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Short introduction to the physics of neutron stars

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Abstract. Here we briefly review several aspects of the physics of neutron stars. In particular, we shortly describe the different types of telescopes employed in their observation, the many astrophysical manifestations of these objects and the measurement of observables such as their masses and radii. A brief summary of their composition, structure equations and equation of state is also presented.

1 Introduction

What is a neutron star?, is a question whose answer is not unique and depends on who is asked. Astronomers would probably answer that these objects are very little stars observed as radio pulsars or sources of X- and γ-rays, whereas particle physicists would say that they are neutrino sources (mainly when they are born) and probably the only places in the Universe where deconfined quark matter may be really abundant. Cosmologists would reply that neutron stars are almost black holes, in the sense that they are very compact objects but not as compact as black holes. Finally, for nuclear physicists they are the biggest neutron-rich nuclei of the Universe with mass numbers of the order $A \sim 10^{56} - 10^{57}$, radii of about $10 - 12$ km, and masses in the range $M \sim 1 - 2M_\odot$ (being $M_\odot \simeq 2 \times 10^{33}$ g the mass of the Sun). Everybody, however, agrees on the fact that neutron stars are a type of stellar compact remnant that can result from the gravitational collapse of an ordinary star with a mass in the range $8 - 25M_\odot$ during a Type-II, Ib or Ic supernova event.

Neutron stars are excellent observatories to test our present knowledge of the fundamental properties of matter under the influence of strong gravitational and magnetic fields at extreme conditions of density, isospin asymmetry and temperature. They offer an interesting interplay between nuclear processes and astrophysical observables. Their study constitutes nowadays one of the most fascinating fields of research that requires expertise from different disciplines like general relativity, high-energy physics, nuclear and hadronic physics, neutrino physics, quantum chromodynamics (QCD), superfluid hydrodynamics, plasma physics or even solid state physics. Enormous theoretical advances has been done in understanding the extreme and unique properties of these exotic objects. Major advances have been also achieved in their observation. The new generation of space X-ray and γ-ray observatories are enabling new observations and breakthrough discoveries (*e.g.,* kHz quasi-periodic oscillations, bursting millisecond pulsars, half-day long X-ray superbursts). The thermal emission from isolated neutron stars provides important information on their cooling history and it allows the determination of their radii. At the same time, improvements in radio telescopes and interferometric techniques have increased the number of known binary pulsars, allowing for extremely precise neutron star mass measurements and tests of general relativity. A large multinational effort has taken place in the last decade to build a new generation of graviational wave detectors which has been recenlty rewarded with the exciting observation of the first signal from the merger of two neutron stars [1].

This work is the result of a lecture given at the 10^{th} *European Summer School on Experimental Nuclear Astrophysics* held at the Laboratori Nazionali del Sud (Catania) in June 2019. To review in a complete and detailed way all the physics of neutron stars in a single lecture is basically an impossible task and, therefore, our scope here is just simply to present a brush-stroke on this topic. The interested reader can find several excellent books and many reviews that comprehensively cover all different aspects of this facinating field [2–6].

The manuscript follows the scheme of the lecture and it is organized in the following way. A brief historical introduction on the idea of neutron stars and the further theoretical and observational developments that followed this idea is presented in Sect. 2. In Sect. 3 we discuss different aspects on the observation of neutron stars whereas their composition, structure equations and equation of state (EoS) are reviewed in Sect. 4. Finally, a summary and few concluding remarks are shortly presented in Sect. 5.

2 Brief historical overview

The possible existence of neutron stars was proposed by Baade and Zwicky [7] in 1934 only two years after the discovery of the neutron by Chadwick [8]. Baade and Zwicky pointed out that a massive object consisting mainly of neutrons at very high density would be much more gravitationally bound than ordinary stars. They also suggested that such objects could be formed in supernova explosions. In 1939, Tolman [9] and, independently, Oppenheimer and Volkoff [10] derived the equations that describe the structure of a static star with spherical symmetry in general relativity, and performed the first theoretical calculation of the equilibrium conditions of neutron stars and their properties assuming an ideal gas of neutrons at high density.

The idea that neutron star cores in massive normal stars might be a source of stellar energy focused the work in neutron stars at that time. However, when the details of thermonuclear fusion became understood this motivation faded. As a consequence, neutron stars were gradually being ignored by the astronomical community for the next 30 years. A reason often given to neglect the neutron star idea was that because of their small area, their residual thermal radiation would be too faint to be observed at astronomical distances with optical telescopes in comparison with ordinary stars. Nevertheless the situation changed in 1967 when the first radio pulsar, named PSR B1919+211, was discovered by Bell and Hewish [11]. The identification of pulsars with neutron stars, however, was not immediately obvious to most astrophysicists. The first argument that observed pulsars were in fact rotating neutron stars with strong surface magnetic fields of the order of $\sim 10^{12}$ G was put forward by Gold [12]. He pointed out that such objects could explain many of the observed features of pulsars, such as, *e.g.*, the remarkable stability of the pulse period. Gold predicted a small increase in the period as rotational energy is lost due to radiation. Shortly after, this was confirmed when a slowdown of the Crab pulsar was discovered. Because of this success and the failure of other models, pulsars were (and are) thought to be highly magnetized rotating neutron stars.

Since 1968, there has been much theoretical work to understand the properties of neutron stars. This was further stimulated by the discovery of pulsating compact X-ray sources ("X-ray pulsars") by the UHURU satellite in 1971. These sources are believed to come from a neutron star in a close binary system which is accreting matter from its ordinary companion star. The evidence for the formation of neutron stars in supernova explosions was provided by the simultaneous discoveries of the Crab and the Vela pulsar in the late fall of 1968, both of which are located in supernova remnants, confirming the prediction of Baade and Zwicky.

The Crab nebula, for instance, is in fact the remnant of the historical supernova explosion observed by Chinese astronomers in 1054 A.D.

A further step in the history of neutron star observation was done in 1974 when Hulse and Taylor [13] discovered the first binary pulsar PSR J1913+16, known popularly simply as the Hulse-Taylor pulsar This system is formed by two neutron stars orbiting around their common center of mass. During the 1980s, 1990s and 2000s several satellites with on board X-ray and γ-ray telescopes devoted to the observation of neutron stars have been launched and more will be launched in the future.

Neutron star history still reserves us many surprises, being the last one the very recent first direct detection of gravitational waves from the merger of two neutron stars [1].

3 Observation of neutron stars

Neutron stars are observed in all bands of the electromagnetic spectrum: radio, infrared, optical, ultraviolet, X-ray and γ-ray. Their observation requires different types of ground-based and on-board telescopes. Radio observations are carried out with ground-based antennas located in different places of the world like, for instance, the *Arecibo radio telescope* in Puerto Rico, the *Green Bank Observatory* in West Virginia, or the *Nançay decimetric radio telescope* in France. Large ground-based telescopes like the *Very Large Telescope* (VLT) in the Atacama desert in Chile can be used to perform observations in the near infrared and the optical bands. Ultraviolet and optical observations can be also performed with the help of the *Hubble-Space Telescope* (HST). Observations in the extreme ultraviolet, X-ray and γ-ray require the use of space observatories. Examples of these observatories are: the *Chandra X-ray Observatory* (CXO), the *X-ray Multi Mirror* (XMM-Newton) and the *Rossi X-ray Timing Explorer* (RXTE) in the case of X-ray observations; and the *High Energy Transient Explorer* (HETE-2), the *International Gamma-Ray Astrophysics Laboratory* (INTEGRAL) and the *Fermi Gamma-ray Space Telescope* (FGST), in the case of γ-ray ones.

Information on the properties of neutron stars can be obtained not only from the observation of their electromagnetic radiation but also through the detection of the neutrinos emitted during the supernova explosion that signals the birth of the star. Some examples of past, present and future neutrino observatories around the world are: the under-ice telescopes AMANDA (*Antarctic Muon And Neutrino Detector Array*) and its successor the *IceCube* observatory both placed in the South Pole; the under-water projects ANTARES (*Astronomy with a Neutrino Telescope and Abyss environmental RESearch*) and the future KM3NET (*Cubic Kilometre Neutrino Telescope*) in the Mediterranean sea; or the underground observatories SNO (*Sudbury Neutrino Observatory*) located 2100 meters underground in the Vale's Creighton Mine in Canada, and the *Kamioka* observatory placed at the Mozumi Mine near the city of Hida in Japan. Gravitational waves, originated either from the oscillation modes of neutron stars or during the coalescence of two neutrons stars such as the event GW170817 recently detected by the Advanced LIGO and Advanced VIRGO collaborations [1], offer a new way of observing these objects and constitute a very valuable new source of information. In particular, constraints on the nuclear EoS can be established by comparing the precise shape of the detected gravitational wave signal, which depends on the masses of the two neutron stars and on their so-called tidal deformabilities, with the different model predictions. Multi-messenger observations of neutron star mergers can potentially provide detailed information on the properties of the merging objects such as their mass and radii, improve our present understanding of the nucleosynthesis of heavy elements in the Universe and enable test of the theory of gravity and dark matter. The interested reader is referred to Ref. [14] for a recent review on this hot and exciting topic.

Neutron stars can be observed either as isolated objects or forming binary systems together with other neutron stars, white dwarfs or ordinary (main-sequence and red giant) stars. Modern theories of binary evolution predict also the existence of binary systems formed by neutron stars and black holes, although this kind of systems have not been discovered yet. The interested reader will find, *e.g.,* in Ref. [5] a detailed description of these objects and their phenomenology.

After fifty years of observations we have collected an enormous amount of data on different neutron star observables that include: masses, radii, rotational periods, surface temperatures, gravitational redshifts, quasiperiodic oscillations, magnetic fields, glitches, timing noise and, very recently, gravitational waves. In the following lines we shortly review how two of then, masses and radii, are measured.

Neutron star masses can be inferred directly from observations of binary systems and likely also from supernova explosions. There are five orbital (or Keplerian) parameters which can be precisely measured in any binary system. These are: the orbital period (P_b), the projection of the pulsar's semimajor axis on the line of sight ($x \equiv a_1 \sin i/c$, where i is inclination of the orbit), the eccentricity of the orbit (e), and the time (T_0) and longitude (ω_0) of the periastron. Using Kepler's Third Law, these parameters can be related to the masses of the neutron star (M_p) and its companion (M_c) though the so-called mass function

$$f(M_p, M_c, i) = \frac{(M_c \sin i)^3}{(M_p + M_c)^2} = \frac{P_b v_1^3}{2\pi G} , \tag{1}$$

where $v_1 = 2\pi a_1 \sin i/P_b$ is the projection of the orbital velocity of the neutron star along the line of sight. If only one mass function can be measured for a binary system, then one cannot proceed further than Eq. (1) without additional assumptions. Fortunately, deviations from the Keplerian orbit due to general relativity effects can be detected. These relativistic corrections are parametrized in terms of one or more post-Keplerian parameters. The most significant ones are: the advance of the periastron of the orbit ($\dot{\omega}$), the combined effect of variations in the transverse Doppler shift and gravitational redshift around an elliptical orbit (γ), the orbital decay due to the emission of quadrupole gravitational radiation ($\dot{P}_b$), and the range (r) and shape (s) parameters that characterizes the Shapiro time delay of the pulsar signal as it propagates through the gravitational field of its companion. These post-Kepletian parameters can be written in terms of measured quantities and the masses of the star and its companion (see *e.g.,* Ref. [15] for the specific expressions). The measurement of any two of these post-Keplerian parameters together with mass function f is sufficient to determine uniquely the masses of the two components of the system.

Neutron star radii are very difficult to measure mainly because neutron stars are very small objects and are very far away from us (*e.g.,* the closest neutron star is probably the object RX J1856.5-3754 which is about 400 light-years from Earth). Direct measurements of radii do not exist. However, a possible way to determine them is to use the thermal emission of low-mass X-ray binaries. The observed X-ray flux (F) and temperature (T), assumed to be originated from a uniform blackbody, together with a determination of the distance (D) of the star can be used to obtained its radius

$$R_\infty = \sqrt{\frac{FD^2}{\sigma T^4}\left(1 - \frac{2GM}{c^2 R}\right)} . \tag{2}$$

Here σ is the Stefan–Boltzmann constant and M the mass of the star. The major uncertainties in the measurement of the radius through Eq. (2) come from the determination of the temperature, which requires the assumption of an atmospheric model (see Sect. 4), and the estimation of the distance of the star. The analysis of present observations from quiescent low-mass X-ray binaries is, however, still controversial (see *e.g.,* Refs. [16, 17]).

4 Composition, structure and EoS

Neutron stars are supported against gravitational collapse mainly by the neutron degeneracy pressure and may have, as already mentioned, typically masses of the order of $1 - 2M_\odot$ and radii within the range $10-12$ km. Such masses and radii yield an averaged density for neutron stars of the order of $\sim 10^{14}$ g/cm^3. However, the expected densities in neutron stars span a rather wide range, and in fact the internal structure of these objects can be described by an "onion"-like structure consisting of several regions. The most external one, the *atmosphere*, is a very thin plasma layer where the observed thermal spectrum of the neutron star is formed. Its thickness varies from some tenths centimeters in hot neutron stars to a few millimeters in the cold ones. The theoretical study of neutron stars atmospheres has been carried out by many authors (see *e.g.,* Ref. [18] and references therein), although current atmosphere models, consisting of hydrogen, helium or carbon, are far from being complete. The main problems are associated with the calculation of the EoS, the ionization equilibrium and the spectral opacity of the atmospheric plasma (see Chapters 2 and 4 of Ref. [5] for a detailed review).

The following region is the *outer crust* which extends from the bottom of the *atmosphere* up to few hundred meters below. It is a solid region where heavy nuclei, mainly around the iron mass number, form a Coulomb lattice in β-equilibrium with a strongly degenerate electron gas which becomes ultrarelativistic at densities $\rho > 10^6$ g/cm^3.

Moving towards the interior of the star the increase of the density induces electron capture processes on nuclei which become more and more neutron rich. When the density reaches a value $\rho = \rho_{drip} \sim 4 \times 10^{11}$ g/cm^3 the only available levels for the neutrons are in the continuum and, thus, they start to "drip out" of the nuclei. The onset of the neutron drip defines the border between the *outer crust* and the next region, the *inner crust*.

The *inner crust* can be about one kilometer thick. The density in this region ranges from ρ_{drip} up to $\sim 0.5\rho_0$. Matter here consist of a mixture of very neutron-rich nuclei arranged in a Coulomb lattice, electrons and free neutrons which are expected to be paired in the s-wave by the nuclear residual interaction and, therefore, to form a superfluid. In addition, in this region, the competition between the nuclear and Coulomb forces makes the nuclei to lose their spherical shapes and to adopt more exotic topologies (droplets, rods, cross-rods, slabs, tubes, bubbles shapes) giving rise to what has been called "nuclear pasta" phase due to their resemblance with the Italian pasta [19].

At densities of about $\sim 10^{14}$ g/cm^3 the nuclear clusters dissolve into their constituents neutrons and protons, and one enters in the *outer core*. This region is a quantum fluid with densities in the range $0.5\rho_0 \leq \rho \leq 2\rho_0$ and a thickness of several kilometers. Matter is mainly composed of p-wave superfluid neutrons with a smaller concentration of s-wave superconducting protons and normal electrons and muons which appear as soon as their chemical potential equals that of the electrons (*i.e.,* $\mu_\mu = \mu_e$). For low-mass neutron stars, whose central densities are found to be less than $2 - 3\rho_0$, the *outer core* actually constitutes the entire core of the object.

The central densities of the more massive stars could easily reach values up to several times ρ_0. In this case, an *inner core* of several kilometers and densities in the range $\rho \geq 2\rho_0$ occupies the central region of the star. Nevertheless, the composition of this region is not well known, and it is still matter of speculation. The different hypotheses include: hyperonic matter, pion or kaon condensates, or deconfined quark matter. In the literature, these additional degrees of freedom are sometimes referred to as *exotic*, and their presence in the inner core is simply consequence of the fact that the star lowers its energy with their appearance. The possible existence of deconfined quark matter is particularly interesting because it establishes a possible link between neutron stars and QCD, the fundamental theory of the strong interaction.

In the previous paragrahs we have described what one can consider more or less the standard internal structure of a neutron star. However, theoreticians have also speculated about a special type of compact stars whose structure does not correspond with the one just described. These objects are the so-called *strange stars* and are thought to be entirely made of a mixture of deconfined up (u), down (d) and strange (s) quarks (*strange quark matter*) with perhaps a small fraction of electrons. Their possible existence is a direct consequence of the Bodmer–Witten–Terezawa hypothesis [20–22] according to which three-flavour uds quark matter in equilibrium with respect to the weak interactions could be the true ground state of strongly interacting matter rather than ^{56}Fe (*i.e.,* $E_{uds} \leq M(^{56}\mathrm{Fe})/56 \simeq 930$ MeV)).

A neutron star is one of the densest objects in the Universe, therefore, Einstein's general relativity theory is needed to determine its structure. Einstein's field equations [23, 24] for a spherical static star take the form of the familiar Tolman–Oppenheimer–Volkoff (or simply TOV) equations [9, 10] which, using units in which $G = c = 1$, read

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r) \,, \tag{3}$$

and

$$\frac{dp(r)}{dr} = -\frac{M(r)\varepsilon(r)}{r^2} \frac{\left(1 + \frac{p(r)}{\varepsilon(r)}\right)\left(1 + \frac{4\pi r^3 p(r)}{M(r)}\right)}{\left(1 - \frac{2M(r)}{r}\right)} \,. \tag{4}$$

These two equations can be easily interpreted. Think of a shell of matter of radius r and thickness dr. Equation (3) gives the mass-energy in this shell. The pressure of the matter exterior to this shell is $p(r)$ and the interior to it $p(r) + dp(r)$. The left side of Eq. (4) is the net force acting outward on the surface of the shell by the pressure difference $dp(r)$ and the first factor on the right is the attractive Newtonian force of gravity acting on the shell by the mass interior to it. The remaining factor on the right side of the equation is the exact correction for general relativity. So these equations express the balance at each r between the internal pressure as it supports the overlying material against the gravitational attraction of the mass-energy interior to r. They are just the equations of hydrostatic equilibrium in general relativity. Neutron stars are, however, rotating objects. We expect the rotation to flatten the star more or less depending on its angular velocity Ω. Spherical symmetry is thereby broken although the star maintains its axial symmetry. This symmetry breaking makes the structure equations of rotating neutron stars much more complicated than those of the non-rotating ones. Due to the lack of space, we cannot present them here and we refer the interested reader *e.g.,* to Refs. [3, 4] for the discussion of the rotating case.

The equation of state, *i.e.,* the relation between the pressure p and the energy density ε, is the manner in which matter enters the equations of stellar structure. For a given EoS, the TOV equations can be integrated from the origin with the boundary conditions $M(r) = 0$ and $p(0) = p_c$, being p_c an arbitrary value, until the pressure becomes zero. Zero pressure can support no overlying material against the gravitational attraction exerted on it from the mass within and, therefore, marks the edge of the star. The point R, where the pressure vanishes, defines the radius of the spherically symmetric star. The integration of Eq. (3) from zero up to this value R gives its gravitational mass.

The only ingredient needed to solve the structure equations of neutron stars is the equation of state of dense matter. Its determination, however, is very challenging due to the wide range of densities, temperatures and isospin asymmetries found in these objects, and it constitutes nowadays one of the main problems in nuclear astrophysics. The main difficulties are associated to our lack of a precise knowledge of the behavior of the in-medium nuclear interaction, and to the very complicated resolution of the so-called nuclear many body problem [25].

Models of the EoS in the neutron star crust are based on reliable experimental data on atomic nuclei, nucleon scattering, and the theory of strongly coupled Coulomb systems. The atomic nuclei in the outer crust are expected to be those studied in the laboratory with a maximum isospin asymmetry $\beta = (N - Z)/A \simeq 0.3$, N, Z and A being, respectively, the neutron, proton and total mass number of an atomic nucleus. In the inner crust nuclei are very neutron rich. Such nuclei, however, do not exist in laboratory because they are beyond the neutron drip line under terrestial conditions. Consequently, our knowledge of the properties of matter under the density and isospin asymmetry conditions characteristic for the inner crust ($10^{11} \leq \rho \leq 10^{14}$ g/cm^3 and $0.3 \leq \beta \leq 0.8$) relies on theoretical models.

At densities $\sim 10^{14}$ g/cm^3 matter becomes a uniform quantum fluid of neutron, protons and electrons. The EoS in the outer core of the neutron star can be calculated in a rather reliable way using models and methods of the nuclear many-body theory which have been applied with some success for the microscopic description of ordinary nuclear structure. However, the reliability of these models and methods decreases when density increases and one enters the inner core region where the true composition of matter is unknown. Theoretical calculations of the nuclear EoS at such extreme densities can be tested exclusively by neutron star observations.

5 Summary and concluding remarks

This work is just a very short overview on the physics of neutron stars where we have tried to present the most remarkable observational and theoretical aspects of this field. Our main intention was to catch the attention on this fascinating topic of the new generation of young students and early-stage researches that attended this school, and motivate them to perform, by their own, more detailed studies. If we have achieved this goal we will feel fully rewarded.

The beginning of the twenty first century has been particularly generous for the physics of neutron stars. Satellite-based telescopes in different frequency bands are providing us an increible wealth of new observational data. New classes of neutron stars have been discovered. The recent observation of gravitational waves originated from the merger of two neutron stars opens, as alreay said, a new era in the observation of neutron stars from which many surprises are expected. Further surprises are also expected from the new generation of ground-based radio telescope arrays. Last but not least, the new generation of exotic beam facilities in France (SPIRAL), Germany (FAIR), Japan (RIKEN, J-PARC), USA (RIA) or the EU (EURISOL) will allow experimental studies of very exotic nuclei which will have a direct impact on the modeling of neutron stars.

The study of neutron stars is probably one of the most interdisciplinary fields in physics. Nowadays, it is becoming more and more clear that the only way to unveil the mysteries of neutron stars requieres the strong interplay and collaboration of observers with theoretitians from different areas of physics. Only with the common effort of different communities it will be possible to reach a coherent description and understanding of these fascinating objects.

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