psi) + a_1^2 - 2) \times \\ &\quad \left(\sin^3 \psi + 7 \sin \psi - 3 \sin \psi \cos^2 \psi + 2 \cos^4 \psi \ln \left(\frac{2}{\sin \psi + 1} - 1 \right) \right). \end{aligned}$$

There are several tensor mesons which can contribute to LS and HFS [30], but in our calculation we take into account only the contribution of $f_2(1270)$ meson, since we know for it more or less reliable values of various parameters, including the coupling constant with the nucleon [28]. Using obtained potentials (39), (40) we can calculate contributions to the energy levels of muonic hydrogen:

$$\Delta E_{f_2}^{Ls}(1S) = -\frac{16\alpha^5 \mu^3 m_1 G_{TNN}}{\pi M_T} \frac{2\sqrt{5}\Gamma_{\gamma\gamma}}{\sqrt{\pi}M_T} \frac{1}{(M_T + 2W)^2} \mathcal{I} = -0.0528 \text{ meV}, \quad (41)$$

$$\Delta E_{f_2}^{Ls}(2S) = -\frac{2\alpha^5 \mu^3 m_1 G_{TNN}}{\pi M_T} \frac{2\sqrt{5}\Gamma_{\gamma\gamma}}{\sqrt{\pi}M_T} \frac{(2M_T^2 + W^2)}{2(M_T + 2W)^4} \mathcal{I} = -0.0066 \text{ meV}, \quad (42)$$

$$\Delta E_{f_2}^{hfs}(1S) = -\frac{8\alpha^5 \mu^3 G_{TNN}}{3\pi^2 M_T m_2} \frac{2\sqrt{5}\Gamma_{\gamma\gamma}}{\sqrt{\pi}M_T} \left(1 - \frac{M_T^2}{(M_T + 2W)^2} \right) \mathcal{J} = -0.0551 \text{ } \mu\text{eV}, \quad (43)$$

$$\Delta E_{f_2}^{hfs}(2S) = -\frac{\alpha^5 \mu^3 G_{TNN}}{3\pi^2 M_T m_2} \frac{2\sqrt{5}\Gamma_{\gamma\gamma}}{\sqrt{\pi}M_T} \left(1 - \frac{M_T^2(2M_T^2 + W^2)}{2(M_T + W)^4} \right) \mathcal{J} = -0.0069 \text{ } \mu\text{eV}, \quad (44)$$

where $W = \mu Z\alpha$. Another tensor meson $a_2(1320)$ can make an order of magnitude smaller contribution to (41)-(44). Such an approximate estimate is due to the fact that we do not know for $a_2(1320)$ exactly the coupling constant with the nucleon, and the estimates of the Reggeon coupling constants with the nucleon differ by an order of magnitude: $f_{2Rpp} = 11.04$, $a_{2Rpp} = 1.68$ [31]. If we use the dipole parameterization for each variable k_1^2 , k_2^2 in TFF, then the numerical results are reduced by about 15 percent.

3 Conclusion

The obtained numerical values of the meson contributions to the Lamb shift and the hyperfine structure show that they are significant and must be taken into account in a more accurate comparison with experimental data. The contribution of the tensor meson to the Lamb shift is comparable to the contribution of the scalar σ -meson. Other tensor mesons apparently make a significantly smaller contribution, since their constant of interaction with the nucleon is much smaller. Experimental data [30] show that all tensor mesons have a significant decay width into a pair of pions, which interact well with the nucleon, therefore, such processes need to be investigated additionally. Work in this direction is in progress.

The work is supported by Russian Science Foundation (grant No. RSF 18-12-00128) and Russian Foundation for Basic Research (grant No. 18-32-00023) (F.A.M.).

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