

# Parallel Numerical Simulation of the Magnetic Moment Reversal within the $\phi_0$ -Josephson Junction Spintronic Model

Stefani Panayotova<sup>1,2,\*</sup>, Maxim Bashashin<sup>3,4,\*\*</sup>, Elena Zemlyanaya<sup>3,4</sup>, Pavlina Atanasova<sup>1,3</sup>, Yury Shukrinov<sup>3,4</sup>, and Ilhom Rahmonov<sup>3,5</sup>

<sup>1</sup>University of Plovdiv Paisii Hilendarski, Tzar Asen 24, 4000 Plovdiv, Bulgaria

<sup>2</sup>Department of Physics, Sofia University, J. Bourchier Blvd. 5, 1164 Sofia, Bulgaria

<sup>3</sup>Joint Institute of Nuclear Research, 141980 Dubna, Moscow Region, Russia

<sup>4</sup>Dubna State University, 141980 Dubna, Russia

<sup>5</sup>Umarov Physical Technical Institute, TAS, 734063 Dushanbe, Tajikistan

**Abstract.** The  $\phi_0$ -Josephson junction model with a coupling between the magnetic moment and the Josephson current in the “superconductor–ferromagnet–superconductor” system has been investigated. Numerical solution of the respective system of nonlinear differential equations is based on the two-stage Gauss–Legendre algorithm. For numerical simulation in a wide range of parameters which requires a significant computer time, a parallel MPI/C++ computer code has been developed. Results of numerical study of the magnetization effect depending on physical parameters, as well as results of methodological calculations demonstrating the efficiency of the parallel implementation, are presented. Calculations have been carried out at the Heterogeneous Platform “HybriLIT” and on the supercomputer “Govorun” of the Multifunctional Information and Computing Complex of the Laboratory of Information Technologies, JINR (Dubna).

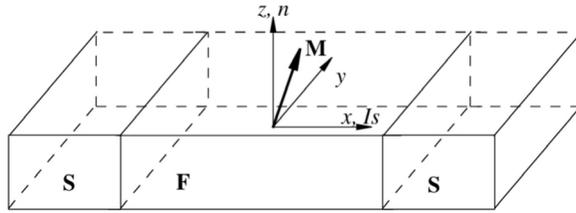
## 1 Introduction

The investigation of the Josephson junction (JJ) systems under external magnetic fields is one of attractive trends of research in the modern superconducting spintronics because of the wide perspectives of practical applications in nanoelectronics [1]. In this contribution we consider the  $\phi_0$ -JJ system where the phase difference is directly coupled with the magnetic moment in the barrier. The relevance of the study of its properties comes from the unique ability to control the magnetic properties of the barrier in Josephson nanostructures by the superconducting current, as well as, in turn, the ability to influence the Josephson current by the magnetic moment of the barrier. Of great interest for various applications is the development of efficient methods of the control of the magnetic moment reversal by means of the superconducting current pulse, as well as the control of the quantum properties of the Josephson nanostructures by superconductivity. One expects that such studies can open up opportunities for the development of new devices, e.g. of the ultra-fast cryogenic memory [2]. Experimental and theoretical studies in these areas are being intensively carried out at present in leading scientific centers of Japan, France, Spain, UK, USA and Germany.

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\*e-mail: stefani.panaiotova93@gmail.com

\*\*e-mail: bashashinmv@jinr.ru



**Figure 1.** Geometry of the system SFS JJ under consideration. Here  $S$  is the superconductor,  $F$  is the ferromagnet,  $M$  is the magnetic moment,  $I_s$  is the Josephson current.

This contribution aims at studying the magnetic reversal effect in a wide range of parameters of the  $\phi_0$ -JJ model. As in our previous works [3], [4], [5], we consider the system of the “superconductor – ferromagnet – superconductor” Josephson junction (SFS JJ), schematically shown in Fig. 1 reproduced from [4]. The three spatial components  $m_x, m_y, m_z$  denote the projections of the magnetic moment  $\mathbf{M}$  along the  $x$ -,  $y$ -,  $z$ -axes, respectively. The ferromagnetic easy axis is directed along the  $z$ -axis, which is also the direction of the spin-orbital potential gradient  $n$ . The  $m_y$  component of the magnetization is connected with the Josephson current  $I_s$ , which is directed along the  $x$ -axis.

## 2 Formulation of the problem

The dynamics of the magnetic moment and the phase difference in the SFS JJ model is described by the following system of equations [3]:

$$\begin{aligned} \frac{dm_x}{dt} &= -\frac{\omega_F}{1 + M\alpha^2} \left\{ (m_y h_z - m_z h_y) + \alpha [m_x (m_x h_x + m_y h_y + m_z h_z) - h_x] \right\} \\ \frac{dm_y}{dt} &= -\frac{\omega_F}{1 + M\alpha^2} \left\{ (m_z h_x - m_x h_z) + \alpha [m_y (m_x h_x + m_y h_y + m_z h_z) - h_y] \right\} \\ \frac{dm_z}{dt} &= -\frac{\omega_F}{1 + M\alpha^2} \left\{ (m_x h_y - m_y h_x) + \alpha [m_z (m_x h_x + m_y h_y + m_z h_z) - h_z] \right\} \\ \frac{d\varphi}{dt} &= \frac{1}{w} \left( I_{\text{pulse}}(t) - \sin(\varphi - r m_y) \right), \quad I_{\text{pulse}}(t) = \begin{cases} A_s, & t \in [t_0 - 1/2\Delta t, t_0 + 1/2\Delta t]; \\ 0, & \text{otherwise,} \end{cases} \end{aligned} \quad (1)$$

where  $t \geq 0$  is the time normalized to  $\omega_F^{-1}$ , where  $\omega_F$  is the resonant frequency of the ferromagnet. The components  $h_x(t), h_y(t), h_z(t)$  of the effective magnetic field are determined by

$$h_x(t) = 0, \quad h_y(t) = Gr \sin(\varphi(t) - r m_y(t)), \quad h_z(t) = m_z(t).$$

All quantities are converted to dimensionless form following [3]. The physical parameters of the system (1) are as follows:  $\alpha$  is the dissipation parameter,  $G$  is the ratio of the Josephson to the magnetic energies,  $r$  is the spin-orbit coupling parameter,  $w = V_F/(I_c R) = \omega_F/\omega_c$ ,  $V_F = \hbar\omega_F/(2e)$ ,  $I_c$  is the critical current,  $R$  is the JJ resistance,  $\omega_c = 2eI_c R/\hbar$  is the characteristic frequency. The parameters  $A_s$ ,  $t_0$ , and  $\Delta t$  characterize the amplitude, the middle point and the duration of the electric current pulse, respectively. The unknown functions in (1) are the components of the magnetic moment  $m_x(t), m_y(t), m_z(t)$  and the phase difference  $\varphi(t)$ . The initial conditions for these functions are as follows:

$$m_x(0) = 0, \quad m_y(0) = 0, \quad m_z(0) = 1, \quad \varphi(0) = 0. \quad (2)$$

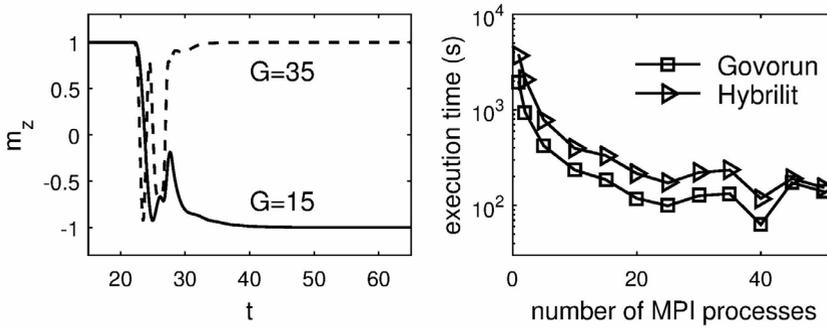
The superconducting current  $I_s(t)$  is calculated through the function  $\varphi(t)$  from

$$I_s(t) = I_c \sin(\varphi(t) - r m_y(t)), \quad t \geq 0.$$

### 3 Numerical approach and results

For the numerical solution of the system (1) of nonlinear differential equations, the implicit two-stage Gauss–Legendre scheme with fixed point iteration at each time step was constructed [5]. The algorithm is incorporated into the *Wolfram Mathematica* user interface developed in [5]. The interface allows the menu to set the necessary input physical and computational parameters, to select the computational scheme (explicit Runge–Kutta method or implicit Gauss–Legendre algorithm) and to choose a way of visualisation of the results.

However, in the case of massive simulations in a wide range of physical parameters, the C++ implementation using the MPI technique for organising the parallel computing is more effective. Using the C++/MPI code, we have made numerical simulations of the magnetization reversal characterized by the change of the  $m_z$ -component of the magnetic field from the initial +1 value to the final -1 value. The possibility of the magnetic moment reversal depending on the physical parameters of the model is of interest for possible applications in quantum electronics [3].



**Figure 2.** Left: the time dependence  $m_z$  for  $\alpha = 0.1$  at values  $G = 15$  (solid) and  $G = 35$  (dashed). Right: the execution time of calculations presented in Table 1 versus the number of parallel MPI processes (logarithmic scale).

The reversal of the magnetic moment is demonstrated in Fig. 2 (left) where the time dependence of  $m_z$  is presented for  $\alpha = 0.1$  and two  $G$  values:  $G = 15$  (solid line), and  $G = 35$  (dashed curve). The values of the other parameters in the calculation here and below are the following:  $\omega_F = 1$ ,  $r = 0.1$ ,  $A_x = 1$ ,  $t_0 = 25$ . One sees that the reversal of the magnetic moment occurs at  $G = 15$ , while at  $G = 35$  the reversal is not observed.

In order to identify the intervals of the parameters  $G$  and  $\alpha$  in which the magnetization reversal occurs, we performed the simulation with different values of  $\alpha$  and found, for each  $\alpha$  in the interval  $\alpha \in [0.01, 0.1]$  8 domains of  $G \in [G_{\min}^i, G_{\max}^i]$  ( $i = 1, 2, \dots, 8$ ) where the magnetic moment is reversed. The calculations were carried out in the  $(\alpha, G)$ -plane with  $\alpha$  varied from  $\alpha_{\min} = 0.01$  to  $\alpha_{\max} = 0.1$  with the stepsize  $\Delta\alpha = 0.01$  and with  $G$  from  $G_{\min} = 0$  to  $G_{\max} = 200$  with the stepsize  $\Delta G = 1$ . The time-increment was taken to be equal  $\Delta t = 0.01$ . In order to make sure that the magnetic reversal has been identified correctly, the numerical simulation was carried out two times for each pair of values  $\alpha$  and  $G$ : up from  $t = 0$  to  $T_{\max} = 1000$  and up from  $t = 0$  to  $T_{\max} = 2000$ . In both cases, the reversal occurrence was determined by checking the condition  $|m_z(T_{\max}) + 1| < \varepsilon$ , where  $\varepsilon = 0.0001$ . Also, the condition  $(m_x^2 + m_y^2 + m_z^2)^{1/2} = 1$  was verified to confirm the stability of the computational scheme. The reversal domains determined in simulations with  $T_{\max} = 1000$  and  $T_{\max} = 2000$  are the same and they are presented in Table 1. It follows from our analysis that the occurrence of magnetic moment reversals under the change of  $G$  is periodic: the alternation of intervals  $G_{\min}^i \leq G \leq G_{\max}^i$  with a reversal and intervals  $G_{\max}^i < G < G_{\min}^{i+1}$  without a reversal is observed as  $G$  increases.

**Table 1.**  $G$ -intervals where the magnetic moment is reversed, for different values of  $\alpha$ . The lines show the intervals  $G \in [G_{\min}^i, G_{\max}^i]$ ,  $i = 1, 2, \dots, 8$  in which the magnetic reversal is found to occur at given  $\alpha$ . The simulations have been done with  $\omega_F = 1$ ,  $r = 0.1$ ,  $A_s = 1$ ,  $t_0 = 25$ .

$\alpha$	1	2	3	4	5	6	7	8
0.01	[9,19]	[36,47]	[58,68]	[80,91]	[108,119]	[130,140]	[152,167]	[180,191]
0.02	[8,19]	[36,47]	[59,69]	[81,96]	[109,120]	[135,147]	[160,170]	[182,196]
0.03	[8,19]	[37,48]	[60,70]	[83,98]	[110,122]	[138,148]	[162,175]	[188,198]
0.04	[9,20]	[37,48]	[61,76]	[89,99]	[111,125]	[139,149]	[164,176]	[189,199]
0.05	[9,20]	[37,49]	[62,77]	[89,100]	[113,127]	[139,150]	[165,177]	[190,199]
0.06	[9,20]	[38,49]	[65,78]	[90,101]	[116,127]	[140,151]	[166,178]	[191,199]
0.07	[9,21]	[38,50]	[67,78]	[91,102]	[117,128]	[141,152]	[167,179]	[191,199]
0.08	[9,22]	[38,50]	[67,79]	[92,103]	[118,128]	[141,153]	[168,179]	[192,199]
0.09	[9,25]	[39,51]	[68,79]	[92,104]	[118,129]	[142,153]	[168,180]	[192,199]
0.10	[9,26]	[39,51]	[68,79]	[93,105]	[118,129]	[142,154]	[168,180]	[193,199]

The derivation of the results in Table 1 required about 1 hour of computer time for the sequential code. The parallel implementation provides a significant speedup of the calculations and allows one to effectively conduct massive simulations in a wide range of parameters of the model. The execution time depending on the number of MPI-processes in Fig. 2 (right) confirms the efficiency of the C++/MPI code. The calculations were performed on the cluster “HybriLIT” and on the supercomputer “Govorun” of the Multifunctional Information and Computing Complex of the JINR Laboratory of Information Technologies (Dubna). In both computers, the execution time decreases according to a hyperbolic law while the number of MPI-processes increases to 25. With a further increase in the number of MPI-processes, the growing of a speedup slows down and the execution time curves oscillate. The minimal execution time and the maximal speedup (about 30 times) is obtained at 40 MPI-processes in both “Govorun” and “HybriLIT” clusters. The corresponding value of effectiveness calculated as the relation of a speedup to the number of MPI-processes is about 75 %.

## 4 Conclusions

The C++/MPI implementation for a parallel simulation of dynamics of the SFS JJ system has been developed that provides a high performance study of the model in a wide range of parameters. A detailed study of the magnetization reversal effect and other properties of the SFS JJ system in various regimes was performed. Preliminary results have been presented in [6].

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