Motion of a Solid Large Spherical Aerosol Particle in a Single-Component Gas under the Influence of Carrier Medium

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Abstract: In the article, a motion of a large solid spherical aerosol particle in a single-component gas has been investigated. Mathematical modeling has been performed considering inertia, particle radius and resistance of the carrier gaseous medium to its motion. The obtained formulas allow determining the time of the particle transition into the final states, and also the distances travelled by the particle. The numerical evaluations solved with the use of these formulas demonstrate that for large initial Reynolds numbers the particle stopping distance can be significant.

1 Introduction

The composition of natural, atmospheric, and industrial aerosols surrounding us [1–7] may include various types of solid aerosol particles [2–6]. These particles can have an arbitrary surface shape, in particular, close to spherical, cylindrical, ellipsoidal, needle, lamellar and other forms [9]. They also have properties that are very important for practical applications, including physicochemical ones. Therefore, aerosol particles are increasingly used in practice [3-6]. They are used, for example, in various fields of technology, including military and space, in medicine[26], in particular in the treatment of wounds, with lesions of the skin and inhalation. Aerosol particles protect agricultural and forest crops from pests, protect and treat animals and birds from parasites and diseases. They are used in everyday life, in military affairs with blackout, etc. Aerosol particles play an important role in the formation of precipitation [27]. Aerosol particles are also widely used in industry, in particular, as catalysts and components of high-energy fuels. Aerosol particles play a well-known role in biology — plant pollen, bacterial and mold spores, and light seeds are transported in nature in the form of aerosols. In connection with this, a comprehensive study of the various properties of aerosol particles, including the laws of their ordered motion in gaseous media, is of great scientific and practical interest [2-6]. In practical applications, they often deal with the ordered movement of aerosol particles that occurs under the influence of fairly well-studied external forces [6,9]. Such forces include, for example, and of molecular nature, thermophoretic [5,6,8-20,24] and photophoretic [15,19,21-23] forces. Thermophoretic force moves particles to areas with lower temperatures. Its appearance is due to the transfer by particles of molecules of an inhomogeneous temperature gas of an uncompensated impulse. The thermophoretic movement of aerosol particles occurs during nonisothermal industrial and natural processes [3,5,6,24]. Such movement of aerosol particles can occur when they are deposited on heat transfer surfaces with lower temperatures, for example, in thermal precipitators, heat and mass exchanger channels [3,6,9], cloud and fog clear zones [4,27]. The thermophoretic movement can, in particular, be used for fine purification of gases, separation of particles by size, obtaining new materials with preset properties, in medicine.

The appearance of the photophoretic force is due to the interaction of the molecules surrounding the gas particles with the particle surface inhomogeneously heated by electromagnetic radiation. This force has a major effect on the movement of particles, called photophoretic. The latter is widely used in technical, medical, environmental and biological applications. The photophoretic force has a significant effect, in particular, on the movement of solid black large and moderately large aerosol particles.

Of great importance is the study of the laws and free motion of particles, which occurs at large initial velocities and Reynolds numbers, since these particles can travel long distances before stopping. Large initial velocities $\vec{V}_i$ and Reynolds numbers Re_i can have aerosol particles of different nature, in particular, shotguns, particles that are formed during dispersion (grinding, spraying) of solids, particles flying from the surface of grinding wheels, particles emitted into the atmosphere by ventilation systems [6,9]. At high

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speeds, particles formed during the high-temperature combustion of fuel in the furnaces can move, internal combustion engines as well particles in the exhaust gases of diesel engines of ships, engines of jet and turbojet aircraft [2-4.9], etc.

It should be noted that aerosol particles moving in gaseous media are affected by the medium resistance which slows down their motion [1,2]. In this process, particles transition from states with the initial Reynolds number Re_i to states with a smaller Reynolds number Re_f. Therefore, when assessing the motion of aerosol particles, the gaseous medium resistance influencing their motion must also be considered.

The resistance force impact ceases when the particle stops. In this case $Re_f = 0$. The distance traveled by the particle to a complete stop is called the stopping distance [1,2]. The length of this distance can greatly depend on the initial Reynolds number Re_i and particle sizes. Thus, knowing the formulas allowing, at known aerosol particle sizes and initial Reynolds numbers Re_i, directly assessing the time the particles take to transition from states with a known initial number Re_i to states with smaller Reynolds numbers Re_f, as well as the distances traveled by particles, in particular, stopping distances, is of significant interest when assessing aerosol particle motion for practical applications considering the impact of the gaseous medium resistance.

The formulas provided below allow, at the initial Reynolds numbers $0 < Re_i \leq 1000$, assessing the motion of large solid spherical aerosol particles with consideration of how their motion is impacted not only by the gaseous medium resistance, but also by their radius.

2 Problem Statement

Free one-dimensional motion of a large solid spherical aerosol particle (with the Knudsen number $Kn = \lambda/R < 0.01$ [5,6], where λ is the average free path length of the gas molecules, R is the particle radius) with the diameter d^*, initial Reynolds number Re_i, and velocity $\vec{V}_i = V_{ix}\vec{n}_x$ occurs in a single-component gas in a horizontal direction along the Ox axis. The particle moving in the gas is affected by the following gas resistance force decelerating its motion (1) [5,6]:

$$\vec{F}_D = F_D\vec{n}_x. \quad (1)$$

Expression (1) for the resistance force includes the scalar coefficient $F_D = (-\pi\mu\nu\beta_D \mathrm{Re}^2)/8$, where $\nu = \mu/\rho_m$ and μ are the kinematic and dynamic viscosity coefficients of the gaseous medium, ρ_m is the gas density, $\mathrm{Re} = (V_x d^*/\nu)$ is the particle Reynolds number, V_x is the projection of the particle velocity vector on the Ox axis, $\vec{n}_x$ is a unit vector in the direction of the Ox axis, β_D is the coefficient of the gas medium resistance to the particle motion that depends on the Reynolds number [1,2].

Coefficient β_D, which is a part of the formula for the resistance force (1), can have a significant influence on its value. The experimental values of the resistance coefficient β_D [4] for large spherical particles in the air depending on the Reynolds number are presented in Table 1. Derivation of analytical expressions for the coefficient β_D is a complicated mathematical problem. Analytical expressions (2) and (3) for the coefficient β_D of large solid spherical particles obtained in Stokes' and Oseen's approximations, respectively, have the following form [2,6]:

$$\beta_D^{(S)} = (24/\mathrm{Re}) \quad (2)$$

$$\beta_D^{(0)} = \beta_D^{(S)}\left[1 + (3\mathrm{Re}/16)\right]. \quad (3)$$

A comparison of the experimental values of the resistance coefficient with the values from theoretical formulas (2) and (3) demonstrates that formula (2) at $\mathrm{Re} > 0.3$ allows obtaining only underestimated values of the resistance coefficient, and formula (3) at $\mathrm{Re} < 1000$ allows finding only overestimated values of the resistance coefficient. The following empirical formula obtained by the authors on the basis of the experimental data provided in [4] gives a relatively good description of the dependence of the experimental values $\beta_D^{(Э)}$ on the Reynolds number at $\mathrm{Re} < 1000$:

$$\beta_D = \beta_D^{(S)}\sqrt{1 + A\mathrm{Re} + B\mathrm{Re}^2} \quad (4)$$

where $A = 0.1865$ and $B = 1.52 \times 10^{-4}$ are empirical constants.

Table 1. The β_D values obtained experimentally and using formulas (2)–(4) depending on the Reynolds number.

Re	Ex.	Formulas		
	$\beta_D^{(Э)}$	$\beta_D^{(S)}$	$\beta_D^{(0)}$	(4)
0.1	240	240	244.6	242.12
0.2	120	120	124.4	122.23
0.3	80	80	84.7	82.22
0.5	49.5	48.2	52.3	50.19
0.7	36.5	34.3	38.75	36.17
1.0	26.5	24.3	28.51	25.91
2	14.6	12.21	16.52	13.93
3	10.4	8.12	12.51	9.91
5	6.9	4.83	9.32	6.72
7	5.3	3.42	7.91	5.19
10	4.1	2.4	6.9	4.06
20	2.55	1.2	5.7	2.62
30	2.00	0.8	5.3	2.07
50	1.50	0.48	4.98	1.58
70	1.27	0.34	4.8	1.31
100	1.07	0.24	4.74	1.11
200	0.77	0.12	4.62	0.82
300	0.65	0.08	4.58	0.66
500	0.55	0.05	4.55	0.54
1000	0.46	0.024	4.52	0.43

It should be noted that formula (4) can be used for analytical solution of aerosol mechanics problems for cases with big and small Reynolds numbers. This will be demonstrated below using an example of solving a problem of free one-dimensional motion in a single-component gas of a large solid spherical aerosol particle with its Reynolds numbers Re < 1000.

2.1 Time of the article transition into states with new Reynolds numbers

Free motion of a large solid spherical aerosol particle in a gas is decelerated by the gaseous medium resistance force (1) [1,2]. The written equation for the motion of the considered particle, taking into account expression (1) for the resistance force, has the following form:

$$m\frac{dV_x}{dt} = -(\pi/8)\beta_D \mu \nu \,\mathrm{Re}^2 \quad (5)$$

In this equation t is the time, $m = \frac{\pi}{6} d^{*3}\rho_p$ is the particle mass, ρ_p is the particle substance density, d^* is the particle diameter. Equation (5) takes the form convenient for integration after entering the Reynolds number into its left side, consideration of expressions for β_D (4), particle mass *m* and $\nu = \mu/\rho_m$. Thereby we obtain:

$$dt = -(\xi/Re\sqrt{1 + A\,\mathrm{Re} + B\,\mathrm{Re}^2}\,)dRe. \quad (6)$$

Here $\xi = d^{*2}\rho_p / 18\mu$ is the relaxation time of the spherical aerosol particle at Re <<1 [2]. After integration of the right side of equation (6) from the initial value Re_i to Re_f, we obtain expression (7) connecting the values of the ratio $\left[\Delta(\mathrm{Re}_f)/\Delta(\mathrm{Re}_i)\right]$ with transition time *t* of the particle from a state with Re_i to a state with Re_f:

$$\exp(t/\xi) = \left[\Delta(\mathrm{Re}_f)/\Delta(\mathrm{Re}_i)\right], \quad (7)$$

$$\Delta(\mathrm{Re}) = \left[2(\sqrt{1 + A\,\mathrm{Re} + B\,\mathrm{Re}^2} + 1) + A\,\mathrm{Re}\right]/\mathrm{Re} \quad (8)$$

Formula (7) with consideration of formula (8) allows, at a known radius, density, particle numbers Re_i and Re_f, directly obtaining the time of its transition to a state with the smaller number Re_f. The data in Table 2 on the dependence of the Re_f variables of steel particles with d^* = 0.4; 0.2; 0.1 cm, airborne at a temperature t^0 = 20ºC and pressure p = 1 atm on time *t* were obtained using these formulas.

These data demonstrate that the transition time *t* can be greatly dependent on the particle diameter.

Table 2. Dependence of the Re_f numbers at Re_i = 1000 on time *t*.

Re_f	d^* = 0.4, cm	d^* = 0.2, cm	d^* = 0.1, cm
	t, c	t, c	t, c
800	6.84	1.71	0.4275
200	59.28	14.82	3.705
25	268.7	67.18	16.79
5	617.5	154.4	38.59
1	1126	281.5	70.38
0.1	1972	493	123.3

Taking the logarithm (7) allows transitioning to the following simpler expression for *t*: $t = \xi \ln\left[\Delta(\mathrm{Re}_f)/\Delta(\mathrm{Re}_i)\right]$.

2.2 The distance traveled by the particle during its transition to a state with new Reynolds numbers

During the transition of the particle from the initial state with Re_i to the state with Re_f,the particle travels a certain distance with the length *S*.

The differential *dS* of the distance S, covered during the time *dt*, equals:

$$dS = V_x dt\,. \quad (9)$$

Considering the above-mentioned expression for *Re* (see the Problem Statement section) connecting the Reynolds number with the projection V_x, differential *dt* (6) and resistance coefficient β_D (4), formula (9) for *dS* was transformed into the following simpler formula (10), which connects the values of the differentials *dS* and *dRe:*

$$dS = -d^*(\hat{S}\sqrt{1 + A\,\mathrm{Re} + B\,\mathrm{Re}^2})d\,\mathrm{Re}\,, \quad (10)$$

where $\hat{S} = (\rho_p / 18\rho_m\sqrt{B})$. During integration of (10) from Re_i to Re_f, the following expression (11), which allows at known values $d^*, \hat{S}, Re_i$ and Re_f determining the traveled path length *S*, was obtained:

$$\exp(S/d^* \cdot \hat{S}) = \left[L(\mathrm{Re}_i)/L(\mathrm{Re}_f)\right], \quad (11)$$

where $\hat{S} = (\rho_p / 18\rho_m\sqrt{B})$,

$$L(\mathrm{Re}) = \left[2\sqrt{B(1 + A\,\mathrm{Re} + B\,\mathrm{Re}^2)} + 2B\,\mathrm{Re} + A\right].$$

For instance, the length *S* traveled by particles until they come to a stop (stopping distance) can be found using formula (11) by substituting Re_f = 0 into it. In this case the formula for S (11) transforms into the following simpler formula (12):

$$\exp(S/d^* \cdot \hat{S}) = \left[L(\mathrm{Re}_i)/(2\sqrt{B} + A)\right], \quad (12)$$

The conducted analysis of the formula for the stopping distance S (12) showed that at initial numbers Re_i <2 the dependence of the stopping distance on the number Re_i is close to linear. And at big initial Reynolds numbers Re_i, it is the dependence of the stopping distance S on $\ln \mathrm{Re}_i$ that is close to linear. It has also been established that the stopping distances S dependent on the initial numbers Re_i and particle diameters d^*, can be significant.

Table 3. The dependence of the stopping distances S on the Re_i numbers and particle diameters d^*.

Re	d^* = 0.015 m	d^* = 0.01 m	d^* = 0.005 m	d^* = 0.002 m
	S, m	S, m	S, m	S, m
1000	252	168	84	33.6
800	223	148	74	29.7
600	198.8	132.5	66.3	26.5
400	162.5	108.3	54.2	21.7
200	112.3	74.9	37.4	14.9
100	75.3	50.2	25.1	10.04
50	48.9	32.6	16.3	6.52
10	15.4	10.3	5.13	2.05
5	8.67	5.78	2.89	1.16
1	1.98	1.32	0.66	0.265

The stopping distances S obtained at the air temperature $t^0 = 20^0 C$ and particle density $\rho_P = 3г/см^3$ are presented in Table 3. The evaluations were performed using formula (12) for the length S, with consideration of the variable $L(\mathrm{Re}_i)$.

It should also be noted that formulas (11) and (12) for the length S after taking the logarithm acquire the following simplest form (13) and (14), respectively:

$$S = d^* \cdot \hat{S} \ln\left[L(\mathrm{Re}_i)/L(\mathrm{Re}_f)\right], \quad (13)$$

$$S = d^* \cdot \hat{S} \ln\left[L(\mathrm{Re}_i)/\left(2\sqrt{B} + A\right)\right] \quad (14)$$

3.Conclusion

In the article, mathematical modeling of the process of free motion in a single-component gas of a large solid spherical aerosol particle is carried out taking into account inertia, particle radius, resistance force of the carrier gaseous medium to its movement. The formulas found in this process of solving the equations of gas dynamics allow for the given radius, density, initial and final Reynolds numbers of the particles to find the time of its transition to final states, as well as the distances covered by the particle, including the braking distance. Using these formulas, data were found Re_i = 1000 (see Table 2) as a function of time t of the final Reynolds numbers of steel particles in air. These data, in particular, show that the time t can strongly depend on the particle diameter.

Assessment of the distance traveled and the braking distances of aerosol particles showed that they can be significant at large initial speeds and Reynolds numbers and can reach several hundred meters at large particle diameters (see Table 3). This circumstance must be taken into account using aerosol particles in practical applications

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