The calculation of astrophysical S factors for E1 and E2 capture data of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction at 300 keV

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Abstract. The astrophysical S factors of E1 and E2 capture data of $^{12}\text{C}(\alpha,\gamma)$ reaction at 300 keV have been calculated based on our recent determined ANC values of 6.92 MeV and 7.12 MeV states of $^{16}\text{O}$ using R-matrix. The S-factors are extrapolated into the range of burning temperature and the S-factors at 300 keV are $S_{E1} = (86 \pm 34)$ keV b and $S_{E2} = (60 \pm 33)$ keV b.

1 Introduction

The most two important thermonuclear reactions by which He is decreased are the $3\alpha \rightarrow ^{12}\text{C}$ and $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reactions in the He-burning stars. At the end of the He-burning phase, the rate of the $3\alpha \rightarrow ^{12}\text{C}$ and $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reactions determines the $^{12}\text{C}/^{16}\text{O}$ abundance ratio. This ratio greatly effects the subsequence nucleo-synthesis in massive stars and final fate of the stars. Due to very low cross-section at astrophysical energy of the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction [1,2,3], large uncertainty arise in the rate of this capture reaction. At 300 keV the cross-section of this reaction is about $10^{-17}$ b and therefore it is very difficult or almost impossible to measure with reasonable accuracy with presently available technique. There are several direct measurements of the $^{12}\text{C}(\alpha,\gamma)$ reaction but the lowest energy measured is 1 MeV [4]. As such, R-matrix extrapolation of the cross-section measured at higher energy to the Gamow energy is a powerful way to solve this problem. In this case the extrapolation is very difficult due to the presence of large uncertainty in the measured $S_{\alpha}$ and $\gamma_\alpha^2$ of the two sub-threshold states 6.92 MeV and 7.12 MeV of $^{16}\text{O}$. R-matrix extrapolation of the E2 and E1 capture data is complicated by the lack of knowledge of the cluster structure of the 6.92 MeV ($2^+$) and 7.12 MeV ($1^-$) states of $^{16}\text{O}$. The Asymptotic Normalization Co-efficient (ANC) of these two states are important inputs to R-matrix extrapolation of the $^{12}\text{C}(\alpha,\gamma)$ reaction at 300 keV where direct measurements could not be performed. Alpha transfer measurements have been usually used to determined the ANC of sub-threshold states that influence capture reaction cross-section. In our previous work, we have extracted the ANC values of the sub-threshold states 6.92 MeV and 7.12 MeV of $^{16}\text{O}$ using a alpha transfer reaction $^{12}\text{C}(^{6}\text{Li},d)$ data [5,6] at sub-coulomb energies.

In this paper we report a study of R-matrix analysis of the E1 and E2 capture data with the ANC of the 6.92 MeV and 7.12 MeV states of $^{16}\text{O}$ determined from $^{12}\text{C}(^{6}\text{Li},d)$ alpha transfer measurements at sub-coulomb energies and calculate astrophysical S factors for E1 and E2 capture data of $^{12}\text{C}(\alpha,\gamma)$ at $T_{9}=0.2$ by R-matrix extrapolation using AZURE2 code [7].

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2 R-matrix analysis and the astrophysical S-factors calculation

The R-matrix theory is a very helpful theoretical framework to calculate compound nuclear cross-sections in presence of resonances. The whole configuration space is divided into two regions, one is inside region at a certain radius where the nuclear and coulomb interactions are present and other region is outside region where only coulomb interaction is present. The amplitudes and the cross-section determines at this radius where the logarithmic derivative of the inside and outside wavefunction is matched. In this R-matrix theory we can write

\[ R = \frac{\gamma^2}{(E_r - E)} \]  

where \( \gamma^2 \) is the reduced width of states of the compound nucleus and \( E_r \) is the resonance energy. The reduced width for bound states are related to the normalization constant of the bound state wavefunction with respect to the Whittaker function at large radial seperation. The reduced width is related with the ANC through the relation

\[ \gamma^2 = \left( \frac{h^2}{8\pi^2\mu R} \right) W^2 C^2 \]  

Where W is the Whittaker function at large radial seperation and \( C^2 \) is the ANC of the bound state wavefunction.

Figure 1. shows the \( S_{E1} \) curve with the \(^{12}\text{C}(\alpha,\gamma)\) E1 capture data \([4]\) have been described by a three level R-matrix fit (\( E_r = -45.1 \) and 2400 keV + "background" level) assuming a radius \( R_0 = 6.5 \) fm for the inner space with the nuclear interaction. All the resonance parameters are kept fixed except the parameters which are describing the background state in the R-matrix calculation. The ANC value of the 7.12 MeV sub-threshold state of \(^{16}\text{O}\) has fixed at the value \( 8.76 \times 10^{14} \) fm\(^{-1/2}\) which was determined from alpha transfer reaction \(^{12}\text{C}(6\text{Li},d)\) data at sub-coulomb energies \([5,6]\).

![Figure 1](image_url)

**Figure 1.** Three level R-matrix fit (solid red line) of E1 capture data of \(^{12}\text{C}(\alpha,\gamma)\) reaction \([4]\) (solid black square with error).

For E2 radiative capture process, similarly four level R-matrix fits have been performed using four \( 2^+ \) levels (6.92 MeV sub-threshold state, 9.85 MeV, 11.52 MeV and a higher background equivalent state which represents the tails of other higher lying \( 2^+ \) states) and
Figure 2. Four level R-matrix fit (solid red line) of E2 capture data of $^{12}$C($\alpha,\gamma$) reaction [4] (solid black square with error).

Table 1. Comparison of astrophysical S-factors for E1 and E2 capture data of $^{12}$C($\alpha,\gamma$) reaction at 300 keV with R-Kunz et al.

<table>
<thead>
<tr>
<th>S-factor</th>
<th>R. Kunz [4]</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{E1}$</td>
<td>(76 ± 20) keV b</td>
<td>(86 ± 34) keV b</td>
</tr>
<tr>
<td>$S_{E2}$</td>
<td>(85 ± 30) keV b</td>
<td>(60 ± 33) keV b</td>
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also all parameters are fixed except parameters of background state. The ANC value of 6.92 MeV state is 134536.24 fm$^{-1/2}$ which is taken from our previous work [5,6].

The extrapolated S-factor curves are shown in figure 1 and figure 2 into the range of burning temperature. The calculated S-factors of E1 and E2 capture data are shown in Table 1.

3 Summary and Conclusions

R-matrix analysis of the E1 and E2 capture data of $^{12}$C($\alpha,\gamma$) reaction was carried out. The fits were performed with fixed ANC values of 6.92 MeV and 7.12 MeV of $^{16}$O which are taken from alpha transfer reaction $^{12}$C($^{6}$Li,d) data at sub-Coulomb energies [5,6]. In this fitting the parameters of background state are taken as freely varying parameters, but all resonance parameters are fixed. The value of E1 and E2 S-factors at 300 keV have been extracted.

References