Terahertz Frequency Comb in Graphene Field-Effect Transistors

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Abstract.

Graphene Field-effect transistors (GFETs) are excellent candidates for all-electric, low-power radiation sources and detectors based on integrated circuit technology. In this work, we show that a hydrodynamic instability can be explored (the Dyakonov–Shur instability) to excite the graphene plasmons. The instability can be sustained with the help of a source-to-drain current and controlled with the gate voltage. It is shown that the plasmons radiate a frequency comb in the Terahertz (THz) range. It is argued how this can pave the stage for a new generation of low power THz sources in integrated-circuit technology.

1 Introduction

Terahertz radiation has numerous applications ranging from sensing and imaging to metrology and spectroscopy [1, 2]; in particular terahertz laser (THL) combs play a prominent role within such technology [3, 4].

However, THL generation still faces significant difficulties, particularly in the low-power bench-top cases, therefore, practical solutions towards inexpensive, compact and easy-to-operate lasing devices are desirable. With the advent of graphene plasmonics, new techniques relying in optical pumping have been put forward [5, 6]. Alongside this, the progress in graphene based transistors [7] paved the way to the possibility for all-electrical miniaturised devices for low power radiation emission and detection.

In this work, we exploit a scheme for the generation of coherent THz frequency combs in graphene field-effect transistors, arising from the Dyakonov–Shur (DS) plasmonic instability [8, 9]. The latter can be excited via the injection of an electric current, thus forgoing the necessity of optical pumping. This opens the possibility to the development of an all-electric, low-consumption stimulated THL, capable of operating at room temperature.

2 Plasmonic instability

2.1 Hydrodynamic model

The electronic flow in graphene is suitable to be modelled by in a hydrodynamic description neglecting viscosity, and assuming also that transport is restricted to occur only in on

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direction, x [10, 11]. This motion is governed by the Euler equations

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}nv = 0 \quad \text{and} \quad \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{F}{m^*} - \frac{1}{m^*n} \frac{\partial P}{\partial x} \quad (1)$$

for the density n and collective velocity v with force F , P the pressure and m^* the carriers mass. The applicability of such hydrodynamic treatment of conduction in graphene is justified by the large mean free path, $l > 0.2 \mu\text{m}$ at room temperature, that assures the ballistic transport approximation. Moreover, throughout this work electrons are considered to be at degenerate Fermi liquid regime insofar that the Fermi level remains below the Van Hove singularities, such conditions in graphene at room temperature translate to $0.025 \text{ eV} \ll E_F \ll 3 \text{ eV}$.

Yet, the fact that electrons in graphene behave as massless fermions pose a difficulty to the development of models with explicit dependency on the mass, in our case the nominal Drude mass $m^* = \hbar \sqrt{\pi n_0}/v_F$, where $v_F = 10^6 \text{ ms}^{-1}$ is the Fermi velocity and n_0 the equilibrium carrier density, is used as an effective mass [10, 12]. Then, the pressure term in (1) can be recast as $P = \hbar v_F \sqrt{\pi n^3}/3$, resorting to the usual Fermi liquid pressure. Moreover, considering a monolayer graphene sheet in a field effect transistor (FET) structure, i.e. placed between two metallic contacts, source and drain, and subject to a gate, as schematised in Fig.1a, the electric force in the electrons is dominated by the imposed potential that screens Coulomb interaction between them. In this conditions the applied bias potential U has the contribution of both the parallel plate capacitance $C_g = \epsilon/d_0$ and the quantum capacitance [13, 14] $C_q = 2e^2 \sqrt{\pi n}/\pi \hbar v_F$ as

$$U = en \left(\frac{1}{C_g} + \frac{1}{C_q} \right). \quad (2)$$

However, for carrier densities in the range $n \gtrsim 10^{12} \text{ cm}^{-2}$, quantum capacity dominates $C_q \gg C_g$ and the potential can be approximated as $U \simeq end_0/\epsilon$. Thus, the acceleration of the FET electron fluid simply reads

$$\frac{-e\nabla U}{m^*} = -\frac{e^2 d}{m^* \epsilon} \frac{\partial n}{\partial x}. \quad (3)$$

Therefore, and keeping the pressure term up to first order, the fluid model can be written in a nondimensional form as:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0 \quad \text{and} \quad \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{S^2}{v_0^2} \frac{\partial n}{\partial x} = 0, \quad (4)$$

where v_0 is the electron mean drift velocity along the graphene channel and $S^2 \equiv \frac{e^2 d_0 n_0}{m^* \epsilon} + \frac{v_F^2}{2}$ can be interpreted as sound velocity of the carriers fluid, as the dispersion relation $\omega = (v_0 \pm S)k$ is akin to a shallow-waters regime; for typical values of graphene properties the ratio S/v_0 scales up to a few tens.

2.2 Dyakonov–Shur instability

The hydrodynamic model (4) describes an instability [8, 15] under the boundary conditions of fixed density at source $n(x=0) = n_0$ and fixed current density at the drain $n(x=L)v(x=L) = n_0 v_0$. Such Dyakonov–Shur instability rises from the multiple reflection of the plasma waves at the boundaries whilst being amplified by the driven current at the drain. The imposed drain direct current guarantees the necessary Doppler shift to the upstream current interfere positively with the downstream current.

According to (4) and the referred boundary conditions the excitation complex frequency $\omega = \omega_r + i\gamma$ can be derived, retrieving [8, 16]

$$\omega_r = \frac{|S^2 - v_0^2|}{2LS} \pi \quad \text{and} \quad \gamma = \frac{S^2 - v_0^2}{2LS} \log \left| \frac{S + v_0}{S - v_0} \right|. \quad (5)$$

Therefore, given the dependence of S with gate voltage, and as $v_0 n_0 = I_{DS} / We$ the frequency can be tuned by the gate voltage and injected drain current, not being solely restricted to the geometric factors of the FET.

3 Radiation emission

The DS plasmonic instability leads to well defined shock waves in the FET channel. One of the most significant implications of such shock waves is that they compel the electrons to not only act collectively but, more importantly, to bunch themselves as the shock profile forces the electronic fluid to concentrate upstream. In that sense the electrons undergo a process with some likelihood to the one in free electron lasers. Then, it is no wonder that the radiation provided by such means exhibits coherence to some extent, indeed the numerical results point to a first order temporal degree of coherence $g^{(1)}(\tau) \gtrsim 0.6$.

Furthermore, the collective oscillation of the bunched electrons is responsible for the THz synchrotron radiation emitted, that can be obtained from the density and current evolution of the fluid.

4 Pulsed stimulation & Frequency Comb

If the excitation of the plasmons is solely achieved by a injection of continuous current at the drain, the instability saturates after a transient time, leading to a continuous wave emission. Although continuous wave is relevant for a number of applications, wide bandwidth frequency combs are quite sought after. Thereby, a scheme was idealised to generate such wave packets.

If one considers that a periodic inversion of polarity is imposed across the channel, in such a way that the time on direct polarisation is enough for the saturation to occur, and then

Figure 1. (a) Schematic diagram for gated graphene transistor. For the DS instability to occur a fixed current I_{DS} is injected at the drain while maintaining the electronic density of the source constant. (b) Example of the instability growth of electronic density at the drain.

suddenly inverted, it is easy to see that the plasma instability would be generated – saturate – and then compelled to decay in time. In this way a wave packet is created with a envelope that can be approximated by:

$$A(t) = \begin{cases} e^{\gamma t} - 1 & \text{if } 0 \leq t < t_s \\ e^{\gamma t_s} - 1 & \text{if } t_s \leq t < t_o \\ e^{-\gamma(t-t_p)} - 1 & \text{if } t_o \leq t < t_p \end{cases} \quad (6)$$

where t_s is the characteristic time for saturation, t_o the duration of the excitation and $t_p = t_o + t_s$ the total duration of the pulse, and that can be seen in Fig.2a. The amplitude envelope Fourier transforms into

$$\hat{A}(\omega) = \gamma \sqrt{\frac{2}{\pi}} \frac{e^{t_s \gamma} \omega \cos \left[\frac{\omega}{2} (t_o - t_s) \right] - \omega \cos \left[\frac{\omega}{2} t_p \right] + e^{t_s \gamma} \gamma \sin \left[\frac{\omega}{2} (t_o - t_s) \right] - \gamma \sin \left[\frac{\omega}{2} t_p \right]}{\omega (\gamma^2 + \omega^2)}, \quad (7)$$

which defines the bandwidth, and general shape, of the frequency comb. The computed full-width at half maximum (FWHM) as function of the pulse duration, t_o , is shown in Fig2b, from where is also evident that the saturation time t_s does not play a significant role in the value of the FWHM.

Figure 2. (a) Example of pulsed excitation of DS instability, using a period $100\tau^*$ with 10% duty cycle. Inset showing the amplitude envelope with the characteristics times. (b) Dependence of FWHM of the spectral envelope from equation (7) with duration of excitation for different saturation times.

The performed simulations showed that the electronic fluid responds as expected to the polarisation inversion originating the pulses that form the frequency comb. In Fig. 3 a paradigmatic example of such frequency combs can be seen, with the central frequency very close to the theoretical value of (5) and the shape of the spectral envelope in accordance with (7), in particular denoting its the non-monotonic silhouette.

5 Conclusion

We make use of a hydrodynamic model to describe a plasmonic instability taking place in a graphene field-effect transistor at room temperature. Our scheme, based on the control of the electron current at the transistor drain, results in the emission of a THz frequency comb. Numerical simulations suggest that the emitted THz radiation is extremely coherent, thus be an appealing candidate for a THz laser source. Our findings point towards a method for the development of tuneable, all-electrical THz antennae, dismissing the usage of external

Figure 3. Frequency comb – Total radiated power FFT. Numerical simulation with $L = 0.75 \mu\text{m}$ and $v_0 = 0.3v_F$, for constant $S/v_0 = 20$, repetition rate of pulses of 5 GHz with a 10% duty cycle. Inset – Spectrum of the continuous wave case for the same parameters.

light sources . This puts graphene plasmonics and, in particular, graphene field-effect transistors in the run for competitive, low-consumption THz devices based on integrated-circuit technology, allowing the fabrication of patch arrays designed to enhance the total radiated power.

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