Asymptotic normalization coefficient for $\alpha + d \rightarrow ^6\text{Li}$ from the peripheral direct capture $d(\alpha, \gamma)^6\text{Li}$ reaction and the astrophysical $S$ factor at Big Bang energies

K.I. Tursunmahatov$^{1, *}$, R. Yarmukhamedov$^{2,**}$, and S.B. Igamov$^{2,***}$

$^1$Department of Physics and Mathematics of Gulistan State University, 120100 Gulistan, Uzbekistan
$^2$Institute of Nuclear Physics, 100214 Tashkent, Uzbekistan

Abstract. The results of the analysis of the new experimental astrophysical $S$ factors $S_{24}^{\text{exp}}(E)$ [D. Trezzi, et al., Astropart. Phys. 89, 57 (2017)] and those measured earlier [R. G. Robertson, et al., Phys. Rev. Lett. 47, 1867 (1981)] for the nuclear-astrophysical $d(\alpha, \gamma)^6\text{Li}$ reaction directly measured at extremely low energies $E$, are presented. New estimates and their uncertainties have been obtained for values of the asymptotic normalization coefficient for $\alpha + d \rightarrow ^6\text{Li}$ and for the direct astrophysical $S$ factors at Big Bang energies.

1 Introduction

At present, the nuclear-astrophysical radiative capture reaction

$$d + \alpha \rightarrow ^6\text{Li} + \gamma$$

is of great interest due to the so-called second lithium puzzle, which is firstly associated with an existence of three order of the discrepancy between the observational and calculated ratios $^6\text{Li}/^7\text{Li}$ [1, 2]. Secondly, it is considered as the only source of the $^6\text{Li}$ production in the standard Big Bang model [3]. But, the amount of the $^6\text{Li}$ production in the Big Bang via the reaction (1) depends in turn on the nuclear cross sections (or respective astrophysical $S$ factors $S_{24}(E)$) at Big Bang energies ($30 \leq E \leq 400$ keV). Despite the impressive improvements in our understanding of the reaction (1) made in the past decades some ambiguities connected with both the extrapolation of the measured astrophysical $S$ factors $S_{24}^{\text{exp}}(E)$ to the energy region ($E \leq 100$ keV) and the theoretical predictions for $S_{24}(E)$ still exist (see, for example, Refs. [1, 4] and references therein) and they may influence the predictions of the Big Bang model [3].

In the present work, the results of the analysis of the experimental astrophysical $S$ factors (ASF) $S_{24}^{\text{exp}}(E)$ [4–6] for the reaction (1) directly measured at extremely low energies $E$ are presented.

*e-mail: tursunmahatovqi@mail.ru
**e-mail: rakhim@inp.uz
***e-mail: igamov@inp.uz
2 Analysis and results

The analysis of the $S_{24}^{\exp}(E)$ is performed within the modified two-body potential method [8]. The method involves two additional conditions, which verify the peripheral character of the direct radiative capture reaction (1). They are conditioned by $R(E, b_0) = \text{const}$ (denoted by the condition $A$ below) for arbitrary variation of the free model parameter $b_0$ for each fixed experimental value of the relative kinetic energy ($E$) of the colliding particles and by the fact that the ratio $C^2_0 = S_{24}^{\exp}(E)/R(E, b_0)$ (denoted by the condition $B$ below) must not depend neither from $b_0$ and nor from the energy $E$ for each experimental point of $E = 93, 120$ and $133$ keV [7] as well as $E = 993$ and $1315$ keV [5], where $R(E, b_0) = S_{24}^{(sp)}(E); b_0)/b_0^2$. Here $S_{24}^{(sp)}$ is a single-particle ASF [9] and $b_0$ is the amplitude of the tail of the radial $s$-component shell-model wave function of the bound $^6\text{Li}$ $[(\alpha + d)]$ state, which is calculated using the Schrödinger equation with the phenomenological Woods-Saxon potential with the geometric parameters $(r_0$ and a diffuseness $a)$, and $C_0$ is the $s$ wave component of the asymptotic normalization coefficient (ANC) for $\alpha + d \rightarrow ^6\text{Li}$, which determines the amplitude of the tail of the radial overlap function for the six-nucleon $^6\text{Li}$ wave function in the $(\alpha + d)$ channel [10]. The value of $b_0$ strongly changes as a function $(r_0, a)$ pair, i.e., $b_0 = b_0(r_0, a)$. Fulfilment of the conditions $A$ and $B$, firstly, it makes possible to remove the model dependence of the calculated direct ASF ($S_{24}^{\text{DC}(E)})$) on the geometric parameters of the adopted potential above both for the two-body bound $(\alpha + d)$ state and the $da$-scattering one and it must not be exceeded the experimental errors. Secondly, it allows us to determine the “indirect” measured ANC ($C_{0}^{\text{exp}}$) and its uncertainty by model-independent way. The determined ANC can then be implemented for obtaining the extrapolated values of $S_{24}(E)$ and their uncertainties within the Big Bang energy range ($30 \leq E \leq 400$ keV), including the range below $30$ keV down to zero, in a self-consistent way, using the same adopted Woods-Saxon potential both for the $(\alpha + d)$ bound state and for the $ad$-scattering one.

The real potential in the Woods-Saxon form with spin-orbital term used in [5] is taken both for the continuum state and for the bound one. We vary the geometric parameters $(r_0$ and $a)$ of the adopted Woods-Saxon potential in the physically acceptable ranges $(r_0$ in $1.13 \pm 0.37$ fm and $a$ in $0.58 \pm 0.72$ fm) with respect to the standard $(r_0 = 1.25$ fm and $a = 0.65$ fm) values. At this, it is used the procedure of the depth adjusted to fit both the binding energy for the two-body bound $(\alpha + d)$ state and the experimental phase shifts for the elastic $da$-scattering within their errors.

The calculation shows that for each the fixed experimental energy $E$ mentioned above such a choice of the limit of variation of the geometric parameters $(r_0$ and $a)$ of the adopted Woods-Saxon potential allows us to supply fulfilment of two the aforementioned conditions for the energies above with the high accuracy. Below, as an example, the $S_{24}^{(sp)}(E; b_0)$, $R_0(E; b_0)$, $Z_0$ and $C_0^2$ dependences on the single-particle ANC $b_0 = b_0(r_0, a)$) within the range $2.369 \leq b_0 \leq 2.858$ fm$^{-1/2}$ are given only for $E = 93$ keV, where $Z_0^{1/2} = C_0/b_0$ and $Z_0$ the $s$ wave spectroscopic factor for the $^6\text{Li}$ nucleus in the $(d + \alpha)$ configuration [10]. They change within the ranges of $2.82 \times 10^{-6} \leq S_{24}^{(sp)} \leq 6.08 \times 10^{-6}$ keV$^{-1}$, $0.66 \leq Z_0 \leq 0.96$, $4.85 \times 10^{-7} \leq R_0 \leq 5.15 \times 10^{-7}$ keV$^{-1}$, and $5.24 \leq C_0^2 \leq 5.57$ fm$^{-1}$. Here the experimental astrophysical $S$ factor ($S_{24}^{\text{exp}}(E)$) at $E = 93$ keV is taken instead of the $S_{24}(E)$. It is seen that the change of the $S_{24}^{(sp)}(E; b_0$) and $Z_0$ within the interval above for $b_0$ is rather noticeably (about 1.45 times), whereas that for the $R_0(E; b_0$) function and $C_0^2$ is rather weak (about 6.06 times). The same dependence is also observed at the other energies above. Besides, for the $^2S_1$, $^3P_1 (j = 0, 1)$ and $^3D_1 (j = 2, 3)$ waves, the phase shifts of the elastic $da$-scattering calculated by variation of the $r_0$ and $a$ parameters within the intervals above are changed within the uncertainty of about $\sim 2-3\%$ and show a rather good agreement with the experimental data (see, Ref. [11] and references therein). As seen from here, the conditions $A$ and $B$ are
Figure 1. The square of the ANC \((C_0^{\text{exp}})^2\) for \(\alpha + d \rightarrow ^6\text{Li}\) obtained in the present work at the experimental energies \(E\) above (a), their weighted mean (the solid line) and its uncertainty (the width of the band) as well as the experimental and calculated astrophysical \(S\) factors (b) for the \(d(\alpha, \gamma)^6\text{Li}\) reaction. The caption of (b) is given in detail in the text.

fulfilled for the reaction (1) with a high accuracy. It follows from here that the reaction (1) at sufficiently low energies is strongly peripheral and the contribution of the nuclear interior \(\alpha\alpha\) interaction region to the calculated astrophysical \(S\) factors is up to 3\%. The calculation shows that the contribution of the \(M1\) transition to the calculated astrophysical \(S\) factor is negligible small (\(\sim 1\textendash2\%)\), whereas, that of the \(E1\)-and \(E2\)-components are important.
Table 1. The ANC square \((C_0^2)\) for \(\alpha + d \rightarrow ^6\text{Li}\) and the modulus square of the respective NVC for the virtual decay \((|G_0|^2)\). Figures in square bracket are experimental and theoretical uncertainty, respectively, whereas, those in bracket are the weighted means and their total uncertainties.

| Method and the reaction | \(C_0^2, (\text{fm}^{-1})\) | \(|G_0|^2, (\text{fm})\) | Refs. |
|-------------------------|-----------------------------|---------------------|-------|
| TBPM \(d(\alpha, \gamma)^6\text{Li}\) analysis\(^1\) | 5.41[0.18; 0.12] | 0.423[0.014; 0.009] | the present work |
| TBCBM \(^{208}\text{Pb}\left( ^6\text{Li}, \alpha d \right) ^{208}\text{Pb}\) with the \(E1\)-and \(E2\)-multipoles\(^2\) | 5.50[0.46; 0.45] | 0.43[0.04; 0.04] | [12] |
| The ACPS\(^3\) of the \(da\)-scattering | 5.37±0.26 | 0.42±0.02 | [13] |
| The dispersion peripheral model with the exchange \(d{^6}\text{Li}\)-scattering | 5.24±0.77 | 0.41±0.06 | [14] |

\(^1\)The two-body potential method(TBPM); \(^2\)The three-body Coulomb breakup method(TBCBM); \(^3\)The analytical continuation for the phase shifts using the Padé-approximation.

The square of the ANC \(((C_0^\text{exp})^2)\) is defined from the condition \(B\) by using the corresponding \(S_{24}^\text{exp}(E)\) for each experimental point of the energy \(E\) mentioned above. The results are displayed in figure 1a. The uncertainty plotted for each the experimental point of the energy \(E\) is the averaged squared error involving the experimental errors of \(S_{24}^\text{exp}(E)\) and the aforesaid uncertainty in the \(R_0(E; b_0)\). The result of the weighted mean and its weighted uncertainty for the \((C_0^\text{exp})^2\) and the respective nuclear vertex constant (NVC) \((|G_0|^2=0.7816C_0^2 (\text{fm}) [10]\) for the virtual decay \(^6\text{Li} \rightarrow \alpha + d\) jointly with those obtained by other authors are presented in table 1. Figure 1b shows the results of the calculation of the full astrophysical \(S\) factors \(S_{24}(E)\), performed within the modified \(R\)-matrix method (see, for example, Ref. [15]) for the full energy range (the solid line). In the calculation, the ANC value presented in the second line of table 1 was used to fix the contribution of the external (direct) amplitude in the full \(R\)-matrix amplitude. Besides, the experimental channel \(\alpha\) width was taken equal to \(\Gamma^\alpha=24\ \text{keV}\) recommended in [17], while the resonance parameter (the \(\gamma\)-ray) was considered as adjustable parameters. The channel radius \(r_c\) is chosen equal to 4.0 fm, which a provides the minimum of \(\chi^2\). In figure 1b, the experimental data are taken from Refs. [7](open triangle symbols), [16](square symbols) and [12] (full circle symbols). There, the solid, dashed and dashed-dotted lines in \((b)\) are our result for the total, \(E2\) and \(E1\) components of the \(S_{24}(E)\), respectively. The width of the band corresponds to the weighted uncertainty for the squared ANC. The resonance parameter (the \(\gamma\)-ray) was found to be \(\Gamma^\gamma=4.0\times10^{-3}\ \text{eV}\), which is in excellent agreement with the experimental data \((\Gamma^\gamma=4.4\times10^{-4}\ \text{eV})\) compiled in [17]. For example, at the most effective Big Bang energy \((E=70\ \text{keV})\), our result is \(S_{24}(70\ \text{keV})=2.424\pm0.081(\text{exp})\pm0.054(\text{theor})[2.424\pm0.097(\text{total})]\ \text{MeV nb}\), which is \(1.6\sigma\) lower than that
of 2.58 MeV nb obtained in [4]. This discrepancy is associated apparently with the model assumption of $Z_0 = 1$ used in [4].

3 Conclusion

From a thorough analysis of the experimental astrophysical $S$ factors for the reaction (1) directly measured in [5, 7] at energies $E = 93–1315$ keV, including the range below 93 keV down to zero, the new estimations are obtained for the weighted mean of the ANC(NVC) for $\alpha + d \rightarrow ^6\text{Li}$ and the astrophysical $S$ factors at the Big Bang energy region, including the range below than 93 keV up to zero, with the overall uncertainty about 4% on the average.

This work has been supported in part by the Ministry of Innovations and Technologies of the Republic of Uzbekistan (grant No. HE F2-14).

References