How do charged environments affect low-energy fusion rates?

Screening effects in laser-induced non-neutral plasmas

Salvatore Simone Perrotta[1,2,3,4,*] and *Aldo* Bonasera[2,4]

[1]Scuola Superiore di Catania, Università degli studi di Catania, Catania, Italy
[2]Cyclotron Insitute, Texas A&M University, College Station, TX, USA
[3]Dipartimento di Fisica e Astronomia, Università degli studi di Catania, Catania, Italy
[4]Laboratori Nazionali del Sud, Istituto Nazionale di Fisica Nucleare, Catania, Italy

Abstract. Globally charged environments can modify the rates of nuclear reactions at the energies relevant for nuclear fusion energy production. This issue was addressed using the screening potential approach, deriving approximate analytical results for some scenarios of interest. The developed model predicts that fusion is hindered for reactions between thermal nuclei, while an enhancement is expected for secondary reactions and cluster Coulomb explosion *beam-target* reactions.

1 Introduction

The cross-section of nuclear fusion reactions induced by charged particles at ultra-low collision energies E (few keV) depends exponentially on E, as the process is dominated by the tunneling through the Coulomb barrier. Consequently, even relatively small energy differences may induce sensible variations in the yields.

In this work, the fusion rate modifications provoked by a positively charged environment are discussed, from the theoretical point of view, for inertial confinement fusion (ICF) [1] and cluster Coulomb explosion [2, 3] plasmas, both important in the field of terrestrial nuclear fusion energy production. The model here presented is discussed in greater detail in [4, 5].

The interactions of the environment with reactants are usually accounted for in an effective manner, by treating the reaction as happening in vacuum and modifying the initial collision energy E by an appropriate *screening potential* U. Precisely, the *screened* cross-section σ_s (the one observed in the environment) is expressed in terms of the *bare* cross-section σ_b (the one for isolated nuclei) as:

$$\sigma_s(E) = \sigma_b(E + U) \tag{1}$$

U represents the total kinetic energy acquired by reactants in their center-of-mass frame through environmental interaction during the collision, and can in general be a function of E itself.

Given a system of particles obeying some energy distribution $\Phi(E)$, the modification in the reaction rate per particle pair, $\langle\sigma v\rangle$, due to the environmental interaction can consequently be computed. It is of interest the *rate enhancement factor*, i.e. the ratio of screened to bare

---

[*] e-mail: perrotta@lns.infn.it

reaction rate, $f_{\sigma v}$. If $\Phi(E)$ is a Maxwell-Boltzmann distribution with temperature T, the reaction is non-resonant (the astrophysical S-factor [4, eq. 1] can be approximated as constant), and U does not depend on E, then $f_{\sigma v}$ can be approximated as (see e.g. [6, eq. 7]):

$$f_{\sigma v} = \frac{\langle \sigma_s v \rangle}{\langle \sigma_b v \rangle} \approx e^{U/T} \tag{2}$$

2 Screening potential calculation scheme

The system is described as a sphere of constant and uniform mass, net charge, and isotopic abundance distribution. Let $V(r)$ be the electrostatic potential energy induced by said charge distribution inside the sphere volume, as a function of the displacement $\underline{r}$ from its center. Consider then two reactants, starting at positions $\underline{r}_+$ and $\underline{r}_-$ and ending as a fused nucleus in $\underline{r}_f$. During the collision, these particles will gain, due to the interaction with the charged environment, in the sphere rest frame, an energy:

$$\Delta E = Z_+ \left[V(r_+) - V(r_f) \right] + Z_- \left[V(r_-) - V(r_f) \right] \tag{3}$$

where $Z_\pm$ are the particles charge numbers. Part of this energy is spent to accelerate the reactants center of mass: let ΔE_{CM} be its energy gain in the sphere rest frame. The screening potential is thus:

$$U = \Delta E - \Delta E_{\mathrm{CM}} \tag{4}$$

For any given configuration of the system under study, eq. (4) may then be inserted in eq. (1) to find the corresponding screened cross-section σ_s. The observed screened reaction rate then corresponds to a proper average of σ_s over all possible system configurations. Here, an approximate result is obtained by first averaging the screening potential over all configurations (and, possibly, on the reactants energy distribution). The resulting $\langle U \rangle$ is then inserted in eq. (2) to find the rate enhancement factor.

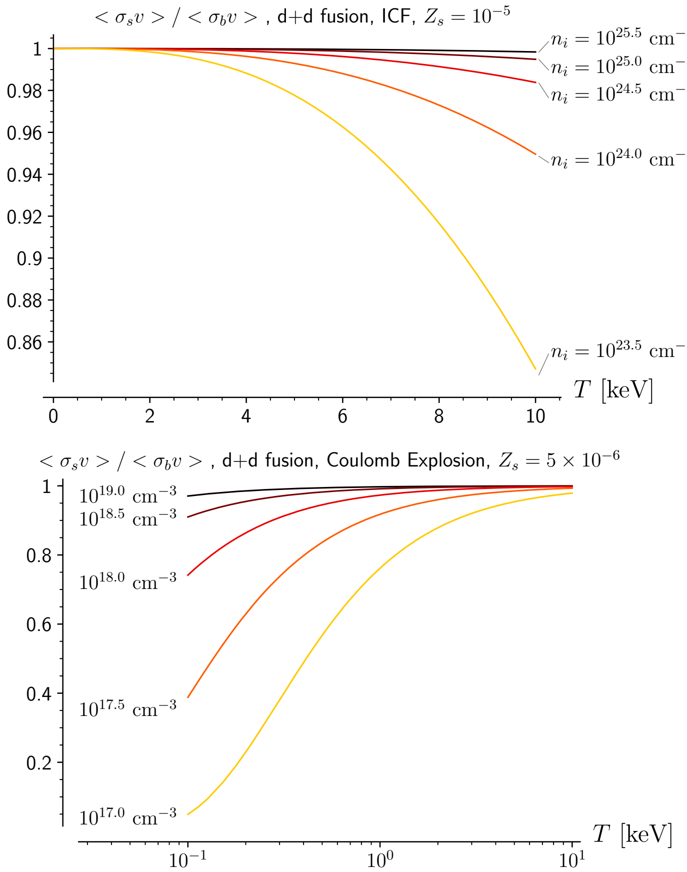
Note that this work only describes the influence of a given charge distribution on the nuclear reaction rates. The effects on the system hydrodynamics are not considered. The charge formation mechanism is not discussed, and no prediction is given for the net charge density. Furthermore, local perturbations to the charge density are ignored, thus excluding the sort of effects already found in a neutral plasma (i.e. Debye screening, see [6]). Finally, it is assumed that all reactants are fully ionized, thus excluding atomic electron screening.

3 Results

Consider a charged plasma modeled as described in section 2, with a total number density of nuclei n_i, and a net charge density of $Z_s n_i$ proton charges. For convenience, let $\tilde{\rho}_s = \frac{4}{3}\pi Z_s n_i$. The average screening potential for a pair of equal reactants with charge number Z, following a Maxwell-Boltzmann energy distribution with temperature T, due only to the plasma net charge, is found to be [4, 5]:

$$\langle U \rangle_{|m_+=m_-, Z_+=Z_-=Z} \approx -0.176 k_e q_e^2 Z \tilde{\rho}_s \frac{\lambda^2}{2} \tag{5}$$

Where $k_e q_e^2 \approx 1.44\,\mathrm{MeV\,fm}$, and λ is a suitably defined mean free path of the reactants in the environment against "significant" deflections. The estimate in [3] was adopted to define λ for cluster Coulomb explosion systems, while for inertial confinement fusion (ICF) plasmas the mean free path against a sequence of small scatterings was considered, see [4, 5]. The


$< \sigma_s v > / < \sigma_b v >$, d+d fusion, ICF, $Z_s = 10^{-5}$

$< \sigma_s v > / < \sigma_b v >$, d+d fusion, Coulomb Explosion, $Z_s = 5 \times 10^{-6}$

Figure 1. Rate enhancement factors from eq. (2), calculated using eq. (5), as a function of the reactants temperature T, for d + d fusion. Each color represents a different number density of nuclei n_i as marked by the labels. Top panel describes an ICF plasma made of only hydrogen isotopes, and bottom panel a cluster Coulomb explosion system. Top and bottom panel refer to a net charge per nucleus respectively of 10^{-5} and 5×10^{-6} proton charges.

negative sign implies that, in this case (reactions between thermal nuclei), the charged plasma hinders fusion. Inserting eq. (5) in eq. (2), the results in figure 1 were deduced.

A similar calculation can be performed assuming that one reactant is at rest with respect to the plasma sphere. This scenario approximately describes secondary reactions and, in Coulomb explosion systems, *beam-target* reactions (as defined e.g. in [2, 3]). It is then found that $\langle U \rangle = 0$: an accurate evaluation of the average screened cross-section, via eq. (1), would thus predict an enhanced fusion rate [4, 5].

In the future, the model here discussed could be improved to include Debye-like screening and, possibly, to allow a comparison of its predictions with existing experimental data and simulations. For instance, [2] reports a reduction of the d + d cross-section extracted from Coulomb explosion measurements, with respect to conventional fixed-target experiments and Trojan Horse Method [7] measurements. The mechanism here described may be sufficient to explain that result, although further studies are needed to deduce the amount of net charge required to this end.

References

[1] S. Pfalzner, *An Introduction to Inertial Confinement Fusion* (CRC Press, 2006)
[2] D. Lattuada et al., Phys. Rev. C **93**, 045808 (2016)
[3] G. Zhang et al., Phys. Lett. A **381**, 1682 (2017)
[4] S.S. Perrotta, A. Bonasera, Nuclear Physics A **989**, 168 (2019)
[5] S.S. Perrotta, honors college thesis, Scuola Superiore di Catania (2019)
[6] E.E. Salpeter, Aust. J. Phys. **7**, 373 (1954)
[7] A. Tumino et al., The Astrophysical Journal **785**, 96 (2014)