Identification of System with Bouc-Wen Hysteresis

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Abstract. A method of adaptive identification of parameters for a system with Bouc-Wen hysteresis has been developed. The method is based on the use of adaptive observers and resolves the problem of stability of the identification system. Adaptive algorithms of identification were obtained using the Lyapunov second method. The stability proof of the adaptive system is based on the application of Lyapunov vector functions. Adaptive algorithms for adjusting model parameters were developed obtained and finiteness of trajectories in the adaptive system was pointed out.

1 Introduction

The Bouc-Wen (BWM) model is widely used to describe a hysteresis [1-11]. System with BWM has the form

$$m\ddot{x}+c\dot{x}+F(x,z,t)=f(t), \qquad (1)$$

$$F(x,z,t)=\alpha kx(t)+(1-\alpha)kdz(t), \qquad (2)$$

$$\dot{z}=d^{-1}\left(a\dot{x}-\beta\left|\dot{x}\right|\left|z\right|^{n}sign(z)-\gamma\dot{x}\left|z\right|^{n}\right), \qquad (3)$$

where $m>0$ is mass, $c>0$ is damping, $F(x,z,t)$ is the recovering force, $d>0$, $n>0$, $k>0$, $\alpha\in(0,1)$, $f(t)$ is exciting force, a,β,γ are some numbers.

Equation (3) is the BWM. Many modifications of BWM [3] have been proposed. Each proposed model considers features of the considered object. The degree of success of the BWM application strongly depends on the identification of its parameters. The solution of the nonlinear equation (3) is the main problem of the BWM identification. A three-level algorithm [4], based on regression analysis, least squares or Gauss-Newton methods, and the extended Kalman filter, is applied to Bouc-Wen model identification. The relevant approach has been applied in [5, 6]. Adaptive algorithms were proposed in [8, 9] for the BWM parameters estimation with the data forgetting [7]. Paper [10] presents an adaptive on-line identification methodology with a variable trace method to adjust the adaptation gain matrix.

Examples [11] are known when BWM parameters estimations do not agree with results obtained for other inputs. Such examples confirm the identification ambiguity which causes instability of the model and emphasize that the Bouc-Wen model should be stable in order to provide an adequate description of a physical process [12].

The analysis of literature shows that accounting for features of the object being examined is critically important for the development of algorithms and procedures for the identification of Bouc-Wen model parameters and that. the main difficulties in the estimation of the BWM parameters are related to the model stability and choice of the input . As a rule, the range, in which BWM parameters are varying, should be specified and all the derivatives of the object are should be measured. However, this is not always happenning that makes the application of the proposed algorithms ineffecient.

In this work, the adaptive identification method based on adaptive observer application [13] is used for the solution of the model (3) stability problem. The systemspecified by the equations (1)-(3) is considered. It is assummedthat the input $f(t)$ and the output $x(t)$ are measured.

2 Problem statement

Let us consider the system S_{BW} (1)-(3). Let y be the output of the system. The set of the experimental data has the form $\mathrm{I}_o=\left\{f(t),y(t),t\in J\right\}$, where $J\subset R$ is the given time slice.

Now we designate the parameters vector of the system as $A=\left[m,c,a,k,\alpha,\beta,\gamma,n\right]^T$.

Problem: to design the adaptive observer for vector estimation A of the system S_{BW} that satisfy the condition

$$\lim_{t\to\infty}\left|\hat{y}(t)-y(t)\right|\le\pi_y, \qquad (4)$$

where $\hat{y}\in R$ is the output of the adaptive observer, $\pi_y\ge 0$.

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3 About identifiability of S_{BW} -system

The identification effectiveness of the system S_{BW} depends on the features of the input $f(t)$. The requirements to $f(t)$ in identification problems are known. The force $f(t)$ satisfies the condition of constant excitation (CE). This condition is necessary, but not sufficient [14]. The input having the CE property cannot ensure the identifiability of the hysteresis structure. The structural identifiability of the hysteresis is guaranteed with the input $f(t)$ having the S-stabilisation property of the system S_{BW} [14]. Conditions of this property verification are given in [14].

4 Adaptive system identification

Let us consider the case $d=1$, $a=1$. Substitute $F(x,z,t)$ in (1), and divide it by $s+\mu$, where $\mu>0$ and does not coincide with roots of the polynomial $s^2+a_1s+a_2$, $s=d/dt$. Then transform (1) to the form

$$\dot{x}=a_1x+a_2p_x+a_3p_z+bp_f\,, \tag{5}$$

$$\begin{aligned}&\dot{p}_x=-\mu p_x+x,\ \dot{p}_f=-\mu p_f+f,\\&\dot{p}_z=-\mu p_z+z,\end{aligned} \tag{6}$$

where $a_1=-(c-\mu m)/m$, $a_2=-(\alpha k-\mu(c-\mu m)/m$, $a_3=-(1-\alpha)k/m$.

Equations (5) and (6) contain only measurable variables, except for z. This complicates the identification process of the system S_{BW} parameters. Now we apply the model

$$\dot{\hat{x}}=-k_x(\hat{x}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{a}_3p_z+\hat{b}p_f \tag{7}$$

to the parameters estimation of (5), where $k_x>0$ is the specified number; $\hat{a}_i(t)$, $i=1,2,3$, and $\hat{b}(t)$ are adjusted parameters.

Let $e=\hat{x}-x$. Obtain the equation for the identification error from (5), (7)

$$\dot{e}=-k_xe+\Delta a_1x+\Delta a_2p_x+\Delta a_3p_z+\Delta bp_f\,, \tag{8}$$

where $\Delta b=\hat{b}(t)-b$, $\Delta a_1=\hat{a}_1(t)-a_1$, $\Delta a_2=\hat{a}_2(t)-a_2$, $\Delta a_3=\hat{a}_3(t)-a_3$.

The (8) is not solvable as the variable z in (6). is unknown Now we shall obtain the current estimation for $z(t)$. Consider model

$$\dot{\hat{x}}_{\bar{z}}=-k_x(\hat{x}_{\bar{z}}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{b}p_f\,. \tag{9}$$

After that we etermine by the misalignment $\varepsilon_z=x-\hat{x}_{\bar{z}}$ and use it for the variable z estimation. Let r ε_z be the current estimation z. After applying the model to the estimation of z, we get

$$\dot{\hat{z}}=-k_z\left(\hat{z}-\varepsilon_z\right)+\tilde{\dot{x}}-\hat{\beta}\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z})-\hat{\gamma}\tilde{\dot{x}}\left|\hat{z}\right|^n\,, \tag{10}$$

where $\tilde{\dot{x}}=\left(x(t+\tau)-x(t)\right)/\tau$, $k_z>0$ is the given number; $\hat{\beta}$, $\hat{\gamma}$ are the hysteresis (3) parameters estimations;, τ is the integration step.

Let us introduce the misalignment $\varepsilon=\hat{z}-\varepsilon_z$ and obtain the equation for ε

$$\begin{aligned}\dot{\varepsilon}=-k_z\varepsilon+\Delta\dot{x}+\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right)+\beta\eta_\beta+\\ \Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^n+\gamma\eta_\gamma\end{aligned}\,, \tag{12}$$

$$\eta_\beta=\left|\dot{x}\right|\left|z\right|^n sign\left(z\right)-\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right),\quad \eta_\gamma=\dot{x}\left|z\right|^n-\tilde{\dot{x}}\left|\hat{z}\right|^n,$$

where $\Delta\dot{x}=\tilde{\dot{x}}-\dot{x}$, $\Delta\beta=\beta-\hat{\beta}$, $\Delta\gamma=\gamma-\hat{\gamma}$.

Let present (7) as

$$\dot{\hat{x}}=-k_x(\hat{x}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{a}_3p_{\hat{z}}+\hat{b}p_f\,,$$

where

$$\dot{p}_{\hat{z}}=-\mu p_{\hat{z}}+\hat{z}\,. \tag{13}$$

Then, (8) can be rewriten as

$$\dot{e}=-k_xe+\Delta a_1x+\Delta a_2p_x+\Delta a_3p_{\hat{z}}+\Delta bp_f\,, \tag{14}$$

and adaptive algorithms is described

$$\begin{aligned}&\Delta\dot{a}_1=-\gamma_1ex,\Delta\dot{a}_2=-\gamma_2e_x,\\&\Delta\dot{a}_3=-\gamma_3ep_{\hat{z}},\Delta b=-\gamma_bep_f,\end{aligned} \tag{15}$$

where $\gamma_i>0, i=1,2,3$; $\gamma_b>0$.

Tuning algorithms for $\Delta\beta$ and $\Delta\gamma$ into (10) have the following form

$$\begin{aligned}&\Delta\dot{\beta}=-\chi_\beta\varepsilon\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right),\\&\Delta\dot{\gamma}=-\chi_\gamma\varepsilon\tilde{\dot{x}}\left|\hat{z}\right|^n,\end{aligned} \tag{16}$$

where $\chi_\beta>0, \chi_\gamma>0$ are parameters ensuring a convergence of algorithms.

Several algorithms are applicable for the indicator n estimation in (10). Their effectiveness depends on several factors. The simple algorithm has the form

$$\dot{\hat{n}}=\begin{cases}-\gamma_n\varepsilon\hat{\beta}\left|\hat{z}\right|^{\hat{n}-1}\hat{z}\tilde{\dot{x}}, if \left|\dfrac{\varepsilon}{\varepsilon_z}\right|\in\left[\upsilon_0,\upsilon_1\right],\\[2ex] 0,\qquad if\left|\dfrac{\varepsilon}{\varepsilon_z}\right|\notin\left[\upsilon_0,\upsilon_1\right],\end{cases} \tag{17}$$

where υ_0,υ_1 are given positive numbers, $\gamma_n>0$.

Remark. Stability of the identification procedure is the main problem the solution of the system with BWM. We proposed the method based on the application of adaptive observers. Another solution to the stability problem is to change the structure of the equation (3). We proposed the equation

$$\dot{z} = d^{-1}\left(-\rho z|\dot{x}|^{\omega} + a\dot{x} - \beta|\dot{x}||z|^{n}\, sign(z) - \gamma\dot{x}|z|^{n}\right)$$

where $\rho > 0$, $\omega > 0$. to describe hysteresis.

5 Properties of adaptive system

Let consider the subsystem AS_X described by (14) and(15). Let $\Delta K(t) \stackrel{\Delta}{=} [\Delta a_1(t), \Delta a_2(t), \Delta a_3(t), \Delta b(t)]^T$,

$$V_K(t) \stackrel{\Delta}{=} 0.5\Delta K^T(t)\Gamma^{-1}\Delta K(t), \qquad (18)$$

$$V(t) = V_e(t) + V_K(t), \qquad (19)$$

where $\Gamma = diag(\gamma_1, \gamma_2, \gamma_3, \gamma_b)$.

Assumption 1. The input $f(t)$ is constantly exciting and limited.

Theorem 1. Let 1) functions (18), $V_K(t)$ are positive definite and satisfy the condition $\inf\limits_{|e|\to\infty} V_e(e) \to \infty$, $\inf\limits_{\|\Delta K\|\to\infty} V_K(\Delta K) \to \infty$; 2) assumption 1 for the system (1)-(3) is satisfied. Then all trajectories of the system AS_X are limited belong area $\mathrm{G}_t = \{(e, \Delta K) : V(t) \le V(t_0)\}$ and the estimation

$$\int_{t_0}^{t} 2k_x V_e(\tau)\, d\tau \le V(t_0) - V(t)$$

is fair.

The theorem 1 shows the limitation of adaptive system trajectories. The asymptotical stability ensuring the system demands to impose additional conditions.

Let $P(t) \stackrel{\Delta}{=} [x(t)\ p_x(t)\ p_{\hat{z}}(t) p_f(t)]^T$.

Definition 1. The vector P is constantly excited with a level ν or have property $\mathcal{PE}_\nu$ if

$$\mathcal{PE}_\nu:\ P(t)P^T(t) \ge \nu I_4$$

fairly for $\exists \nu > 0$ and $\forall t \ge t_0$ on some interval $T > 0$, where $I_4 \in R^4$ is the unity matrix.

If the vector $P(t)$ has property $\mathcal{PE}_\nu$ then we will write $P(t) \in \mathcal{PE}_\nu$.

The system S_{BW} is stable, and the input $f(t)$ is restricted. Therefore, present the property $\mathcal{PE}_\nu$ for the matrix $B_P(t) = P(t)P^T(t)$ as

$$\mathcal{PE}_{\nu,\bar{\nu}}:\ \nu I_4 \le B_P(t) \le \bar{\nu} I_4 \quad \forall t \ge t_0, \qquad (20)$$

where $\bar{\nu} > 0$ is some number.

Let us the estimation to $V_K(t)$ be fair

$$0.5\,\beta_l^{-1}(\Gamma)\|\Delta K(t)\|^2 \le V_K(t) \le 0.5\,\beta_1^{-1}(\Gamma)\|\Delta K(t)\|^2, \qquad (21)$$

where $\beta_1(\Gamma)$, $\beta_l(\Gamma)$ are minimum and maximum eigenvalues of the matrix Γ.

Now we apply inequalities (20), (21) and obtain estimations for $\dot{V}_e, \dot{V}_K$

$$\dot{V}_e \le -k_x V_e + \frac{\bar{\nu}\beta_l(\Gamma)}{k_x} V_K, \qquad (22)$$

$$\dot{V}_K \le -\frac{3}{4}\vartheta\nu\beta_1(\Gamma)V_K + \frac{8}{3}\vartheta V_e, \qquad (23)$$

Estimates (22), (23) are obtained applying the approach [15].

Theorem 2. Let conditions be satisfied 1) positive definite Lyapunov functions $V_e(t)$ and $V_K(t)$ allow the indefinitely small highest limit at $|e(t)| \to 0$, $\|\Delta K(t)\| \to 0$; 2) $P(t) \in \mathcal{PE}_{\nu,\bar{\nu}}$; 3) equality

$$e\Delta K^T P = \vartheta\left(\Delta K^T B \Delta K + e^2\right)$$

is fair in the area $O_\nu(O)$ with $0 < \vartheta$, where $O = \{0,\ 0^{3m}\} \subset R \times R^{3m} \times J_{0,\infty}$, O_ν is some neighbourhood of the point O; 4) the function $V_K(t)$ satisfies (21); 5) $\dot{V}_\varepsilon, \dot{V}_K$ satisfy the system of inequalities

$$\begin{bmatrix}\dot{V}_e \\ \dot{V}_K\end{bmatrix} \le \underbrace{\begin{bmatrix} -k_x & \dfrac{\bar{\nu}\ \beta_l(\Gamma)}{k_x} \\ \dfrac{8}{3}\vartheta & -\dfrac{3\nu\vartheta\beta_1(\Gamma)}{4}\end{bmatrix}}_{A_V}\begin{bmatrix}V_e \\ V_K\end{bmatrix};$$

6) the upper solution for $V_{e,K}(t) = [V_e(t)\ V_K(t)]^T$ satisfies to the comparison equation $\dot{S} = A_V S$ if

$$V_\rho(t) \le s_\rho(t) \quad \forall(t \ge t_0) \,\&\, (V_\rho(t_0) \le s_\rho(t_0)),$$

where $\rho = e, K$, $S = [s_e\ s_K]^T$, $A_V \in R^{2\times 2}$ is M-matrix. Then the system AS_X is exponentially stable with the estimation

$$V_{e,K}(t) \le e^{A_V(t-t_0)} S(t_0),$$

if

$$k_x \ge \frac{4}{3}\sqrt{\frac{2\bar{\nu}\beta_l(\Gamma)}{\nu\beta_1(\Gamma)}}. \qquad (24)$$

Theorem 2 shows that the adaptive system AS_X gives the true estimates for parameters of the system (1). This is fair at the fulfilment of conditions (24). We assume that the variable $p_{\hat{z}}$ restricted.

The boundedness of the variable $\hat{x}_{\bar{z}}$ follows from the boundedness of the system AS_X trajectories.

Let us consider subsystem AS_Z described by equations (12), (16) and
introduce Lyapunov functions

$$V_{\varepsilon\beta\gamma}(t) = V_\varepsilon(t) + V_{\beta,\gamma}(t),$$
$$V_{\beta,\gamma}(t) = 0.5\chi_\beta^{-1}\left(\Delta\beta(t)\right)^2 + 0.5\chi_\gamma^{-1}\left(\Delta\gamma(t)\right)^2. \quad (25)$$

Theorem 3. Let
(1) functions $V_\varepsilon(t) = 0.5\varepsilon^2(t)$, $V_{\beta,\gamma}(t)$ are positive definite and satisfy conditions

$$\inf_{|\varepsilon|\to\infty} V_\varepsilon(\varepsilon) \to \infty, \quad \inf_{\|[\Delta\beta,\Delta\gamma]\|\to\infty} V_{\beta,\gamma}\left(\Delta\beta,\Delta\gamma\right) \to \infty;$$

(2) the function $V_{\varepsilon\beta\gamma}(t)$ has the form (25); 3) the function

$$\tilde{g}_1(t) = \sup_{\varepsilon\in\Omega} \frac{|\varepsilon|^{n+1}(t)}{V_\varepsilon(t,\varepsilon)}, \quad g_1 = \sup_{\varepsilon\in\Omega} \tilde{g}_1(t), \quad (26)$$

exists, where Ω is the definition range of the subsystem AS_Z; (4) $|\Delta\dot{x}| \le \delta_\Delta$, $\delta_\Delta \ge 0$; 5) $|\dot{x}| \le \upsilon$, $\upsilon > 0$; 6) the assumption 1 holds for the system (1)-(3). Then all trajectories of the system AS_Z are bounded, belonging to the area $G_\varepsilon = \left\{\left(\varepsilon,\Delta\beta,\Delta\gamma\right) : V_{\varepsilon\beta\gamma}(t) \le V_{\varepsilon\beta\gamma}\left(t_0\right)\right\}$, and the estimation

$$\int_{t_0}^{t}\left(k_z - \upsilon(\beta+\gamma)g_1\right)V_\varepsilon(\tau)d\tau + \frac{1}{2\left(k_z - \upsilon(\beta+\gamma)g_1\right)\left(t-t_0\right)}\left(\delta_\Delta\right)^2 \le V_{\varepsilon\beta\gamma}(t_0) - V_{\varepsilon\beta\gamma}(t) \quad (27)$$

is fair if

$$k_z > \upsilon(\beta+\gamma)g_1. \quad (28)$$

Hence, the boundedness of trajectories in the adaptive system is proved. The analysis showed that the subsystem AS_X is asymptotically stable. The prove of trajectories boundedness for the subsystem AS_Z is a more complex problem in parametrical and output spaces. This problem is solvable if the condition (28) is satisfied. The estimation (27) shows that the quality of processes in the AS_Z-system depends on the output derivative of the S_{BW}-system. The following result given more exact estimations for processes in the AS_Z-system.

Theorem 4. Let (1) positive definite Lyapunov functions $V_{\beta,\gamma}(t)$ and $V_\varepsilon(t)$ allow the indefinitely small higher limit $\left\|\left[\Delta\beta(t),\Delta\gamma(t)\right]\right\| \to 0$ to $|\varepsilon(t)| \to 0$; (2) $P(t) \in \mathcal{PE}_{\nu,\bar{\nu}}$; (3) such $c_1 > 0, c_2 > 0$ exist that conditions

$$\varepsilon\Delta\gamma\tilde{x}|\hat{z}|^n = c_2\left[\left(\Delta\gamma\right)^2\left(\tilde{x}|\hat{z}|^n\right)^2 + \varepsilon^2\right],$$

$$\varepsilon\Delta\beta|\tilde{x}||\hat{z}|^n\, sign\left(\hat{z}\right) = c_1\left[\left(\Delta\beta\right)^2\left(|\tilde{x}||\hat{z}|^n\right)^2 + \varepsilon^2\right]$$

are satisfied in the area $O_\nu(O)$, where $O = \{0,\ 0^2\} \subset R \times R^2 \times J_{0,\infty}$, O_ν is some neighbourhood of the point O; (4) inequality $\left(\varepsilon - \varepsilon_z\right)^{2n} \ge c_z$ holds for almost all t where $c_z \ge 0$; (5) such $\pi_x \ge 0$ and $\omega > 0$ exist that $\left(\tilde{\dot{x}}\right)^2 \ge \pi_x$ и $|\varepsilon - \varepsilon_z| \le \omega|\varepsilon|$; (6) the function

$$g_2(t) = \sup_{\varepsilon\in\Omega} \frac{|\varepsilon|^{2(n+1)}(t)}{V_\varepsilon(t,\varepsilon)}, \quad g_2 = \sup_{\varepsilon\in\Omega} \tilde{g}_2(t)$$

exists, where Ω the subsystem AS_Z definition domain; (7) $\dot{V}_\varepsilon$, $\dot{V}_{\beta,\gamma}$ satisfy the system of inequalities

$$\begin{bmatrix}\dot{V}_\varepsilon \\ \dot{V}_{\beta,\gamma}\end{bmatrix} \le \underbrace{\begin{bmatrix}-\left(k_z - 2\tilde{\upsilon}g_1 - \omega\upsilon g_2\right) & \lambda\chi\omega\upsilon \\ c & -\dfrac{d_s}{2}\end{bmatrix}}_{A_\varepsilon}\begin{bmatrix}V_\varepsilon \\ V_{\beta,\gamma}\end{bmatrix} + \underbrace{\begin{bmatrix}\dfrac{1}{2k_z} \\ 0\end{bmatrix}}_{B_\varepsilon}\left(\delta_\Delta\right)^2;$$

(8) the upper solution for $V_{\varepsilon,\beta,\gamma} = \left[V_\varepsilon(t)\, V_{\beta,\gamma}(t)\right]^T$ satisfies to the equation

$$\dot{\tilde{S}} = A_\varepsilon\tilde{S} + B_\varepsilon\left(\delta_\Delta\right)^2$$

If

$$V_{\tilde{\rho}}(t) \le \tilde{s}_{\tilde{\rho}}(t) \quad \forall\left(t \ge t_0\right) \& \left(V_{\tilde{\rho}}\left(t_0\right) \le \tilde{s}_{\tilde{\rho}}\left(t_0\right)\right),$$

where $\tilde{\rho} = \varepsilon, \beta, \gamma$, $\tilde{S} = \left[\tilde{s}_\varepsilon\ \tilde{s}_{\beta,\gamma}\right]^T$, $A_\varepsilon \in R^{2\times2}$ is M-matrix. Then the system AS_Z is exponentially dissipative with the estimation

$$V_{\varepsilon,\beta,\gamma}(t) \le e^{A_\varepsilon(t-t_0)}\tilde{S}(t_0) + \left(\delta_\Delta\right)^2\int_{t_0}^{T} e^{A_\varepsilon(t-\tau)}B_\varepsilon d\tau,$$

if $\left(k_z - 2\tilde{\upsilon}g_1 - \omega\upsilon g_2\right)d_s > 2c\lambda\chi\omega\upsilon$, $k_z > 2\tilde{\upsilon}g_1 - \omega\upsilon g_2$, $d_s > 0$,

$$\overline{\chi} = \min\left(\chi_\beta, \chi_\gamma\right), \overline{c} = \min\left(c_1, c_2\right),$$
$$\chi = \max\left(\chi_\beta, \chi_\gamma\right), d_s = \chi\pi_x\overline{c}c_z.$$

We have shown that the system AS_Z is exponentially dissipative. The area of the dissipativity depends on the informational set I_o of the S_{BW} -system. The obtained results justify the application of adaptive observers for the S_{BW} -system identification.

6 Simulation results

Let us consider the system (5)-(8) with parameters $n = 1.5$, $c = 2$, $m = 1$, $\beta = 0.5$, $\alpha = 0.7$, $k = 0.6$. Let $d = a = 1$. The exciting force $f(t) = 2 - 2\sin\left(0.15\pi t\right)$. The system S_{BW} modelled with initial conditions $x(0) = 1$, $\dot{x}(0) = 0$, $z(0) = 1$. Form the set I_o. The system phase portrait and output of the hysteresis are shown in Fig. 1.

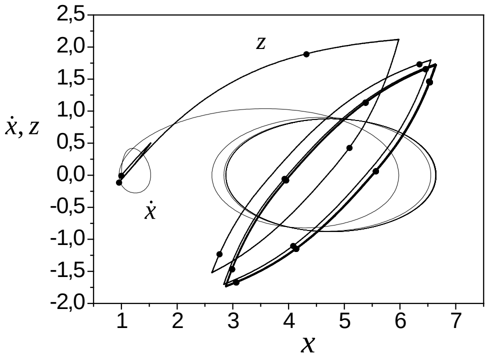

Fig. 1. System phase portrait and hysteresis change.

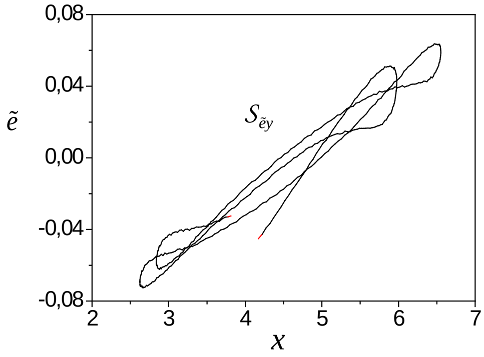

Fig. 2. System phase portrait and hysteresis change.

Let us construct the framework $S_{\tilde{e}y}$ (Fig. 2), using the method [14], and evaluate the structural identifiability of the system S_{BW}. The variable $\tilde{e} \in R$ is equal $\tilde{e} = \dot{x} - \hat{\dot{x}}_h$. $\hat{\dot{x}}_h$ is the estimation of the steady state (process) in the S_{BW} -system for $\forall t \geq 9.85$ s, and $\tilde{e}$ is the hysteresis output estimation. Fig. 1 and 2 show that definition ranges z and $\tilde{e}$ coincides. The analysis of $S_{\tilde{e}y}$ shows that the system S_{BW} is structurally identifiable, and the input $f(t)$ is S-stabilizing.

Let usonsider the identification of the system S_{BW} parameters. After that we determine by the parameter μ of the system Eq. (13) using the transient process analysis for $\tilde{e}$ and $t < 9.85$ s. Now we alculate Lyapunov exponents (LE) [16]. The estimation for the maximum LE is –0.9. Therefore, we set $\mu = 0.8$. Initial conditions in (6) are equal to zero.

The results of the work of the adaptive system are presented in Figs. 3-7. Parameters k_x, k_z are equal to 2.5 and 0.75. The tuning process of AS_X -systems (the model (7)) parameters are shown in Fig. 3, Fig. 4 shows the tuning parameters of the model (10).

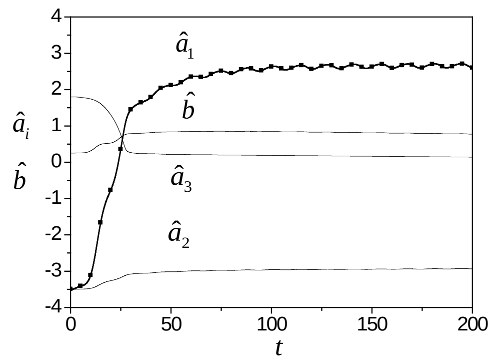

Fig. 3. Tuning of model (7) parameters.

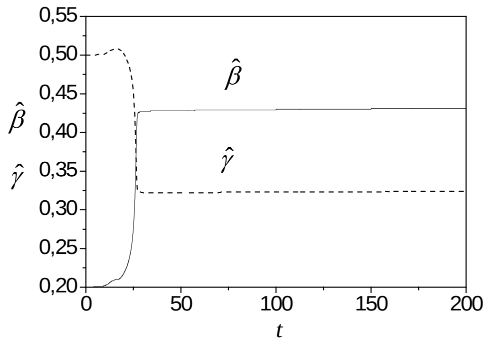

Fig. 4. Tuning of model (10) parameters.

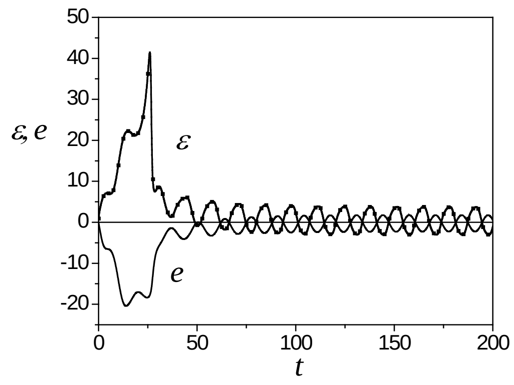

Fig. 5. Outputs modification of systems AS_X, AS_Z.

The modification of identification errors e, ε are shown in Fig. 5. We can see that the accuracy of obtained estimations depends on numbers of tuned parameters and the $\dot{x}$ level, and properties $f(t)$. Obtained results confirm statements of theorems 3, 4. The results of AS_Z-system work influence the tuning processes in the AS_X-system. Gain coefficients in (15), (16) and (17) are $\chi_\beta = 0.0000002$, $\chi_\gamma = 0.0000002$, $\gamma_4 = 0.00005$ $\gamma_1 = 0.0002$, $\gamma_2 = 0.00001$, $\gamma_3 = 0.00002$.

The hysteresis output estimation process is shown in Fig. 6.

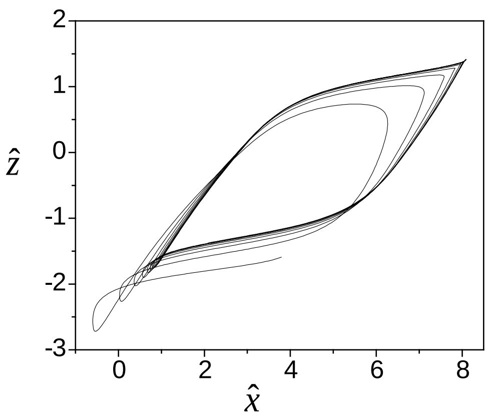

Fig. 6. Hysteresis estimation at adaptation of AS_{BW}-system

So, simulation results confirm the exponential dissipativity of the designed system.

7 Conclusion

The adaptive parameter identification method for the system with Bouc-Wen hysteresis has been developed. The method is based on the application of adaptive observers and solves the stability problem. Adaptive algorithms of model parameter tuning have been designed, and the boundedness of trajectories in the adaptive system is shown. Estimates of uncertainty, which are used for the tuning of the hysteresis model parameters, have been obtained.

We show that the boundary of the system exponential dissipativity area is determined by the system output derivative level.

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