

# Nonlinear interactions of the radion field

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**Abstract.** We study the Randall-Sundrum model with two branes. The Standard Model fields are localized on the brane with a negative tension, the gravitational field and scalar Goldberg-Wise field propagate in the multidimensional bulk. For this model we construct the Lagrangian for the scalar fluctuations of the gravitational and scalar fields above a background solution. We obtain an effective four-dimensional Lagrangian, which contains nonlinear self-interaction terms of the radion field up to the fourth order and nonlinear interactions of the radion and the Standard Model fields.

## 1 Introduction

The radion field and a massive radion, the lowest Kaluza-Klein (KK) mode, are a prediction of stabilized brane world models. Strictly speaking the radion field comes from the scalar fluctuations of the metric in extra dimension and the stabilizing scalar field [1]. The Randall-Sundrum model [2] with stabilizing scalar field proposed by Goldberger and Wise [3] and worked out by DeWolfe, Freedman, Gubser and Karch [4]. This model takes into account the backreaction of the metric on the scalar field by solving exactly the coupled equations for these fields. It turns out that the radion might be significantly lighter than all the other KK modes propagating in the multidimensional bulk. Due to its origin the radion couples to the trace of the energy-momentum tensor of the Standard Model. As a result, the interaction Lagrangian of the radion and the Standard Model fields is similar to the Lagrangian of the Standard Model Higgs interactions. In the case of on-shell fermions the interaction Lagrangian is the same as for the Higgs boson, but for off-shell fermions additional terms have to be taken into consideration. However, it was shown [5,6] that many processes involving the radion and SM fermions are similar to the processes involving the fermions and the Higgs boson. The investigation of Higgs boson production is an important task for experimental measurements. This question becomes important, when one wants to measure the scalar potential, which is obviously what one needs to know eventually to understand the electroweak symmetry breaking. However, the presence of the radion can complicate the problem of the Higgs potential research due to the similarity of the Higgs boson and the radion properties. Therefore it becomes important to perform a more detailed study of the radion self-interaction, in particular, to obtain the radion potential and nonlinear interactions of the radion with the Standard Model fields.

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## 2 Background solution

In space-time  $M_4 \times S^1/Z_2$  we consider five-dimensional gravity, which interacts with two branes and the scalar field  $\phi$ . We denote by  $\{x^M\} = \{x^\mu, y\}$ ,  $M = 0, 1, 2, 3, 4$ , the coordinates in this space-time, where  $\{x^\mu\}$ ,  $\mu = 0, 1, 2, 3$ , are four-dimensional coordinates, and  $y \equiv x^4$  is the coordinate of extra dimension, which forms the orbifold  $S^1/Z_2$ . The branes are located at the fixed points of the orbifold  $y = 0$  and  $y = L$ . The Standard Model fields are localized on the brane  $y = L$ . The action of the model is

$$S = S_1 + S_2, \tag{1}$$

where  $S_1$  and  $S_2$  are

$$S_1 = \int d^4x \int_{-L}^L dy \left( 2M^3 R - \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right) \sqrt{-g}, \tag{2}$$

$$S_2 = - \int_{y=0} d^4x \lambda_0(\phi) \sqrt{-\tilde{g}} + \int_{y=L} d^4x (-\lambda_L(\phi) + L_{SM}) \sqrt{-\tilde{g}}. \tag{3}$$

Here  $M$  is a fundamental five-dimensional energy scale,  $R$  is the five-dimensional scalar curvature,  $\phi$  is the Goldberg-Wise field,  $V(\phi)$  is a self-interaction potential,  $\lambda_0(\phi)$ ,  $\lambda_L(\phi)$  are the additional scalar field potentials on the branes,  $\tilde{g}_{\mu\nu}$  is the induced metric on the branes,  $\tilde{g} = \det \tilde{g}_{\mu\nu}$ , and  $L_{SM}$  is the Standard Model Lagrangian. The signature of the metric is  $g_{MN}$  is  $(-, +, +, +, +)$ . A background solution for this system was found in [4]:

$$g_{\mu\nu} \equiv \gamma_{\mu\nu} = e^{-2A(y)} \eta_{\mu\nu}, \quad g_{\mu 4} = 0, \quad g_{44} = 1, \tag{4}$$

$$A(y) = k|y| + \frac{\varphi_0^2}{48M^3} e^{-2u|y|} - \left( kL + \frac{\varphi_0^2}{48M^3} e^{-2uL} \right), \quad \phi(y) \equiv \varphi(y) = \varphi_0 e^{-2u|y|},$$

Here  $k, u, \varphi_0$  are parameters,  $V(\phi)$  and  $\lambda_{0,L}(\phi)$  are the corresponding potentials:

$$V(\phi) = -24M^3 k^2 + \frac{(u+4k)u}{2} \phi^2 - \frac{u^2}{24M^3} \phi^4, \tag{5}$$

$$\lambda_0(\phi) = 24M^3 k - u\varphi_0^2 - 2u\varphi_0(\phi - \varphi_0) + \beta_0^2(\phi - \varphi_0)^2, \tag{6}$$

$$\lambda_L(\phi) = 24M^3 k - u\varphi_L^2 + 2u\varphi_L(\phi - \varphi_L) + \beta_L^2(\phi - \varphi_L)^2, \quad \varphi_L = \varphi_0 e^{-uL}. \tag{7}$$

Consider small fluctuations above the background solution:

$$g_{MN} = \gamma_{MN} + \frac{1}{\sqrt{2M^3}} h_{MN}, \tag{8}$$

$$\phi(x, y) = \varphi(y) + \frac{1}{\sqrt{2M^3}} f. \tag{9}$$

The second variation Lagrangian of the model (up to quadratic terms in  $h_{MN}$  and  $f$ ) and linearized equations of motion have been found in [7]. It has been also shown that there exists a gauge such that:  $g = e^{-2A} h_{44}$  and

$$h_{\mu\nu} = b_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} g, \quad \mu 4 = 0, \quad f = 3M^3 \frac{e^{2A}}{\varphi'} g', \tag{10}$$

Then we get the following equations of motion for the fields  $b_{\mu\nu}$  and  $g$ :

$$\frac{1}{2} (\square b_{\mu\nu} + b''_{\mu\nu}) - (2(A')^2 - A'') b_{\mu\nu} = 0, \tag{11}$$

$$g'' + 2 \left( A' - \frac{\varphi''}{\varphi'} \right) g' - \frac{(\varphi')^2}{6M^3} - \square g = 0. \tag{12}$$

The Kaluza-Klein towers arise, when we decompose the fields  $b_{\mu\nu}$  and  $g$  as follows:

$$b_{\mu\nu}(x, y) = \sum_{n=0}^{\infty} b_{\mu\nu}^{(n)}(x)\omega_n(y), \quad (13)$$

$$g(x, y) = \sum_{n=0}^{\infty} g_n(x)\psi_n(y), \quad (14)$$

Below we study only the radion, which is the lightest scalar mode, and neglect all the other heavier modes. We also denote  $\psi_1$  by  $\psi$ , and the radion  $g_1$  by  $r$ . Thus, we obtain the following equations (up to the terms of the order  $1/2M^3$ ) for the fields  $g_{MN}$  and  $\phi$

$$g_{\mu\nu} = e^{-2A} (1 - \chi r) g_{\mu\nu}^{(0)}, \quad g_{\mu\nu}^{(0)} = \eta_{\mu\nu} + b_{\mu\nu}^{(0)}(x)N_0, \quad (15)$$

$$g_{\mu 4} = 0, \quad g_{44} = (1 + 2\chi r), \quad \phi = \varphi + \Phi r, \quad (16)$$

where

$$\chi = \frac{1}{2\sqrt{2M^3}} e^{2A} \psi, \quad \Phi = \frac{3}{2} \frac{\sqrt{2M^3} e^{2A}}{\varphi'} \psi', \quad (17)$$

The mass and the wave function of the radion are obtained from the boundary value problem on the segment  $(0, L)$ :

$$\psi'' + \left(2A' - 2\frac{\varphi''}{\varphi'}\right)\psi' - \frac{(\varphi')^2}{6M^3}\psi = -\mu^2 e^{2A}\psi, \quad (18)$$

$$(\beta_0^2 + u)\psi' + \mu^2 e^{2A}\psi|_{y=0} = 0, \quad (19)$$

$$(\beta_L^2 - u)\psi' - \mu^2 e^{2A}\psi|_{y=L} = 0. \quad (20)$$

Taking into account  $uL \ll 1$ ,  $\tilde{k}L \simeq 35$ , and  $\frac{u}{\tilde{k}} \ll e^{\tilde{k}L} \sim 10^{15}$  (see for example [3]), we finally obtain the mass spectrum of the scalar modes:

$$\frac{\mu^2}{\tilde{k}^2} \simeq \frac{4(\alpha + 1)(\beta_L^2 - u)(\alpha - 1 - u/\tilde{k})}{4(\alpha + 1)\tilde{k} + (\beta_L^2 - u)(\alpha - 1 - u/\tilde{k})} \simeq \frac{4(\beta_L^2 - u)(\alpha - 1)}{\beta_L^2 + 4\tilde{k}} \quad (21)$$

$$\mu^2 \simeq \frac{2(\beta_L^2 - u)u^2}{3} \frac{\hat{k}^2 \varphi_0^2}{\beta_L^2 + 4\tilde{k}}. \quad (22)$$

### 3 Effective radion Lagrangian with nonlinear self-interaction terms

Combining the gravitational part of five-dimensional Lagrangian (2) and the expression for five-dimensional metric (15),(16), we obtain

$$\begin{aligned} \sqrt{-g}R(g) = & \partial^\mu \left[ e^{-2A} \sqrt{-g^{(0)}} g_{\mu\nu}^{(0)} \frac{(1 + 8\chi r)\chi}{\sqrt{1 + 2\chi r}} \partial^\nu r \right] - \\ & \partial_4 \left[ e^{-2A} \sqrt{-g^{(0)}} \frac{4(1 - \chi r)\chi}{\sqrt{1 + 2\chi r}} \partial_4(e^{-2A}(1 - \chi r)) \right] + \\ & \sqrt{-g^{(0)}} \left\{ \frac{3}{\sqrt{1 + 2\chi r}} (e^{-2A}(1 - \chi r))'^2 - e^{-2A} \frac{3(1 - 4\chi r)\chi^2}{2\sqrt{1 + 2\chi r}(1 - \chi r)} g_{\mu\nu}^{(0)} \partial^\mu r \partial^\nu r \right\} + \\ & e^{-2A}(1 - \chi r) \sqrt{1 + 2\chi r} \sqrt{-g^{(0)}} R(g^{(0)}) \end{aligned} \quad (23)$$

$$\begin{aligned}
 & e^{-4A}(1-\chi)^2 \sqrt{-g^{(0)}} \sqrt{1+2\chi r} \left( \frac{1}{2} \Phi^2 \frac{2^{2A}}{1-\chi} g_{\mu\nu}^{(0)} \partial^\mu r \partial^\nu r + \frac{(\varphi' + \Phi' r)^2}{2(1+2\chi)} \right) + \\
 & e^{-4A}(1-\chi)^2 \sqrt{-g^{(0)}} \{ \lambda_0 (\varphi + \Phi r) \delta(y) + \lambda_L (\varphi + \Phi r) \delta(y-L) \}.
 \end{aligned} \tag{24}$$

Summing (23) and (24), we obtain an expression for the effective action of the radion:

$$S_{eff} = \int d^4x \left( -g_{\mu\nu}^{(0)} \partial^\mu r \partial^\nu r P(r) - U(r) \right), \tag{25}$$

where  $P(r)$  and  $U(r)$  are non-polynomial functions of  $r$ :

$$P(r) = \frac{1}{2} \int_{-L}^L \left\{ 2M^3 \frac{3(1-4\chi r)}{\sqrt{1+2\chi r}(1-\chi r)} \chi^2 + (1-\chi r) \sqrt{1+2\chi r} \Phi^2 \right\} e^{-2A} dy \tag{26}$$

$$\begin{aligned}
 U(r) = & \int_{-L}^L \left\{ 2M^3 \frac{3}{\sqrt{1+2\chi r}} \left( e^{-2A}(1-\chi r) \right)'^2 - \right. \\
 & e^{-4A}(1-\chi r)^2 \sqrt{1+2\chi r} \left( \frac{(\varphi + \Phi r)'^2}{2(1+2\chi r)} + V(\varphi + \Phi r) \right) - \\
 & \left. e^{-4A}(1-\chi r)^2 (\lambda_0 (\varphi + \Phi r) \delta(y) + \lambda_L (\varphi + \Phi r) \delta(y-L)) \right\} dy.
 \end{aligned} \tag{27}$$

Next we find the expansion of these functions in a series in  $r$  up to the terms of the fourth order. Then for  $P(r)$  and  $U(r)$  we have

$$P(r) = \alpha_0 + \alpha_1 r + \alpha_2 r^2 \tag{28}$$

$$U(r) = \beta_0 + \beta_1 r + \beta_2 r^2 + \beta_3 r^3 + \beta_4 r^4. \tag{29}$$

where

$$\alpha_0 = \frac{1}{2} \int_{-L}^L \{6M^3\chi^2 + \Phi^2\} e^{-2A} dy \quad (30)$$

$$\alpha_1 = -12M^3 \int_{-L}^L \chi^3 e^{-2A} dy \quad (31)$$

$$\alpha_2 = \frac{1}{4} \int_{-L}^L \{18M^3\chi^4 - 3\Phi^2\chi^2\} e^{-2A} dy \quad (32)$$

$$\beta_0 = \int_{-L}^L \left\{ 24M^3 A'^2 - \frac{\varphi'^2}{2} - V(\varphi) - 12M^3 \left( k - \frac{u\varphi^2}{12M^3} (\delta(y) - \delta(y-L)) \right) \right\} e^{-4A} dy \quad (33)$$

$$\beta_1 = \int_{-L}^L \left\{ \left( 2t_0 + V(\varphi) + \frac{3\varphi'^2}{2} - 24M^3 A' \chi' \right) - \frac{dV(\varphi)}{d\varphi} \Phi - \varphi' \Phi' + 2u\varphi\Phi(\delta(y) - \delta(y-L)) \right\} e^{-4A} dy \quad (34)$$

$$\beta_2 = \int_{-L}^L \left\{ -3M^3 \mu^2 \chi^2 - \frac{\mu^2}{2} \Phi^2 \right\} e^{-2A} dy \quad (35)$$

$$\beta_3 = \int_{-L}^L \left\{ \frac{9M^3 \mu^2}{u\varphi} \left( \frac{2M^3 \mu^2 e^{2A}}{u\varphi} + u\varphi \right) \chi^3 + \frac{M^3 \mu^2 e^{2A}}{u\varphi} \left( \frac{u\varphi^2}{2M^3} - 6(2k+u) \right) \chi^2 \Phi + 2\mu^2 \chi \Phi^2 \right\} e^{-2A} dy \quad (36)$$

$$\beta_4 = \int_{-L}^L \left\{ \left[ \frac{63M^3 \mu^4 e^{4A}}{2u^2 \varphi^2} - 15M^3 \mu^2 e^{2A} + 54M^3 k^2 - \frac{u}{16} \left( 72k + 2u - \frac{3u\varphi^2}{2M^3} \right) \varphi^2 \right] \chi^4 + \left[ \frac{M^3 \mu^2 e^{2A}}{u\varphi^2} \left( 12k + 21u - \frac{u\varphi^2}{2M^3} \right) + \frac{u^2}{2} \varphi \right] \chi^3 \Phi + \frac{1}{4} \left[ -6\mu^2 e^{2A} + 2(6k-u)u - \frac{u^2 \varphi^2}{2M^3} \right] \chi^2 \Phi^2 + \frac{u^2 \Phi^4}{24M^3} \right\} e^{-4A} dy. \quad (37)$$

Now we introduce dimensionless small parameters:

$$\xi \equiv \frac{\mu}{\bar{k}}, \quad \sigma \equiv \frac{u}{\bar{k}}, \quad \rho \equiv \hat{k}^2 \varphi_0^2. \quad (38)$$

and expand expressions (30)-(37) in series in these parameters. After evaluation the integrals in equations (30)-(37), we obtain the following expression for the Lagrangian  $L_r$  of the self-interaction of the radion field

$$L_r = - \left( \frac{1}{2} + \frac{\bar{\alpha}_1}{\Lambda} + \frac{\bar{\alpha}_2}{\Lambda^2} \right) \eta^{\mu\nu} \partial_\mu r \partial_\nu r - \mu^2 \left( \frac{1}{2} r^2 + \frac{\bar{\beta}_3}{\Lambda} r^3 + \frac{\bar{\beta}_4}{\Lambda^2} r^4 \right), \quad (39)$$

where (up to  $O(\sigma^3)$  and  $O(\xi^3)$ )

$$\bar{\alpha}_1 = -2 + \frac{1}{12} (2e^{2uL} - 1) \rho \sigma^2 - \frac{1}{4} \left( 1 - \sigma + \frac{3}{4} \left[ 1 - \frac{1}{18} (4e^{2Lu} - 1) \rho \right] \sigma^2 \right) e^{2uL} \xi^2 \quad (40)$$

$$\bar{\alpha}_2 = \left( 1 - \frac{1}{18} (3e^{2uL} - 1) \rho \sigma^2 \right) + \left( \frac{5}{16} - \frac{17}{64} \sigma + \frac{45}{256} \left[ 1 - \frac{4}{405} (27e^{2Lu} - 4) \rho \right] \sigma^2 \right) e^{2uL} \xi^2 \quad (41)$$

$$\begin{aligned} \bar{\beta}_3 = & \frac{3}{2} + e^{2uL} + \frac{1}{12} (e^{2Lu} - 1) \rho \sigma + \frac{1}{48} (3 + 2e^{2Lu} - 4e^{4Lu}) \rho \sigma^2 + \\ & \left\{ -\frac{e^{2Lu} - 1}{12\sigma} + \frac{1}{144} (-6 + e^{2Lu} + 18e^{4Lu}) - \frac{1}{864} (2(-9 + 28e^{2Lu} + 54e^{4Lu}) - \right. \\ & 15(e^{2Lu} - 1)e^{2Lu}\rho)\sigma + \\ & \left. \frac{1}{10368} (2(-54 + 355e^{2Lu} + 486e^{4Lu}) + 3(12 + 67e^{2Lu} - 44e^{4Lu} - 72e^{6Lu})\rho)\sigma^2 \right\} \xi^2 \end{aligned} \quad (42)$$

$$\begin{aligned} \bar{\beta}_4 = & \left\{ 18 + 3(1 - e^{-2Lu})\rho\sigma - \frac{1}{24} [36e^{2Lu} + 38e^{-2Lu} - (3 - 6e^{-2Lu} + 3e^{-4Lu})\rho] \rho\sigma^2 \right\} \frac{1}{\xi^2} + \\ & \frac{1}{12} (-31 + 11e^{2Lu}) + \left[ -\frac{185}{36} e^{2Lu} - \frac{5}{24} + \left( \frac{19}{72} e^{2Lu} - \frac{5}{36} - \frac{1}{8} e^{-2Lu} \right) \rho \right] \sigma + \\ & \left[ \frac{1433}{432} e^{2Lu} + \frac{5}{48} - \frac{1}{72} \left( 19e^{4Lu} + \frac{193}{6} e^{2Lu} + \frac{169}{12} + \frac{49}{4} e^{-2Lu} \right) \rho + \right. \\ & \left. \frac{1}{192} (3e^{2Lu} - 5 + e^{-2Lu} + e^{-4Lu}) \rho^2 \right] \sigma^2 + \\ & \left\{ \frac{15e^{2Lu}}{4\rho\sigma^2} + \frac{77760e^{2Lu}}{27648\rho\sigma} + \frac{3456e^{2Lu}}{27648\sigma} - \frac{630e^{2Lu}}{384\rho} + \frac{-382e^{2Lu} + 29e^{4Lu} - 939}{384} + \right. \\ & \left. \left[ \frac{225e^{2Lu}}{256\rho} + \frac{-692e^{4Lu} + 372e^{2Lu} + 2169}{1152} + \frac{459e^{-2Lu} + 41e^{2Lu} - 83e^{4Lu} - 345}{2304} \right] \rho \right\} \sigma + \\ & \left[ -\frac{465e^{2Lu}}{1024\rho} + \frac{13100e^{4Lu} + 3165e^{2Lu} - 17577}{13824} + \frac{534e^{6Lu} + 3688e^{4Lu} + 1817e^{2Lu} + 6954 - 4329e^{-2Lu}}{27648} \rho - \right. \\ & \left. \frac{(e^{2Lu} - 1)^2 (3 + 2e^{-2Lu} + 51e^{-4Lu})}{6144} \rho^2 \right] \sigma^2 \left\} \xi^2 \end{aligned} \quad (43)$$

In what follows we consider the interaction of the radion with the Standard Model fields up to the terms  $r^2$ . It is well known that the linear part of the interaction of the radion with the fields of matter is a convolution with the energy-momentum tensor of these fields:

$$L_{rSM} = \frac{1}{2} \int_{-L}^L \chi(y) r \eta^{\mu\nu} T_{\mu\nu} \delta(y - L) dy = \frac{1}{2} \chi(L) r \eta^{\mu\nu} T_{\mu\nu} \equiv \frac{1}{\Lambda} r T_{\mu}^{\mu}. \quad (44)$$

The next order term can be obtained from  $\sqrt{-\tilde{g}}L_{SM}$  by expanding it in the field of the radion. Thus, for the scalar field we get

$$\begin{aligned}\sqrt{-\tilde{g}}L_{h,\tilde{g}} &= \sqrt{-\tilde{g}}\left\{-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu h\partial_\nu h - U(h)\right\} = \\ &= \left\{-\frac{1}{2}e^{-2A}(1-\chi r)\eta^{\mu\nu}\partial_\mu h\partial_\nu h - e^{-4A}(1-\chi r)^2 U(h)\right\} = \\ &= \left\{L_{h,\eta} + \chi r\left[\frac{1}{2}e^{-2A}\eta^{\mu\nu}\partial_\mu h\partial_\nu h + 2e^{-4A}U(h)\right] - \chi^2 r^2 e^{-4A}U(h)\right\},\end{aligned}\quad (45)$$

Since on the brane  $y = L$  the function  $A(L) = 0$ , finally we obtain

$$L_{rh} = -\frac{r}{\Lambda}T_\mu^\mu(h) - \frac{r^2}{\Lambda^2}U(h).\quad (46)$$

Similarly, for a vector field we have

$$\begin{aligned}\sqrt{-\tilde{g}}L_{V,\tilde{g}} &= \sqrt{-\tilde{g}}\left\{-\frac{1}{4}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + \frac{m_V^2}{2}\tilde{g}^{\mu\nu}V_\mu V_\nu\right\} = \\ &= \left\{-\frac{1}{4}\eta^{\mu\nu}\eta^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + e^{-2A}(1-\chi r)\frac{m_V^2}{2}\eta^{\mu\nu}V_\mu V_\nu\right\}.\end{aligned}\quad (47)$$

In our approximation, we finally obtain the following expression for the Lagrangian of the vector field interaction with the radion

$$L_{rV} = -\frac{r}{\Lambda}T_\mu^\mu(V).\quad (48)$$

And finally, we obtain the following Lagrangian of the spinor field

$$\sqrt{-\tilde{g}}L_{\Psi,\tilde{g}} = \sqrt{-\tilde{g}}\left[\frac{i}{2}\bar{\Psi}\gamma^\mu(\nabla_\mu\Psi) - \frac{i}{2}(\nabla_\mu\bar{\Psi})\gamma^\mu\Psi + m_\Psi\bar{\Psi}\Psi\right],\quad (49)$$

Using formula (50) that holds in the case of a conformal metric

$$\bar{\Psi}\gamma^\mu(\nabla_\mu\Psi) - (\nabla_\mu\bar{\Psi})\gamma^\mu\Psi = \frac{e^A}{\sqrt{1-\chi r}}(\bar{\Psi}\gamma^\mu(D_\mu\Psi) - (D_\mu\bar{\Psi})\gamma^\mu\Psi),\quad (50)$$

we obtain the Lagrangian of the radion interaction with the spinor field

$$L_{r\Psi} = -\frac{r}{\Lambda}T_\mu^\mu(\Psi) + \frac{r^2}{\Lambda^2}\left[\frac{3}{4}(i\bar{\Psi}\gamma^\mu(D_\mu\Psi) - i(D_\mu\bar{\Psi})\gamma^\mu\Psi) + 4m_\Psi\bar{\Psi}\Psi\right].\quad (51)$$

## 4 Conclusion

In the present paper we have derived an effective four-dimensional Lagrangian, which contains nonlinear self-interaction terms of the radion field up to the fourth order and nonlinear interactions of the radion with the Standard Model fields. This question becomes important, because the problem of finding the Higgs potential is complicated due to the similarity of the Higgs boson and the radion properties. An analysis of the radion self-interaction potential obtained after the reduction shows that the radion field is in a true minimum. Thus, the model is consistent. Indeed, the self-interaction fourth-order potential could have several minima, if the equation

$$\frac{dU}{d\varphi} = \{\mu^2 + 3\lambda_3\varphi + 4\lambda_4\varphi^2\}\varphi = 0\quad (52)$$

had more than one real solution. However this is not the case, because its discriminant is negative:

$$D = 9\lambda_3^2 - 16\lambda_4\mu^2 = 16\mu^2\lambda_4 \left( \frac{9}{16} \frac{\mu^2}{\tilde{k}^2} \frac{\tilde{\beta}_3^2}{\tilde{\beta}'_4} - 1 \right) < 0 \quad (53)$$

## References

- [1] C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D **V.63**, P. 065002 (2001)
- [2] L. Randall, R. Sundrum, Phys. Rev. Lett. **V.83**, P. 3370 (1999)
- [3] W. D. Goldberger, M. B. Wise, Phys. Rev. Lett. **V.83**, P. 4922 (1999)
- [4] O. DeWolfe, D. Z. Freedman, S. S. Gubser, A. Karch, Phys. Rev. D. **V.83**, P. 046008 (2000)
- [5] E. Boos, S. Keizerov, E. Rakhmetov, K. Svirina, Phys. Rev. D **V.90**, P. 095026 (2014)
- [6] E. Boos, S. Keizerov, E. Rakhmetov, K. Svirina, Phys. Rev. D **V.94**, P. 024047 (2016)
- [7] W. D. Goldberger, M. B. Wise, Phys. Lett. B **V.475**, P. 275 (2000)