Asymptotic normalization coefficient for $\alpha + d \rightarrow {}^6\mathrm{Li}$ from the peripheral direct capture $d(\alpha, \gamma)^6\mathrm{Li}$ reaction and the astrophysical S factor at Big Bang energies

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Abstract. The results of the analysis of the new experimental astrophysical S factors $S_{24}^{\exp}(E)$ [D. Trezzi, et al., *Astropart. Phys.* **89**, 57 (2017)] and those measured earlier [R. G. Robertson, et al., *Phys. Rev. Lett.* **47**, 1867 (1981)] for the nuclear-astrophysical $d(\alpha, \gamma)^6$Li reaction directly measured at extremely low energies E, are presented. New estimates and their uncertainties have been obtained for values of the asymptotic normalization coefficient for $\alpha + d \rightarrow {}^6$Li and for the direct astrophysical S factors at Big Bang energies.

1 Introduction

At present, the nuclear-astrophysical radiative capture reaction

$$d + \alpha \rightarrow {}^6\mathrm{Li} + \gamma \tag{1}$$

is of great interest due to the so-called second lithium puzzle, which is firstly associated with an existence of three order of the discrepancy between the observational and calculated ratios ^{6}Li/^{7}Li [1, 2]. Secondly, it is considered as the only source of the ^{6}Li production in the standard Big Bang model [3]. But, the amount of the ^{6}Li production in the Big Bang via the reaction (1) depends in turn on the nuclear cross sections (or respective astrophysical S factors $S_{24}(E)$) at Big Bang energies ($30 \lesssim E \lesssim 400$ keV). Despite the impressive improvements in our understanding of the reaction (1) made in the past decades some ambiguities connected with both the extrapolation of the measured astrophysical S factors $S_{24}^{\exp}(E)$ to the energy region ($E \lesssim 100$ keV) and the theoretical predictions for $S_{24}(E)$ still exist (see, for example, Refs. [1, 4] and references therein) and they may influence the predictions of the Big Bang model [3].

In the present work, the results of the analysis of the experimental astrophysical S factors (ASF) $S_{24}^{\exp}(E)$ [4–6] for the reaction (1) directly measured at extremely low energies E are presented.

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2 Analysis and results

The analysis of the $S^{\rm exp}_{24}(E)$ is performed within the modified two-body potential method [8]. The method involves two additional conditions, which verify the peripheral character of the direct radiative capture reaction (1). They are conditioned by $\mathcal{R}(E,\, b_0)$= const (denoted by the condition $\mathcal{A}$ below) for arbitrary variation of the free model parameter b_0 for each fixed experimental value of the relative kinetic energy (E) of the colliding particles and by the fact that the ratio $C_0^2= S^{\rm exp}_{24}(E)/\mathcal{R}(E,\, b_0)$ (denoted by the condition $\mathcal{B}$ below) must not depend neither from b_0 and nor from the energy E for each experimental point of E= 93, 120 and 133 keV [7] as well as E= 993 and 1315 keV [5], where $\mathcal{R}(E,\, b_0)=S^{(sp)}_{24}(E_i;\, b_0)/b_0^2$. Here $S^{(sp)}_{24}$ is a single-particle ASF [9] and b_0 is the amplitude of the tail of the radial s-component shell-model wave function of the bound ^{6}Li [=($\alpha + d$)] state, which is calculated using the Schrödinger equation with the phenomenological Woods-Saxon potential with the geometric parameters (a radius r_0 and a diffuseness a), and C_0 is the s wave component of the asymptotic normalization coefficient (ANC) for $\alpha + d \to {}^6$Li, which determines the amplitude of the tail of the radial overlap function for the six-nucleonic ^{6}Li wave function in the ($\alpha + d$) channel [10]. The value of b_0 strongly changes as a function (r_0, a) pair, i.e., b_0=$b_0(r_0,\, a)$. Fulfilment of the conditions $\mathcal{A}$ and $\mathcal{B}$, firstly, it makes possible to remove the model dependence of the calculated direct ASF ($S^{\rm DC}_{24}(E)$) on the geometric parameters of the adopted potential above both for the two-body bound ($\alpha + d$) state and the $d\alpha$-scattering one and it must not be exceeded the experimental errors. Secondly, it allows us to determine the "indirect" measured ANC ($(C^{\rm exp}_0)^2$) and its uncertainty by model-independent way. The determined ANC can then be implemented for obtaining the extrapolated values of $S_{24}(E)$ and their uncertainties within the Big Bang energy range (30≤ E ≤400 keV), including the range below 30 keV down to zero, in a self-consistent way, using the same adopted Wods-Saxon potential both for the ($\alpha + d$) bound state and for the αd-scattering one.

The real potential in the Woods-Saxon form with spin-orbital term used in [5] is taken both for the continuum state and for the bound one. We vary the geometric parameters (r_0 and a) of the adopted Woods-Saxon potential in the physically acceptable ranges (r_0 in 1.13÷1.37 fm and a in 0.58÷0.72 fm) with respect to the standard (r_0= 1.25 fm and a= 0.65 fm) values. At this, it is used the procedure of the depth adjusted to fit both the binding energy for the two-body bound ($\alpha + d$) state and the experimental phase shifts for the elastic $d\alpha$-scattering within their errors.

The calcuation shows that for each the fixed experimental energy E mentoned above such a choice of the limit of variation of the geomentic parameters (r_0 and a) of the adopted Woods-Saxon potental allows us to supply fulfilment of two the aforementioned conditions for the energies above with the high accuracy. Below, as an example, the $S^{(sp)}_{24}(E;\, b_0)$, $\mathcal{R}_0(E;\, b_0)$, Z_0 and C_0^2 dependences on the single-particle ANC b_0[=$b_0(r_0,\, a)$] within the range 2.369≤ b_0 ≤2.858 fm$^{-1/2}$ are given only for E= 93 keV, where $Z_0^{1/2}$=C_0/b_0 and Z_0 the s wave spectroscopic factor for the ^{6}Li nucleus in the ($d + \alpha$) configuration [10]. They change within the ranges of 2.82x10^{-6} ≤ $S^{(sp)}_{24}$ ≤4.08x10^{-6} keV·b, 0.66≤ Z_0 ≤0.96, 4.85x10^{-7} ≤ $\mathcal{R}_0$ ≤5.15x10^{-7} keV·b·fm^{-1} and 5.24≤ C_0^2 ≤5.57 fm^{-1}. Here the experimental astrophysical S factor ($S^{\rm exp}_{24}(E)$) at E= 93 keV is taken instead of the $S_{24}(E)$. It is seen that the change of the $S^{(sp)}_{24}(E;\, b_0)$ and Z_0 within the interval above for b_0 is rather noticeably (about 1.45 times), whereas that for the $\mathcal{R}_0(E;\, b_0)$ function and C_0^2 is rather weak (about 1.06 times). The same dependence is also observed at the other energies above. Besides, for the $^2S_1$, $^3P_{\rm j}$ (j= 0, 1 and 2) and $^3D_{\rm j}$ (j= 2 and 3) waves, the phase shifts of the elastic $d\alpha$-scattering calculated by variation of the r_0 and a parameters within the intervals above are changed within the uncertainty of about ~2-3% and show a rather good agreement with the experimental data (see, Ref. [11] and references therein). As seen from here, the conditions $\mathcal{A}$ and $\mathcal{B}$ are

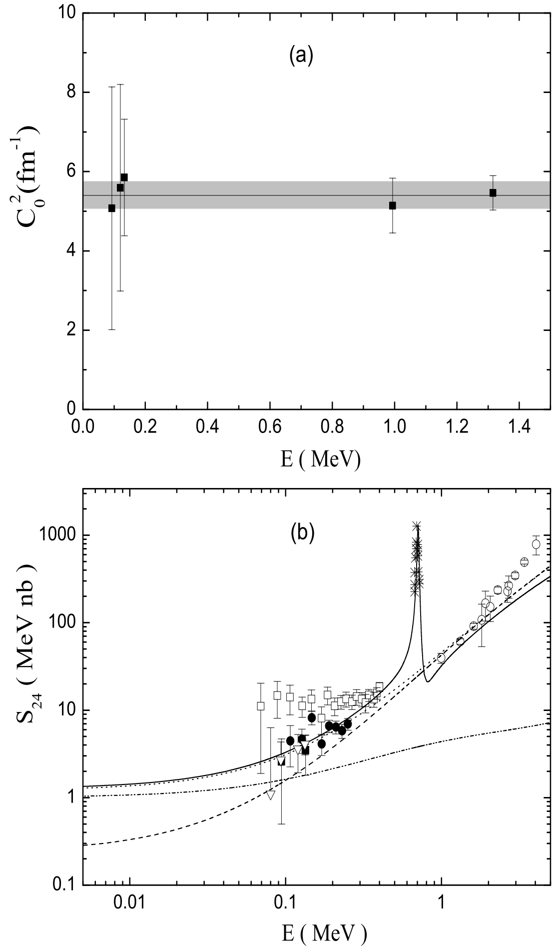

Figure 1. The square of the ANC ($(C_0^{\exp})^2$) for $\alpha + d \rightarrow {}^6$Li obtained in the present work at the experimental energies E above (a), their weighted mean (the solid line) and its uncertainty (the width of the band) as well as the experimental and calculated astrophysical S factors (b) for the $d(\alpha, \gamma)^6$Li reaction. The caption of (b) is given in detail in the text.

fulfilled for the reaction (1) with a high accuracy. It follows from here that the reaction (1) at sufficiently low energies is strongly peripheral and the contribution of the nuclear interior $d\alpha$ interaction region to the calculated astrophysical S factors is up to 3%. The calculation shows that the contribution of the $M1$ transition to the calculated astrophysical S factor is negligible small (~ 1–2%), whereas, that of the $E1$-and $E2$-components are important.

Table 1. The ANC square (C_0^2) for $\alpha + d \rightarrow {}^6$Li and the modulus square of the respective NVC for the virtual decay ($| G_0 |^2$). Figures in square bracket are experimental and theoretical uncertainty, respectively, whereas, those in bracket are the weighted means and their total uncertainties.

The method and the reaction	C_0^2, (fm^{-1})	$\|G_0\|^2$, (fm)	Refs.
TBPM the $d(\alpha,\gamma)\,{}^6$Li analysis$^{1)}$	5.41[0.18; 0.12] (5.41±0.21)	0.423[0.014; 0.009] (0.423±0.017)	the present work
TBCBM ^{208}Pb(^{6}Li, αd)^{208}Pb with the $E1$-and $E2$-multipoles$^{2)}$	5.50[0.46; 0.45] (5.50±0.64)	0.43[0.04; 0.04] (0.43±0.05)	[12]
The ACPS$^{3)}$ of the $d\alpha$-scattering	5.37±0.26	0.42±0.02	[13]
The dispersion peripheral model with the exchange $d\,{}^6$Li-scattering	5.24±0.77	0.41±0.06	[14]

$^{1)}$The two-body potential method(TBPM); $^{2)}$The three-body Coulomb breakup method(TBCBM);$^{3)}$The analytical continuation for the phase shifts using the Padé-approximation.

The square of the ANC ($(C_0^{\exp})^2$) is defined from the condition $\mathcal{B}$ by using the corresponding $S_{24}^{\exp}(E)$ for each experimental point of the energy E mentioned above. The results are displayed in figure 1a. The uncertainty plotted for each the experimental point of the energy E is the averaged squared error involving the experimental errors of the $S_{24}^{\exp}(E)$ and the aforesaid uncertainty in the $\mathcal{R}_0(E;\ b_0)$. The result of the weighted mean and its weighted uncertainty for the $(C_0^{\exp})^2$ and the respective nuclear vertex constant (NVC) ($| G_0^{\exp} |^2$=$0.7816C_0^2$ (fm) [10]) for the virtual decay ^{6}Li $\rightarrow \alpha + d$ jointly with those obtained by other authors are presented in table 1. Figure 1b shows the results of the calculation of the full astrophysical S factors $S_{24}(E)$, performed within the modified R-matrix method (see, for example, Ref. [15]) for the full energy range (the solid line). In the calculation, the ANC value presented in the second line of table 1 was used to fix the contribution of the external (direct) amplitude in the full R-matrix amplitude. Besides, the experimental channel α width was taken equal to Γ^{α}=24 keV recommended in [17], while the resonance parameter (the γ-ray) was considered as adjustable parameters. The channel radius r_c is chosen equal to 4.0 fm, which a provides the minimum of χ^2. In figure 1b, the experimental data are taken from Refs. [7](open triangle symbols), [16](square symbols) and [12] (full circle symbols). There, the solid, dashed and dashed-dotted lines in (b) are our result for the total, $E2$ and $E1$ components of the $S_{24}(E)$, respectively. The width of the band corresponds to the weighted uncertainty for the squared ANC. The resonance parameter (the γ-ray) was found to be Γ^{γ}=4.0×10^{-4} eV, which is in excellent agreement with the experimental data (Γ^{γ}=4.4×10^{-4} eV) compiled in [17]. For example, at the most effective Big Bang energy (E= 70 keV), our result is S_{24}(70 keV)= 2.424±0.081(exp)±0.054(theor)[2.424±0.097(total)] MeV nb, which is 1.6σ lower than that of 2.58 MeV nb obtained in [4]. This discrepancy is associated apparently with the model assumption of Z_0= 1 used in [4].

3 Conclusion

From a thorough analysis of the experimental astrophysical S factors for the reaction (1) directly measured in [5, 7] at energies E=93–1315 keV, including the range below 93 keV down to zero, the new estimations are obtained for the weighted mean of the ANC(NVC) for $\alpha + d \rightarrow {}^6$Li and the astrophysical S factors at the Big Bang energy region, including the range below than 93 keV up to zero, with the overall uncertainty about 4% on the average.

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