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Particle-Hole Cluster Shell Model States in ^{8}Be, a Challenge for Ab-Intio Nuclear Theories

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Abstract. We propose the first application of the Cluster Shell Model (CSM) of Della Rocca and Iachello to particle-hole (p-h) states in ^{8}Be. We demonstrate a few essential features of the CSM in ^{8}Be: 1) All predicted p-h states of the CSM, and only the predicted p-h states, are observed near thresholds and up to 19.5 MeV in ^{8}Be. 2) The states are observed in the predicted order, with positive parity states below negative parity states. 3) Some of the p-h states are already known to have the rotational structure predicted for the deformed p-h states. 4) The rotational structures observed at high excitations in the p-h bands in ^{8}Be, resemble the ground state bands of ^{8}Be, ^{9}Be and ^{9}B, with similar moment of inertia. Based on these observations we contemplate new measurements of the spectroscopy of ^{8}Be at energies above 19.5 MeV, where our knowledge of states in ^{8}Be is scarce. We examine the observed B(M1)s and B(E2)s in these nuclei and contemplate a measurement of the B(E2) of the isobaric Analog transition in ^{8}B. We discuss the observed rotational structure in ^{8}Be as a challenge to ab-initio calculations that searched for "emerging rotational structures at high excitations" in beryllium nuclei, and reveal rotational structure at high excitations in ^{10}Be and ^{12}Be but not in ^{8}Be.

1 Introduction

The nuclei ^{12}C and ^{8}Be have currently attracted much attention. On one hand the algebraic cluster model (ACM) of Bijker and Iachello [1], presents some of the most interesting application of geometrical point group symmetries to cluster states, and on the other hand ab-initio theories such as the no-core shell model (NCCI) [2] and the quantum Monte Carlo (GFMC) [3], can be applied to these light nuclei as testing grounds. But, perhaps one of the most intriguing aspect of these new theoretical development is the prediction and observation in ^{12}C of the mixed parity ground state rotational band including the 4^+ and 4^- parity doublet, and the states of $J^\pi = 0^+, 2^+, 3^-, 4^\pm$ and 5^- [4].

1.1 Ab-Initio No Core Shell Model

Ab-initio no-core shell model (NCCI) [2] and the quantum Monte Carlo (GFMC) [3] calculations reached an impressive level of accomplishment where they can be compared to the spectra of light nuclei at very high excitations, as shown in Fig. 1. However, these ab-initio calculations appear at the present time to have difficulties with calculating negative parity states. Of interest for this paper is the study of the "emergence of rotational bands at high excitation energies" in ab-initio no core shell model calculations [5] as shown in Fig. 2. We note that these ab-initio calculations reveal the emergence of positive parity rotational bands at high excitations in ^{10}Be and ^{12}Be, but not in ^{8}Be.

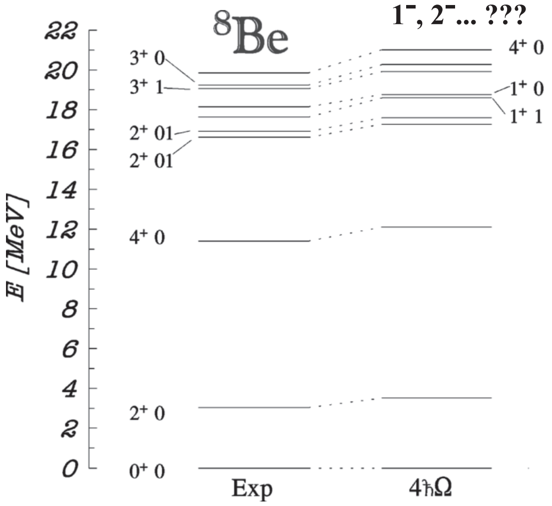

Figure 1. Ab-initio shell model calculations [2] of (only) positive parity states extended to very high excitations in ^{8}Be.

1.2 The Cluster Shell Model

The ACM [1] was recently extended to describe single particle motion in molecular orbits within the cluster shell model (CSM) of Della Rocca and Iachello [6]. In Fig. 3 we show the Nilsson-like (neutron) single particle states in ^{9}Be as a function the two alpha-particles separation distance (β), as predicted by the CSM [6]. However, while the CSM is a Nilsson-like model, it predicts molecular single particle orbits, hence the occurrence of rotational bands

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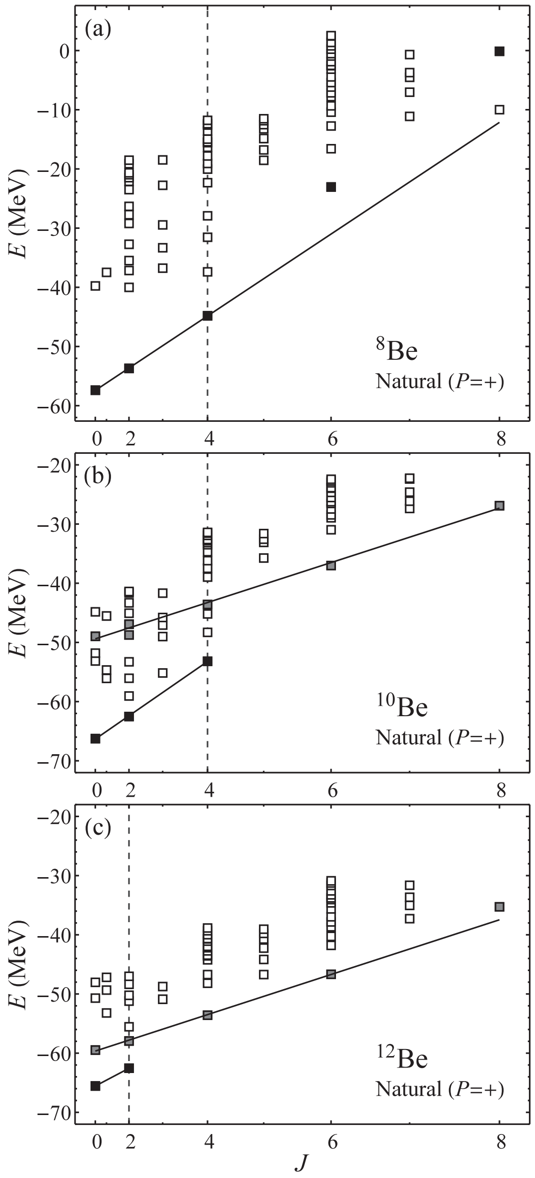

Figure 2. Excited states rotational bands of (only) positive parity states revealed in ab-initio no-core configuration interaction calculations [5] of the beryllium isotopes. Note the emergence of such rotational bands at high excitations in ^{10}Be and ^{12}Be, but not in ^{8}Be.

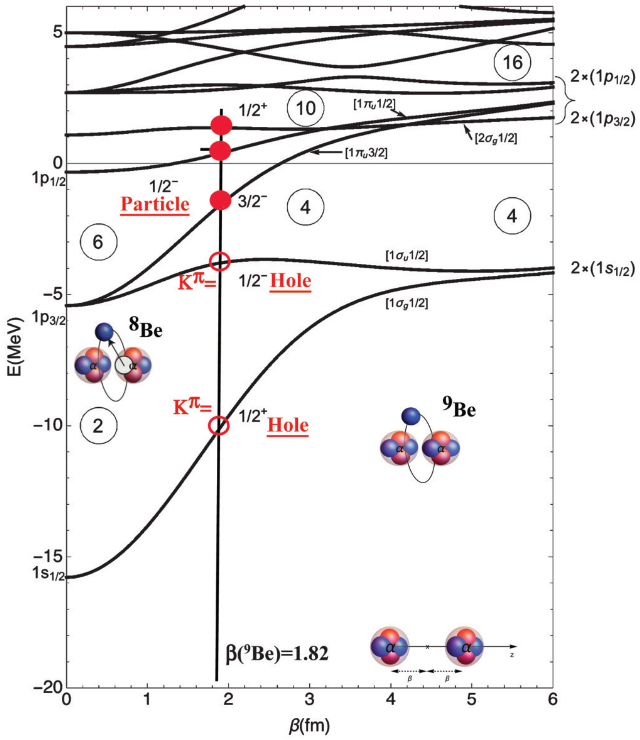

Figure 3. Nilsson like single (neutron) particle states in ^{9}Be predicted by the CSM [6] as a function of the separation distance of the alpha-particles (β). The particle (hole) states in ^{8}Be are indicated by full (open) circles.

with parity doublets [6], which are not predicted in the Nilsson Model. In addition the CSM predicts similarities and relationships between rotational bands in ^{9}Be and ^{8}Be [6], which are not evident in the Nilsson model.

2 CSM p-h states in ^{8}Be

The lowest particle-Hole (p-h) states in ^{8}Be arise in the CSM as holes in the K$^\pi$= 1/2$^-$ and 1/2$^+$ proton or neutron molecular orbits, excited to the K$^\pi$ = 3/2$^-$ orbit. Hence the lowest p-h sates in ^{8}Be are expected to be with K$^\pi$ = 2$^+$, 1$^+$, 2$^-$ and 1$^-$, with the positive parity states at lower excitations. In addition, both isospin 0 and 1 are predicted, corresponding to neutron (^{9}Be like) and proton (^{9}B like) excitations. In Fig. 4 we show all the known states in ^{8}Be below 19.5 MeV, we also add three selected states above 20 MeV. In the energy range below 19.5 MeV, aside from the ground state $\alpha + \alpha$ band, all p-h states predicted by the CSM are observed at these high excitations, and only the predicted states are observed. The predicted order of the negative parity states above the positive parity is also observed. Hence, we tentatively denote the states by the K$^\pi$ classification of the CSM.

The phenomenological agreement with the CSM of the known eigen states of ^{8}Be up to 19.5 MeV, encourage us on one hand to consider measurements of the spectroscopy of ^{8}Be above 20 MeV [7], in order to clarify the spectroscopy of ^{8}Be above 20 MeV [8, 9]. On the other hand, the observed spectrum gives very strong motivation to calculations of p-h states in 8Be, in the frame of the CSM.

Two additional important features of p-h CSM states in ^{8}Be are highlighted: First, since the p-h states are deformed, a rotational band is predicted on top of each p-h state. Second, the rotational band structure is predicted to be very similar to the ground state rotational bands observed in ^{9}Be and ^{9}B. In addition, the ground state rotational bands in ^{9}Be and ^{9}B were already demonstrated [6] to be similar to the K$^\pi$ = 0$^+$ ground state rotational band of ^{8}Be, also shown in Fig. 4. Hence, the p-h bands in ^{8}Be, the g.s. rotational bands in ^{9}Be and ^{9}B, the g.s. rotational band in ^{8}Be, are all predicted to have very similar properties including similar moment of inertia and related B(E2) and B(M1) values.

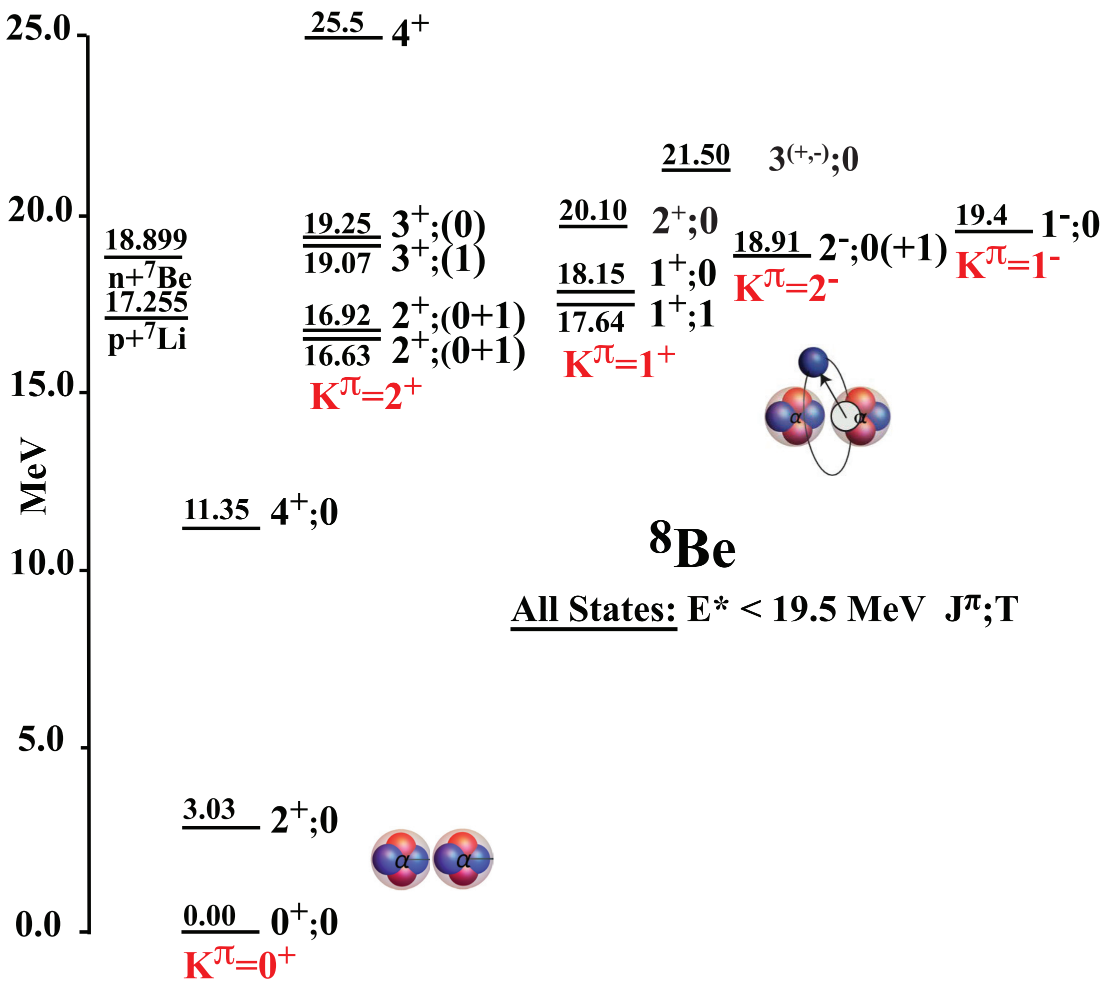

Figure 4. Measured states in ^{8}Be up to 19.5 MeV, with the (tentative) K^π assignments in the CSM indicated in red.

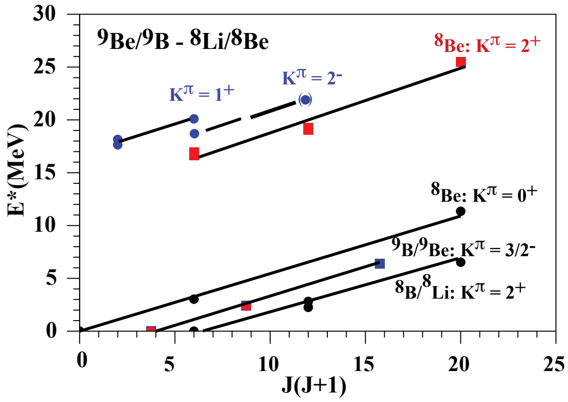

Figure 5. Rotational band structures in ^{8}Be and the isobaric Analog ^{8}B and ^{8}Li, and ^{9}Be and ^{9}B. The tentative 3^- state at 21.5 MeV is from [9]

Table 1. B(M1)s in ^{8}Li (isobaric analog of ^{8}Be) and ^{9}Be

$E_i \rightarrow E_f$ (MeV)	$J^\pi;T \rightarrow J^\pi;T$	B(M1) (W.u.)
2.26→ 0.0	$3^+;1 \rightarrow 2^+;1$	0.29±0.12
2.4294→ 0.0	$\frac{5}{2}^-;\frac{1}{2} \rightarrow \frac{3}{2}^-;\frac{1}{2}$	0.3±0.03

Table 2. B(E2)s in ^{8}Be and ^{9}Be

$E_i \rightarrow E_f$ (MeV)	$J^\pi;T \rightarrow J^\pi;T$	B(E2) (W.u.)
11.35→ 3.03	$4^+;0 \rightarrow 2^+;0$	25±8.4
2.4294→ 0.0	$\frac{5}{2}^-;\frac{1}{2} \rightarrow \frac{3}{2}^-;\frac{1}{2}$	24.4±1.8
6.38→0.0	$\frac{7}{2}^-;\frac{1}{2} \rightarrow \frac{3}{2};\frac{1}{2};\frac{1}{2}$	8.5±3.7

In Fig. 5 we show the $K^\pi = 0^+$ g.s. rotational band in ^{8}Be, the $K^\pi = 3/2^-$ g.s. rotational bands in ^{9}Be and ^{9}B, the $K^\pi = 2^+$ (and $K^\pi = 1^-$ and 2^-) p-h bands in ^{8}Be, and the $K^\pi = 2^+$ g.s. rotational band in a ^{8}Li (and ^{8}B), the isobaric-analog of the $K^\pi = 2^+$ p-h band in ^{8}Be. The similarity of the observed moments of inertia is striking (with differences smaller or equal to 7%).

In Table I we compare the measured B(M1) in the $K^\pi = 3/2^-$ g.s. rotational bands in ^{9}Be and the $K^\pi = 2^+$ g.s. rotational band in ^{8}Li, the isobaric-analog of the $K^\pi = 2^+$ p-h band in ^{8}Be. The similarity of the B(M1) is indeed striking. We however note that when including the Clebsch-Gordon coefficients, slightly different B(M1)s are predicted due to the different spins involved.

The similarity of the isobaric-Analog transition in ^{8}Li and ^{9}Be, shown in Table I, encourage us to consider the B(E2) of these nuclei. A strong B(E2: $3^+ \rightarrow 2^+$) in ^{8}B or ^{8}Li, will further demonstrate the rotational character of the isobaric Analog p-h band in ^{8}Be shown in Fig. 5. In Table II we list measured B(E2) in ^{8}Be and ^{9}Be. Based on the similarity of the rotational structure in the beryllium isotopes we expect the B(E2: $3^+ \rightarrow 2^+$) in ^{8}B or ^{8}Li to be approximately 25 W.u. We are engaged in developing a proposal to measure the Coulomb excitation of the ^{8}B to measure the B(E2: $3^+ \rightarrow 2^+$).

3 Conclusions

We identified a rotational structure in ^{8}Be at very high excitations above 16 MeV. The observed structure is indicative of particle-hole states of the cluster shell model (CSM). They arise from a particle and a hole moving in molecular orbits. The description of molecular states requires many $\hbar\omega$ model space and as such they pose a challenge to ab-intio no core shell model calculations, that searched for the “emergence" (in NCCI calculations) of such rotational structure at high excitations in ^{8}Be.

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