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Estimates of the B_c wave function from the CDF and LHCb production data

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Abstract. In the framework of perturbative QCD and nonrelativistic bound state formalism, we calculate the production of B_c and B_c^* mesons at the conditions of the CDF and LHCb experiments. We derive first estimations for the B_c wave function from a comparison with the available data.

1 Introduction

The family of $B_c^{(*)}$ mesons is an interesting though poorly explored part of the quarkonium world. Although some properties of these mesons may look apparently different from the ones of the hidden-flavor onium states, their inner structure must be similar and driven by the same physics. Studying the $B_c^{(*)}$ properties is important on its own and can provide an additional cross check of the exploited theoretical models.

The flavor composition of $B_c^{(*)}$ mesons excludes the convenient strong and electromagnetic decays channels that could be used as a prompt measure of the nonrelativistic wave function. Instead, we will try to obtain an estimate of this essential parameter via considering the production process. We will rely on the data collected by the CDF Collaboration at the Fermilab Tevatron at 1.8 TeV [1] and 1.96 TeV [2] and by the LHCb Collaboration at CERN at 7 TeV [3] and 8 TeV [4].

The proposed method was tested in a number of calculations [5, 6] by performing a global fit of the inclusive ψ', χ_c, and J/ψ production. All of the meson wave functions were taken there as free parameters, and the returned values were close to those obtained from the decays $\psi'\to\mu^+\mu^-$, $\chi_{c2}\to\gamma\gamma$, $J/\psi\to\mu^+\mu^-$.

2 Theoretical framework

In the theory, the production of $B_c^{(*)}$ mesons at the LHCb conditions is dominated by the $O(\alpha_s)^4$ partonic subprocess

$$g + g \to B_c^{(*)} + b + \bar{c}, \tag{1}$$

where $B_c^{(*)}$ may denote either pseudoscalar B_c (spin=0) or vector B_c^* (spin=1) bound state of the charm and beauty quarks. The evaluation of the relevant 36 Feynman diagrams is straightforward and is described in every detail in Ref. [7]. The only innovation made in

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the present calculation is in using the k_t-factorization approach. The advantage of the latter comes from the ease of including the initial state radiation corrections that are efficiently taken into account in the form of the evolution of unintegrated, or transverse momentum dependent (TMD) gluon densities. Then, in accordance with the k_t-factorization prescription [8], the initial off-shell gluon spin density matrix is taken in the form $\overline{\epsilon_g^\mu \epsilon_g^{*\nu}} = k_T^\mu k_T^\nu / |k_T|^2$, where k_T is the component of the gluon momentum perpendicular to the beam axis. In the limit when $k_T \to 0$, this expression converges to the ordinary $\overline{\epsilon_g^\mu \epsilon_g^{*\nu}} = -g^{\mu\nu}/2$, and we recover the results of collinear approach [9–11]. This work is the first calculation of the $B_c^{(*)}$ hadronic production with k_t-factorization.

he perturbative part of the calculation is performed according to the formula

$$d\sigma(pp \to B_c b\bar{c}X) = \frac{\alpha_s^4}{12\,\hat{s}^2}\,|\mathcal{R}(0)|^2 \frac{1}{4}\sum_{\text{spins}} \frac{1}{64}\sum_{\text{colors}} \left|\mathcal{M}(gg \to B_c b\bar{c})\right|^2$$

$$\times\, \mathcal{F}_g(x_1, k_{1T}^2, \mu^2)\, \mathcal{F}_g(x_2, k_{2T}^2, \mu^2)\, dk_{1T}^2\, dk_{2T}^2\, dp_{B_cT}^2\, dp_{cT}^2\, dy_{B_c}\, dy_b\, dy_{\bar{c}}\, \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi}\frac{d\phi_{B_c}}{2\pi}\frac{d\phi_c}{2\pi}, \quad (2)$$

and we use the JH'2013 (set 2) [12] parametrization for the TMD gluon distribution $\mathcal{F}_g(x_i, k_{iT}^2, \mu^2)$. The factorization scale is set to $\mu_F^2 = \hat{s} + q_T^2$, where $\hat{s}$ is the total energy of the partonic subprocess and q_T is the net non-zero transverse momentum of the incoming gluon pair. This special choice for the factorization scale is connected with the evolution details of the TMD gluon density (see [12]). The renormalization scale is set, by default, to $\mu_R^2 = m_{B_cT}^2 \equiv p_{B_cT}^2 + m_{B_c}^2$. We do not exclude that this setting overestimates the momentum transfer in the hard subprocess, and so, will also consider an alternative choice. An extended discussion on this point will be given in the next section.

The absolute normalization of the cross section depends on a singe non-perturbative parameter: the radial (color singlet) wave function at the origin of the coordinate space $|\mathcal{R}(0)|^2$ [13–15]. The so called color octet contributions are nowhere important as they are suppressed by the relative velocity counting rules. Note by the way that the gluon fragmentation mechanism (known to dominate the production of J/ψ mesons at high transverse momenta) is not applicable to our case because of the flavor composition of B_c mesons.

To perform a comparison with the data (see below) we also need to calculate the production of the ordinary B^+ mesons. Again, we do that in the k_t-factorization approach with the same gluon density [12] as above, and with Peterson [16] fragmentation function with $\epsilon = 0.0126$ for the formation of B^+ mesons from b-quarks. The consistency of this setting was shown in a previous publication [17].

3 Numerical results and discussion

The data we wish to compare with are presented in the form of the ratio of the $B_c^{(*)}$ to B^+ production cross sections times the relevant branching fractions. All these results accumulate the statistics from both B_c and B_c^* mesons and include also their charge conjugate states.

At $\sqrt{s} = 1.8$ TeV,the CDF Collaboration reports [1] for the fiducial phase space defined as $p_T^{B_c} > 6$ GeV, $p_T^{B^+} > 6$ GeV, $|y^{B_c}| < 1$, $|y^{B^+}| < 1$:

$$\frac{\sigma(B_c)\, Br(B_c \to J/\psi\, l\nu)}{\sigma(B^+)\, Br(B^+ \to J/\psi\, K)} = 0.132 \pm {}^{0.061}_{0052}\,. \quad (3)$$

Hereafter, in the experimental references B_c will denote a combined sample of B_c and B_c^* mesons. Within the specified kinematic cuts, we obtain from Eq.(2):

$$\begin{aligned}\sigma^{theor}(B_c^+) &= |\mathcal{R}(0)|^2 \cdot 0.248 \text{ nb/GeV}^3,\\ \sigma^{theor}(B_c^{*+}) &= |\mathcal{R}(0)|^2 \cdot 0.515 \text{ nb/GeV}^3,\\ \sigma^{theor}(B_c^+ + B_c^{*+}) &= |\mathcal{R}(0)|^2 \cdot 0.763 \text{ nb/GeV}^3. \end{aligned} \tag{4}$$

We also have for the production of B^+ mesons

$$\sigma^{theor}(B^+)\, Br(B^+ \to J/\psi\, K^+) = 3.56 \text{ nb}, \tag{5}$$

where we have used the decay branching fraction value

$$Br(B^+ \to J/\psi\, K^+) = 1.026 \cdot 10^{-3} \tag{6}$$

taken from the Particle Data book [18].

At $\sqrt{s} = 1.96$ TeV,the CDF Collaboration reports [2] for $p_T^{B_c} > 6$ GeV, $p_T^{B^+} > 6$ GeV, $|y^{B_c}| < 0.6$, and $|y^{B^+}| < 0.6$:

$$\frac{\sigma(B_c)\, Br(B_c \to J/\psi\, l\nu)}{\sigma(B^+)\, Br(B^+ \to J/\psi\, K)} = 0.211 \pm {}^{0.024}_{0.023} \tag{7}$$

Within the above cuts, we obtain

$$\begin{aligned}\sigma^{theor}(B_c^+) &= |\mathcal{R}(0)|^2 \cdot 0.175 \text{ nb/GeV}^3,\\ \sigma^{theor}(B_c^{*+}) &= |\mathcal{R}(0)|^2 \cdot 0.363 \text{ nb/GeV}^3,\\ \sigma^{theor}(B_c^+ + B_c^{*+}) &= |\mathcal{R}(0)|^2 \cdot 0.538 \text{ nb/GeV}^3, \end{aligned} \tag{8}$$

and

$$\sigma^{theor}(B^+)\, Br(B^+ \to J/\psi\, K^+) = 2.43 \text{ nb}. \tag{9}$$

At $\sqrt{s} = 7$ TeV,the LHCb Collaboration reports [3] for $p_T^{B_c} > 4$ GeV, $p_T^{B^+} > 4$ GeV, $2.0 < y^{B_c} < 4.5$, and $2.0 < y^{B^+} < 4.5$:

$$\frac{\sigma(B_c)\, Br(B_c \to J/\psi\, \pi^+)}{\sigma(B^+)\, Br(B^+ \to J/\psi\, K)} = 0.0061 \pm 0.0012. \tag{10}$$

Under these conditions, we have:

$$\begin{aligned}\sigma^{theor}(B_c^+) &= |\mathcal{R}(0)|^2 \cdot 1.23 \text{ nb/GeV}^3,\\ \sigma^{theor}(B_c^{*+}) &= |\mathcal{R}(0)|^2 \cdot 1.80 \text{ nb/GeV}^3,\\ \sigma^{theor}(B_c^+ + B_c^{*+}) &= |\mathcal{R}(0)|^2 \cdot 3.03 \text{ nb/GeV}^3, \end{aligned} \tag{11}$$

and

$$\sigma^{theor}(B^+)\, Br(B^+ \to J/\psi\, K^+) = 9.04 \text{ nb}. \tag{12}$$

Finally, at $\sqrt{s} = 8$ TeV,the LHCb Collaboration reports [3] for $p_T^{B_c} < 20$ GeV, $p_T^{B^+} < 20$ GeV, $2.0 < y^{B_c} < 4.5$, and $2.0 < y^{B^+} < 4.5$:

$$\frac{\sigma(B_c)\, Br(B_c \to J/\psi\, \pi^+)}{\sigma(B^+)\, Br(B^+ \to J/\psi\, K)} = 0.0068 \pm 0.0002; \tag{13}$$

and our predictions read:

$$\begin{aligned} \sigma^{theor}(B_c^+) &= |\mathcal{R}(0)|^2 \cdot 4.92 \text{ nb/GeV}^3, \\ \sigma^{theor}(B_c^{*+}) &= |\mathcal{R}(0)|^2 \cdot 5.63 \text{ nb/GeV}^3, \\ \sigma^{theor}(B_c^+{+}B_c^{*+}) &= |\mathcal{R}(0)|^2 \cdot 10.55 \text{ nb/GeV}^3, \end{aligned} \quad (14)$$

and

$$\sigma^{theor}(B^+)\, Br(B^+ \to J/\psi\, K^+) = 32.66 \text{ nb}. \quad (15)$$

The above data have to be combined with the experimentally measured [19] ratio of the branching fractions

$$Br(B_c \to J/\psi\, \pi^+)/Br(B_c \to J/\psi\, \mu\nu) = 0.047 \quad (16)$$

and with the theoretically calculated [20] decay branching fraction

$$Br(B_c \to J/\psi\, \pi^+) = 0.0033. \quad (17)$$

(The original [20] prediction of 0.0029 was corrected [4] to 0.0033 for the latest measurement of the B_c lifetime.)

Making the necessary substitutions and comparing Eqs. (3), (7), (10), and (13) with theoretical predictions we deduce the following estimations for the radial wave function:

$$\begin{aligned} |\mathcal{R}(0)|^2 &= 4.39 \pm 2.00 \text{ GeV}^3 && \text{Ref. [1]} \\ |\mathcal{R}(0)|^2 &= 6.79 \pm 0.08 \text{ GeV}^3 && \text{Ref. [2]} \\ |\mathcal{R}(0)|^2 &= 5.52 \pm 0.11 \text{ GeV}^3 && \text{Ref. [3]} \\ |\mathcal{R}(0)|^2 &= 6.32 \pm 0.18 \text{ GeV}^3 && \text{Ref. [4]} \end{aligned} \quad (18)$$

These can be summarised in a mean-square average value

$$|\mathcal{R}(0)|^2 = 5.78 \text{ GeV}^3 \quad (19)$$

with an error of ± 0.64 GeV^3 and ± 1.07 GeV^3 at the 60% and 80% confidence level, respectively.

We conclude our analysis with showing the ratio (13) in the differential form, as a function of the transverse momentum for 3 separate rapidity intervals (see Fig. 1). The calculations and the data [4] are in good agreement in shape, thus indicating that the hard scattering partonic subprocesses are calculated correctly. The choice of the TMD gluon density is unimportant since the gluon distributions cancel out in the ratio. The sensitivity to the renormalization scale is high, as a reflection of the fourth power of $\alpha_S(\mu_R^2)$ in the key subprocess (1). The central values of the cross sections (green histograms) correspond to the conventional choice $\mu_R^2 = m_{B_cT}^2$; the theoretical uncertainty band (yellow area in Fig. 1) is obtained by varying μ_R around its default value by a factor of 2.

Our extracted values of $|\mathcal{R}(0)|^2$ are systematically higher than the predictions [21] of potential models which range from 1.508 GeV^3 for the logarithmic potential [22], through 1.642 GeV^3 and 1.710 GeV^3 for the Buchmuller-Tye [23] and power low [24] potentials up to 3.102 GeV^3 for the Cornell potential [25]. An even lower value of the wave function is obtained in the lattice calculation of Ref. [26] which reports $f_{B_c} = 0.427$ GeV, equivalent to $|\mathcal{R}(0)|^2 = 1.2$ GeV^3.

This systematic discrepancy may be taken as an evidence of the importance of radiative corrections. The latter are known to be large, yielding a factor of $1 - 16\alpha_s/3\pi$ [27]. (The QCD correction factor is obtained by transcription from QED; see, for example, [28].)

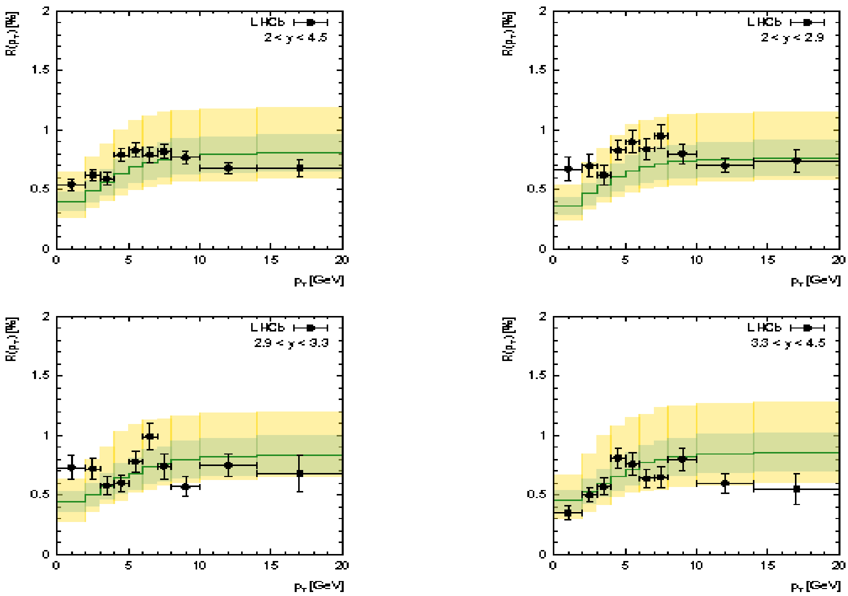

Figure 1. The ratio of the $B_c^{(*)}$ to B^+ production cross sections (13) as a function of the transverse momentum for three different rapidity intervals. Green histograms correspond to the conventional choice $\alpha_S^4(m_{B_cT}^2)$. Grey band indicates the statistical uncertainty in the determination of the $B_c^{(*)}$ radial wave function; yellow band represents uncertainties coming from the renormalization scale. The histograms reflect the replacement $\alpha_S^4(m_{B_cT}^2) \to \alpha_S^2(m_{B_cT}^2)\,\alpha_S^2(m_{cT}^2)$. Experimental points are from Ref. [4].

Another possible interpretation may guess that the conventional choice of renormalization scale μ_R somehow overestimates the momentum transfer in the hard process. Indeed, in addition to the initial gluons which bring all of the subprocess energy $\hat{s}$, there are internal gluons lines carrying some lower fraction of the energy-momentum. For example, for a gluon splitting into c-quarks (which further assemble with b-quarks to form B_c's) it looks more reasonable to use $\mu_R^2 = m_{cT}^2$ rather than $m_{B_cT}^2$ value. One can hardly tell a rigorous definition of an unique scale suitable for all Feynman diagrams, but the conventional choice $\mu_R^2 = m_{B_cT}^2$ looks too high. Under accepting a compromise expression $\alpha_S^4 \to \alpha_S^2(m_{B_cT}^2)\,\alpha_S^2(m_{cT}^2)$, the estimated cross sections increase by approximately a factor of two, thus requiring a lower value of the wave function to fit the data. Performing the fitting procedure as described above (solid histograms in Fig. 1), we obtain

$$|\mathcal{R}(0)|^2 = 3.02\ \mathrm{GeV}^3 \tag{20}$$

with an error of ± 0.25 GeV^3 and ± 0.50 GeV^3 at the 60% and 80% confidence level, respectively. The latter is in closer agreement with the predictions of potential models, though still shows tension with the lattice result.

4 Conclusions

Our note presents the first attempt to evaluate the $B_c^{(*)}$ wave function by considering the $B_c^{(*)}$ production data. We find that the ambiguity in the choice of the renormalization scale causes numerical uncertainties that are too large to declare a 'real measurement'. We only can judge on the consistency or inconsistency of the fitted values with model prdictions.

We argue for a choice of renormalization scale μ_R^2 different from its conventional definition. We show that the estimates obtained from the $B_c^{(*)}$ production cross sections under our assumption $\alpha_S^4 \to \alpha_S^2(m_{B_cT}^2)\,\alpha_S^2(m_{cT}^2)$ are good in shape and are nearly consistent with the predictions of potential models (though, probably, not with the lattice calculation).

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