

# Discovery of collective states in $^{208}\text{Pb}$ by complete spectroscopy

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**Abstract.** Most neutron and proton bound states in  $^{208}\text{Pb}$  are described in the shell model as one-particle one-hole configurations. Besides precise data obtained with the Q3D magnetic spectrograph of the MLL (Garching, Germany) an important reaction is the inelastic proton scattering via isobaric analog resonances in  $^{209}\text{Bi}$ . It yields amplitudes of neutron one-particle one-hole configurations with relative signs in each state of  $^{208}\text{Pb}$ . The orthogonality, normality, and sum rule relations allow to investigate the completeness of the transformation matrices of one-particle one-hole configurations describing the states in  $^{208}\text{Pb}$  with spins from  $0^-$  to  $14^-$  and  $0^+$  to  $12^+$ . By this method amplitudes of unobservable one-particle one-hole configurations can be determined. The comparison of spin, parity, and dominant particle-hole components thus derived in up to 30 states of a certain spin to shell model calculations allows to identify states described as collective excitations of the entire nucleus.

## 1 Introduction

$^{208}\text{Pb}$  is the heaviest nucleus where more than 500 states are rather well known [1]. The excitation energies of most neutron bound states ( $S(n) = 7368\text{ keV}$ ) and part of the proton bound states ( $S(n) = 8004\text{ keV}$ ) are known with an uncertainty of less than 1 keV. Most states are one-particle one-hole (1p1h) configurations but three dozen states consist definitely of other configurations [2–5].

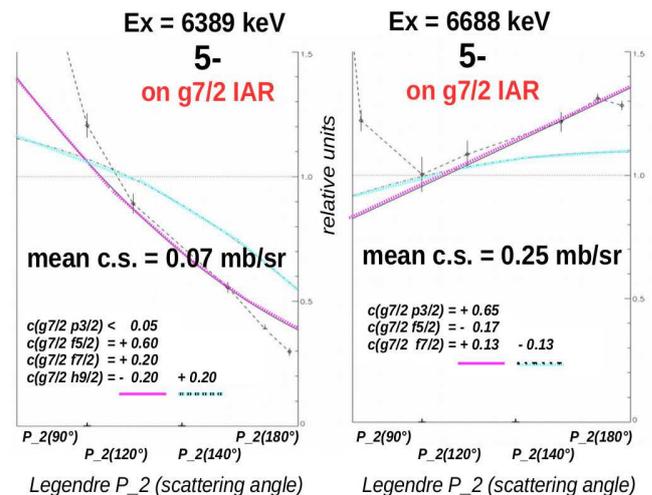
Knowledge of spin, parity, and dominant configuration up to a certain excitation energy defines complete spectroscopy. It is rather good for states at  $E_x < 7.2\text{ MeV}$ . States at  $6.2 < E_x < 7.2\text{ MeV}$  [6, 7] are being analyzed from reconstructed data presented in [8]. States at  $E_x < 6.2\text{ MeV}$  are completely known with spin, parity, and dominant configuration [3]. Complete spectroscopy allows to find collective states in the heavy nucleus  $^{208}\text{Pb}$  not described as 1p1h configurations.

## 2 Experimental observations

Spectroscopy of  $^{208}\text{Pb}$  started immediately after the discovery of radioactivity. Rather soon the high excitation energy of the lowest state in  $^{208}\text{Pb}$  was recognized. The energy of two and a half MeV is the almost the highest across the whole nuclear chart. It is now known with a precision of 10 eV [1]. In 1954 the spin of the lowest state was determined as  $3^-$  [9] which is again rather unique across the whole nuclear chart.

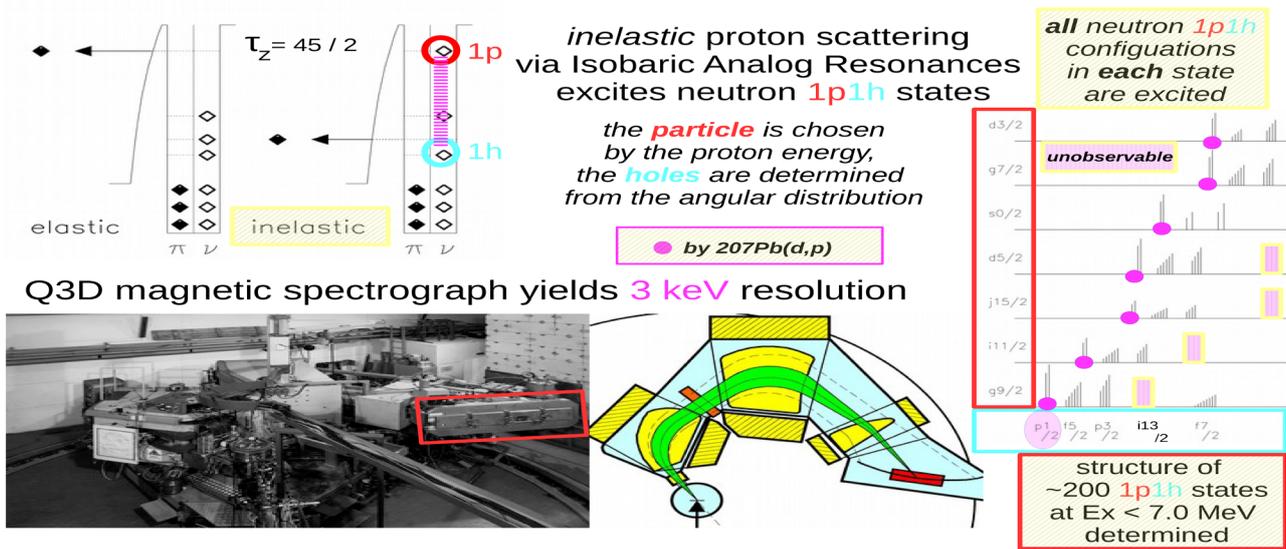
An important event was the discovery of isobaric analog resonances (IAR) in heavy nuclei [10]. From 1965 until 1969 important data were obtained for  $^{208}\text{Pb}$  from the study of the proton decay of IARs in  $^{209}\text{Bi}$  [1, 6–8, 11]. Yet only with the finishing of the Q3D mag-

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**Figure 1.** Angular distributions for  $^{208}\text{Pb}(p, p')$  measured at the MPI-Heidelberg in 1969 using semiconductor detectors (12 keV resolution) at scattering angles  $90^\circ < \Theta < 170^\circ$  on the  $g_{7/2}$  IAR in  $^{209}\text{Bi}$  [8]. Calculations of the differential cross section with known s.p. widths [7] yield up to four amplitudes for each state. The sensitivity is extremely high. The shown amplitudes  $-1 < c(LJ^{+1}l_j^{-1}) < +1$  for the configurations (left frame)  $g_{7/2}p_{3/2}$ ,  $g_{7/2}f_{5/2}$ ,  $g_{7/2}f_{7/2}$ ,  $g_{7/2}h_{9/2}$  and (right frame)  $g_{7/2}p_{3/2}$ ,  $g_{7/2}f_{5/2}$ ,  $g_{7/2}f_{7/2}$  and the angular distribution at backward angles  $140^\circ \leq \Theta \leq 170^\circ$ . Reversing the sign of one amplitude as indicated does not yield a fit. Note that instead of the scattering angle  $\Theta$  the Legendre polynomial  $P_2(\Theta)$  is used as abscissa.

netic spectrograph (Fig. 2) at the Maier-Leibnitz Laboratorium (MLL) at Garching (Germany) higher excited states ( $4.0 < E_x < 8.0\text{ MeV}$ ) could be studied.



**Figure 2.** (upper and right frames) Schematic presentation of the reaction  $^{208}\text{Pb}(p,p')$  via IARs in  $^{209}\text{Bi}$ . (lower left frames) The quadrupole 3-dipole (Q3D) magnetic spectrograph at the MLL (Garching, Germany).

The resolution of 3 keV is barely sufficient to resolve the states in  $^{208}\text{Pb}$  with a mean distance of 10 keV. The resolution is limited to 3 keV because the  $M$ -electrons in lead have such binding energies.  $L$ -electrons and  $K$ -electrons with binding energies of 15 and 88 keV produce annoying satellites to each peak.

Using the Q3D magnetic spectrograph from 2003 until now most negative parity states at  $E_x < 7.2$  MeV and nearly all states at  $E_x < 6.2$  MeV became known. Five states were described as neutron and proton vibrations [2]. Ten states were recognized as tetrahedral rotations and vibrations [4]. Two dozen states were described by the coupling of  $1p1h$  configurations to the  $3^-$  yrast state [5].

The question how to discern other collective states in  $^{208}\text{Pb}$  from  $1p1h$  states can be only solved by complete spectroscopy. It means that up to a certain limit in excitation energy all states must be found and for each of them spin, parity and dominant structure must be determined.

### 3 Complete spectroscopy

Complete spectroscopy is approached by studying several different reactions, especially  $^{208}\text{Pb}(p,p')$ ,  $^{208}\text{Pb}(d,d')$ ,  $^{208}\text{Pb}(\alpha,\alpha')$ ,  $^{207}\text{Pb}(d,p)$  and  $^{209}\text{Bi}(d,^3\text{He})$ . However, the most important reaction is the inelastic proton scattering via IARs in  $^{209}\text{Bi}$ . It allows to determine amplitudes with relative signs of all neutron one-particle one-hole configurations in each state.

In  $^{208}\text{Pb}$  there are 44 excess neutrons. By adding one proton an IAR in  $^{209}\text{Bi}$  is created with a typical energy of 15-20 MeV and a width of about 300 keV. In the subsequent proton decay all neutron particle-hole configurations in each state of  $^{208}\text{Pb}$  are excited. The decay protons act

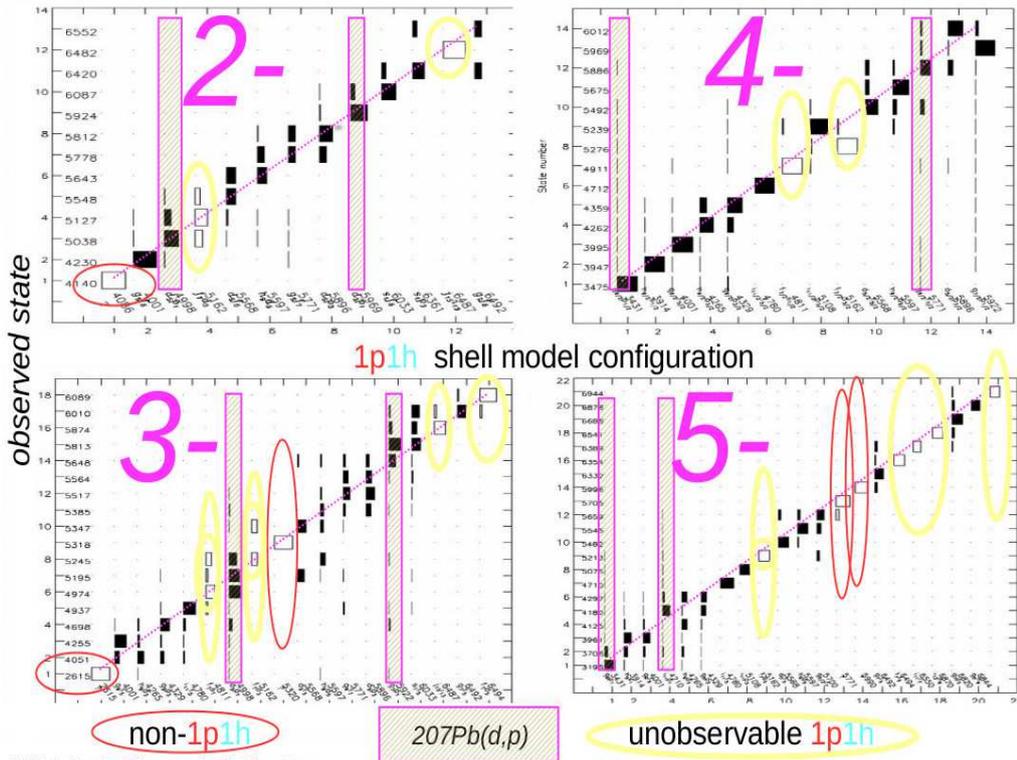
in a coherent manner to create interference patterns in the angular distributions (Fig. 1).

The knowledge of some amplitudes with relative signs allows to investigate the orthogonality relations among the states. Together with the normality and sum rule relations amplitudes of unobservable configurations can be determined. They are unobservable either because the cross section is too low or because there is no target as is the case for proton particle-hole configurations built with a particle in a higher orbit.

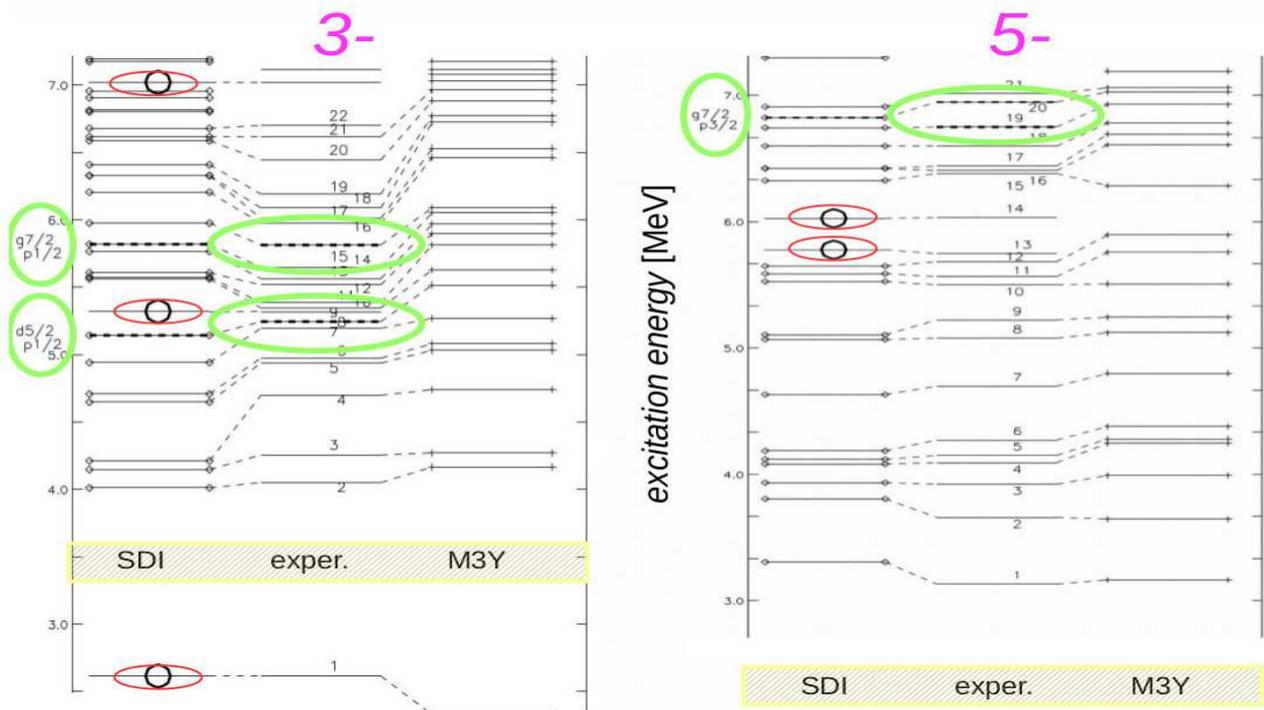
Fig. 1 shows angular distributions of  $^{208}\text{Pb}(p,p')$  for two states with spin  $5^-$  taken on the  $g_{7/2}$  IAR in  $^{209}\text{Bi}$ . It demonstrates the high sensitivity of the inelastic proton scattering via IARs. Similar angular distributions for about hundred states were measured from which spin, parity, and several amplitudes were derived [1, 3, 4, 6, 7].

Fig. 3 shows transformation matrices from  $1p1h$  configurations to states with spins  $2^-, 3^-, 4^-$ , and  $5^-$ . Transformation matrices for states with spins  $0^-, 1^-, 6^-, 7^-, 8^-$ , and  $1^+, 5^+, 6^+, 7^+, 8^+, 9^+, 10^+$  look similar. Most large amplitudes are centered close to the diagonal, only sometimes the order of two configurations is exchanged. For states with spins  $12^-, 13^-, 14^-$ , and  $0^+, 2^+, 3^+, 4^+, 11^+$ , and  $12^+$  no amplitudes could be measured. No state with spin  $9^-, 10^-,$  or  $11^-$  is known [1].

Fig. 4 compares level schemes for the  $3^-$  and  $5^-$  states at  $E_x < 7.2$  MeV to shell model calculations. One calculation (SDI) uses the surface delta interaction [12], another one (M3Y) the Michigan 3-Yukawa interaction [13]. The SDI calculates the multipole splitting and assumes no further residual interaction, the M3Y employs the mixing among  $1p1h$ ,  $2p2h$ , and  $3p3h$  configurations.



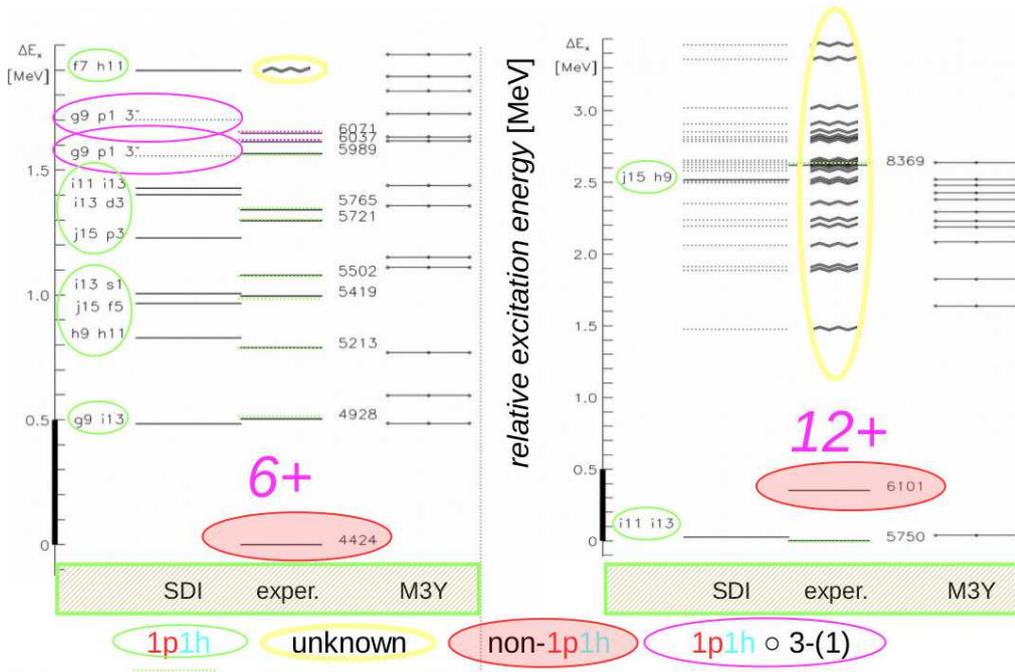
**Figure 3.** Transformation matrices from 1p1h configurations to states with spins  $2^-$ ,  $3^-$ ,  $4^-$ , and  $5^-$ . The normalized amplitudes are shown in the range  $0 < 1$ ; black for measured values, white for values (unobservable configurations marked yellow) determined by using the orthogonality, normality, and sum rule relations. Amplitudes obtained in a redundant manner by  $^{207}\text{Pb}(d,p)$  are marked by a frame. Amplitudes of states not described by 1p1h configurations are marked – the  $2^-$  yrast,  $3^-$  yrast states, another  $3^-$  and two  $5^-$  states at  $E_x \approx 5.5$  MeV.



**Figure 4.** Comparison of level schemes for the  $3^-$  and  $5^-$  states at  $E_x < 7.2$  MeV to shell model calculations with SDI [12] and M3Y [13]. Three  $3^-$  and two  $5^-$  states are not easily explained with M3Y by 1p1h configurations and with SDI not at all.

Both by SDI and M3Y describe the excitation energies for most states with spins  $3^-$  and  $5^-$  with a mean deviation of 30 keV, also for most states with spins  $0^+$ ,  $1^-$ ,  $2^-$ ,  $4^-$ ,  $6^-$ ,  $7^-$ ,  $8^-$ ,  $12^-$ ,  $13^-$ ,  $14^-$ , and  $3^+$ ,  $5^+$ ,  $7^+$ ,  $8^+$ ,  $9^+$ ,  $10^+$ ,  $11^+$ , and the  $12^+$  yrast state. Fig. 5 shows level schemes

for the  $6^+$  states at  $E_x < 6.2$  MeV and for the  $12^+$  states at  $E_x < 10$  MeV (see also Figs. 5 and 9 in [3]). Fig. 3 in [5] shows the level scheme for all positive parity states at  $E_x < 6.5$  MeV, see also [14]. The  $6^+$  yrast and  $12^+$  yrast states evidently appear in addition to 1p1h configurations.



**Figure 5.** Comparison of level schemes for the  $6^+$  and  $12^+$  states at  $E_x < 6.2$  MeV to shell model calculations with SDI [3] and M3Y [13]. (The energies are shown relative to the yrast state.) The  $6^+$  yrast and  $12^+$  yrast states are not explained by any existing model. Two  $6^+$  states at  $E_x \approx 6.0$  MeV (1.6 MeV above the yrast state) are described by the coupling of  $g_{9/2}p_{1/2}$  to the  $3^-$  state.  $2p2h$  configurations with spin  $12^+$  are predicted more than 1.5 MeV above the yrast state (wavy lines).

## 4 Summary

In the heavy nucleus  $^{208}\text{Pb}$  several different classes of excitations are observed,

- overwhelmingly  $1p1h$  configurations,
- two dozen states with  $1p1h$  configurations coupled to the  $3^-$  yrast state,
- five neutron and proton pairing vibrations,
- ten members of tetrahedral rotating and vibrating bands,
- the  $6^+$  yrast and  $12^+$  yrast states unexplained,
- one more  $3^-$  state and two more  $5^-$  states in the region at  $5.2 < E_x < 6.05$  MeV not explained by any existing model.

Still a few positive parity states below  $E_x = 6.2$  MeV are not yet identified and above  $E_x = 6.2$  MeV most positive parity states are unknown. A mysterious question is about a  $2^+$  state near the ground state predicted as the coupling of both intruders  $j_{15/2}$  and  $i_{13/2}$  to the  $3^-$  yrast state [5].

## 5 Outlook

Urgently needed is the theoretical description of dodecahedral and icosahedral configurations similar to the tetrahedral configurations by some algebraic cluster model.

On the experimental side the neutron capture on  $^{207}\text{Pb}$  and the investigation of the subsequent  $\gamma$ -cascade is needed. Modern equipment exists but studying  $\gamma$ -transitions among the more than 500 states needs a great effort.

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