

# Cosmology with Thermal Sunyaev Zeldovich Power Spectrum and Cluster Counts: Consistency, Tensions and Prospects

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**Abstract.** In this proceeding I summarise the current status of cosmological constraints obtained from current SZ data, focusing on the Planck thermal SZ power spectrum and cluster counts. I discuss the consistency between Planck SZ data and other SZ cluster or galaxy surveys as well as the apparent discrepancy between SZ and CMB for the amplitude of matter clustering  $\sigma_8$ . Finally I discuss forecasted constraints on massive neutrinos and the X-ray mass bias in the context of future SZ power spectrum measurements.

## 1 Introduction

The thermal Sunyaev Zeldovich (tSZ) power spectrum and galaxy cluster counts detected via the SZ effect are competitive probes of cosmology. The amplitude of the tSZ power spectrum is set by the electron pressure profile projected along the line-of-sight and integrated over the halo mass function at all redshift, with heavier clusters contributing more to the large-scale power while the abundance of less massive clusters determines the amplitude at small scales. Cluster counts are simply the number of objects at different redshifts and different masses seen by a given experiment, i.e., the halo-mass function convoluted with the detection probability of the survey. Since the halo mass function depends essentially on the mean matter density and the amplitude of matter clustering, both observables are strongly sensitive to the cosmological parameters  $\Omega_m$  and  $\sigma_8$  and can be used to measure them [1–3].

In recent years, the Atacama Cosmology Telescope, the South Pole Telescope and the Planck Collaboration have carried out parameter extraction analyses based on their respective SZ datasets [4–7]. In particular, the Planck Collaboration obtained the first all-sky map of the Compton parameter  $y$  and its associated power spectrum up to multipole  $\ell \approx 10^3$  [7]. Along with the  $y$ -map power spectrum, the Planck Collaboration delivered a catalogue of 438 clusters with signal-to-noise  $\xi > 6$  and their associated redshift measured by other X-ray or optical observations [8].

Given a cosmological model, some information on the cluster masses is required in order to predict the tSZ power spectrum and cluster counts. Indeed, the halo mass function is function of the cluster masses and therefore one needs to know what are the projected pressure profile and signal-to-noise relevant to each mass. In the original analyses, the Planck Collaboration uses X-ray data from the XMM-Newton survey to obtain a universal pressure profile

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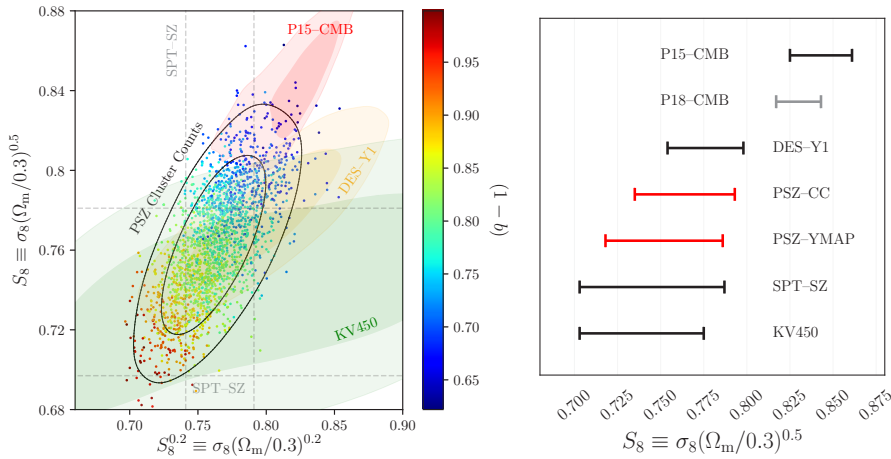
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which depends on the radius  $r_{500}$ , itself a simple function of the over-density mass  $M_{500}$  (defined with respect to the critical density of the universe), as well as a  $Y_{500} - M_{500}$  scaling relation for computing the signal-to-noise associated with a cluster of a given mass. Hence, the cluster mass entering the pressure profile and the signal-to-noise is an ‘X-ray mass’. The standard procedure to infer the mass of a cluster from X-ray observation relies on several assumptions. One of them is the assumption of hydrostatic equilibrium. The validity of these assumptions has been studied extensively with hydrodynamical simulations and the assumption of hydrostatic equilibrium is known to lead to an underestimation of the true mass by about 20% (see, e.g. [9] and reference therein). To account for this X-ray mass bias, one can use a rescaled mass  $M/B$  with  $B \approx 1.25$  in the calculations of the pressure profile and cluster signal-to-noise, where  $M$  is the ‘true mass’ that enters the halo mass function. Aside from departure from hydrostatic equilibrium there are other aspects of the analysis of X-ray data that can lead to additional mass bias such as the non-sphericity of the ICM or the X-ray temperature calibration. Therefore it is relevant to let the mass bias vary within some realistic prior range in the maximum likelihood analysis of the Planck SZ data.

Three aspects of the modelling of the tSZ power spectrum and cluster counts deserve particular attention. First, since the HMF is generally presented in terms of the overdensity mass  $M_{200m}$  (with respect to the mean matter density of the universe) it is necessary to use concentration-mass relation to translate the mass to  $M_{500}$ , which is used in the pressure profile and  $Y - M$  relation. However, different concentration-mass relations available in the literature yield a wide scatter for the tSZ power spectrum and cluster counts. A good compromise is to avoid the mass conversion by directly interpolating the HMF at  $M_{500}$ , as was done in the original Planck analysis [10, 11]. Second, the Compton  $y$  parameter fluctuations across the sky are non-Gaussian due to the Poissonian noise from the cluster distribution. Hence, in addition to the Gaussian sampling variance it is important to take into account the trispectrum term in the covariance matrix of the tSZ power spectrum [10, 12]. Third, when considering models with massive neutrinos, one should compute the Halo Mass Function (HMF) with the baryon and CDM density instead of the total matter density and with the baryon and CDM transfer functions instead of the total matter transfer functions, see [13] and reference therein.

## 2 Consistency between SZ data and tension with CMB

Taking care of these technical aspects, the best-fitting tSZ power spectrum obtained from the Planck  $y$ -map after marginalisation over the foreground nuisance parameters is consistent with the ACT and SPT measurements at  $\ell = 3000$ . Moreover, the constraints on cosmological parameters from the Planck tSZ power spectrum analysis and Planck cluster counts are nearly identical. Finally, with a Gaussian prior on the mass bias centred at 20% with a ten percent relative width, the constraints on  $S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$  are consistent with constraints from SPT-SZ, DES-Y1 and KV450 at the half- $\sigma$  level. However, the constraints on  $S_8$  from clusters and the DES-Y1 and KV450 galaxy surveys are lower than the Planck CMB constraints, in tension at the 1- $\sigma$  level. A larger mass bias than the 20% motivated by simulation results studying departure from hydrostatistical equilibrium could explain the discrepancy between Planck SZ and Planck CMB cosmological constraints. Performing a joint analysis of Planck SZ plus Planck CMB, which amounts to fixing the cosmology to Planck CMB, yields a mass bias of about 40%. One outstanding issue related to the X-ray data is the temperature calibration: ICM temperatures measured by Chandra disagree with those from XMM, with Chandra temperatures being lower than XMM temperatures, especially at large masses. Assuming Chandra temperature to be the reference would effectively be equivalent to a mass dependent X-ray mass bias with  $B(M) \approx 20\%$  for the heaviest clusters of the Planck sample, alleviating the tension between Planck SZ and CMB. Another effect that could contribute to alleviating



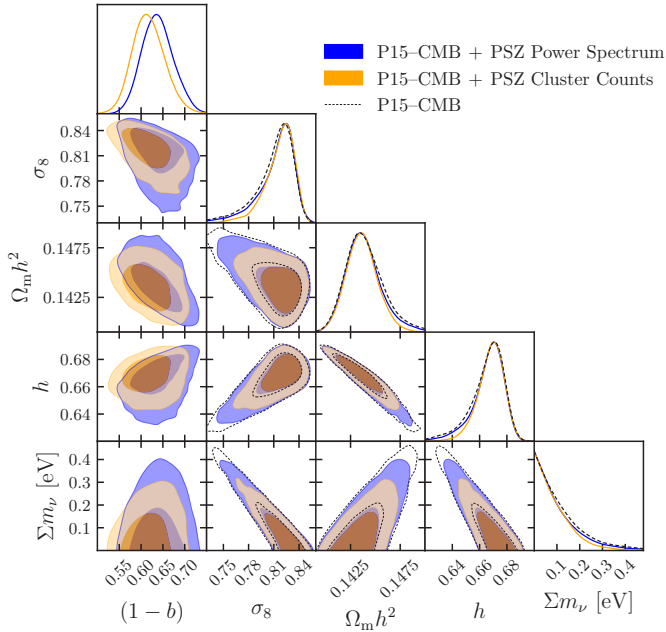
**Figure 1.** (Left panel:) Marginalised 2d joint posterior probability distributions with 68% CL and 95% CL contours from the Planck cluster counts analysis with a Gaussian prior on the X-ray mass bias  $B$ . (Right panel:) Marginalised 1- $\sigma$  constraints on  $S_8$  from several datasets (CC: cluster counts; YMAP: SZ power spectrum). Figures from [13].

this tension is the presence of significant higher-order terms, associated with relativistic electron temperature, in the spectral template of the SZ effect used in the component separation to extract the  $y$ -map and in the Matched Multifrequency Filter for the cluster detection [14]. Finally, as discussed extensively in the preceeding by Ruppin *et al* in the present volume and in [15], the discrepancy may also come from an inaccurate modelling of the ICM pressure profiles

Nevertheless, the remarkable agreement between Planck SZ, SPT-SZ, DES-Y1 and KV450 constraints on  $S_8$  still favours an X-ray mass bias of about 20% in the Planck SZ analyses. This motivates searching for an explanation of the discrepancy between these low redshift probes and the CMB that lies on the side of an extended cosmological model rather than details of the X-ray temperature calibration and relativistic SZ.

While keeping the amplitude of scalar curvature perturbations  $A_s$  fixed to its value determined by measurement of the primordial CMB temperature anisotropy power spectrum, there are two straightforward extended cosmological models that leads to a lower  $\sigma_8$  compared to  $\Lambda$ CDM. The first is cosmology with a dark energy that has an equation of state less negative than -1, i.e.,  $w$ CDM with  $w > -1$ . Indeed, dark energy with  $w > -1$  starts dominating earlier compared to a cosmological constant, slowing down structure formation over a longer period of time. The second is a cosmology with more-massive neutrinos, dubbed  $\nu\Lambda$ CDM, since the thermal motion of massive neutrinos smears out matter clustering on small scales.

To study whether these extensions can bring galaxy clusters and CMB into consistency, one can perform a joint analysis of the CMB and SZ data in  $w$ CDM,  $\nu\Lambda$ CDM, or even the combination  $\nu w$ CDM. Doing this with Planck SZ and Planck CMB does not hint towards massive neutrinos or  $w < -1$ . For instance in  $\nu\Lambda$ CDM, on Figure 2 one can see that rather than favouring a large neutrino mass, the data combination favours a larger mass bias, yielding  $b = 0.37 \pm 0.04$  (68%CL) or  $\approx 40\%$  (while the 2- $\sigma$  limit on the neutrino mass is actually tighter than with CMB alone). If we impose a Gaussian prior on the bias centred at 20% with 10% relative width, the posterior probability distribution peaks at a lower value,  $b = 0.32 \pm 0.04$



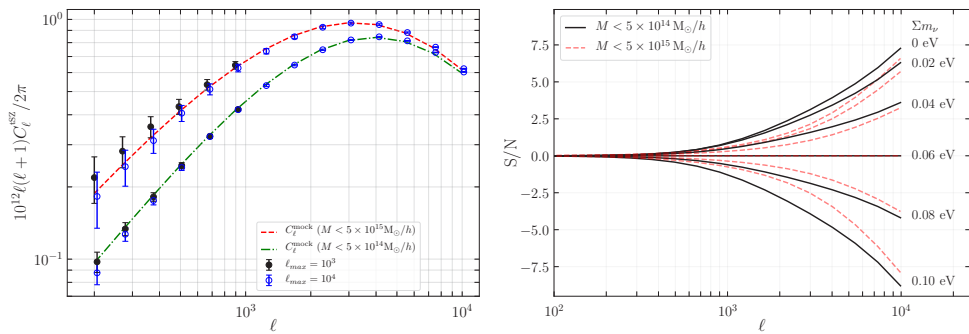
**Figure 2.** Marginalised (1d and 2d) joint posterior probability distributions with 68% CL and 95% CL contours (for a sub-set of parameters) obtained from the Planck y-map power spectrum (blue) and Planck SZ cluster counts (orange) combined with Planck 2015 primary CMB in  $\nu\Lambda$ CDM. Contours from Planck 2015 primary CMB alone are the dashed lines. Figures from [13].

(68%CL) and the probability distribution for the neutrino mass extends towards slightly larger neutrino masses, namely  $\Sigma m_\nu < 0.37$  eV (95%CL) with cluster counts and  $\Sigma m_\nu < 0.39$  eV (95%CL) with SZ power spectrum compared to  $\Sigma m_\nu < 0.35$  eV (95%CL) with CMB alone, however it does not peak at any particular value. Although these analyses with Planck SZ data do not hint towards extended cosmological models, the current uncertainty on the mass bias makes it difficult to conclude that massive neutrinos or dark energy are definitely not solutions to the discrepancy between constraints from galaxy clusters and primordial CMB anisotropy. See [13] for further details on these analyses.

This discussion also illustrates the fact that the constraining power of SZ clusters and power spectrum depends upon our knowledge of the cluster masses, i.e., the X-ray mass bias in the case of the Planck SZ data.

### 3 Prospects

A promising way to fix the cluster mass scale is to infer cluster masses from gravitation lensing of the CMB or background source galaxies. This has already been done for some of the Planck clusters using lensing data from projects such as CCCP, WtG and HSC [see 6, and references therein]. Recently, adding the Planck CMB weak lensing mass measurements to the Planck SZ cluster count likelihood, Zubeldia and Challinor obtained a constraint on the X-ray mass bias  $b = 0.29 \pm 0.10$  (68%CL) [16]. While the uncertainty is still relatively large, future experiments such as Euclid, LSST, CMB-S4 and Simons Observatory (SO) are expected to allow for a sub-percent accuracy on the mass bias. Then SZ cluster counts from



**Figure 3.** (Left panel) Mock SZ power spectra with error bars for  $\ell_{max} = 10^3$  and  $\ell_{max} = 10^4$  for two different values of  $M_{max}$ . (Right panel) Variation of signal-to-noise ‘S/N’ with respect to a fiducial total neutrino mass at  $\Sigma m_{\nu} = 0.06$  eV for two different values of  $M_{max}$ . Figures from [13].

CMB-S4 and SO with  $\sim 10^4$  clusters can in principle detect the minimal neutrino mass in the normal hierarchy  $\Sigma m_{\nu} = 0.06$  eV at high significance, thus providing an independent probe of the neutrino masses, competitive with future CMB lensing power spectrum analyses [17, 18].

To assess the constraining power of future SZ power spectrum measurements one can use a mock likelihood. The mock SZ power spectrum data points are drawn from the probability distribution of the SZ power spectrum. This probability distribution can be computed numerically as it approximates a multivariate Gaussian distribution at sufficiently large multipoles  $\ell > 10^2$  [19]. The left panel of Figure 3 shows several power spectra obtained with this procedure up to  $\ell_{max} = 10^3$  and  $\ell_{max} = 10^4$ . Assuming that the measurements are only limited by cosmic variance, one can then perform a maximum likelihood analysis using the MCMC method with a simple covariance matrix containing only the Gaussian sampling variance and the trispectrum. Combining this mock SZ power spectrum with a mock CMB-S4 likelihood for primary CMB anisotropy, the constraints are competitive only if the high- $\ell$  part of the SZ power spectrum is used. To further improve the constraining power of SZ power spectrum, one can mask the heaviest clusters in order to decrease the non-Gaussian variance and thus increase the ‘signal-to-noise’ (See right panel of Figure 3). With a relative uncertainty of 10% on the mass bias one then obtains  $\sigma(\Sigma m_{\nu}) = 31$  meV and with a 1% relative uncertainty one obtains  $\sigma(\Sigma m_{\nu}) = 29$  meV. These numbers become  $\sigma(\Sigma m_{\nu}) = 24$  meV when mock DESI-BAO data is added, which represents a  $\approx 14\%$  improvement over CMB plus BAO without SZ. In fact, for  $\ell_{max} = 10^4$ , the precision on the X-ray mass bias does not affect the total neutrino mass constraint when SZ power spectrum is combined with CMB-S4 and BAO. This is because once the amplitude of primordial curvature perturbations is fixed, the effect of neutrino mass on the SZ power spectrum at small scales differs from the effect at large scales, whereas the effect of the bias is close to a scale-invariant shift of the amplitude. Hence, with  $\ell_{max} = 10^4$  the X-ray mass bias and total neutrino mass are no longer as degenerate as for  $\ell_{max} = 10^3$ .

The SZ power spectrum at small scales is not only determined by cosmological expansion and perturbations but also by the details of the ICM via the pressure profile. Hence, it may be challenging to use the SZ power spectrum up to  $\ell_{max} = 10^4$  as an independent probe of the total neutrino mass. However, the small-scale SZ power spectrum can instead be used as a powerful probe of the ICM properties: the mock cosmic variance limited SZ power spectrum

experiments easily yield a measurement of the X-ray mass bias at the 2% precision level when information from mock CMB-S4 experiments is used to constrain cosmological parameters.

## 4 Conclusion

To conclude, current cosmological constraints from SZ data are hindered by the uncertainty on the cluster mass scale. Hence, it is difficult to reach definitive conclusions regarding consistency or tension between SZ and CMB constraints on matter clustering. In the next decade, weak lensing mass measurement and SZ observations at high resolution will shed sufficient light on the cluster mass scale and SZ will be used as an independent cosmological probe. Other promising avenues for SZ science include the measurement and characterisation of low density regions around galaxy clusters, e.g., with the NIKA2 camera [20], and within the filaments between galaxy clusters [21] and cosmic voids [22]. In addition to enable competitive constraints on the fundamental properties of the universe, future SZ data will become an exquisite probe of baryonic processes that shape the intra cluster medium.

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