

B-anomalies in $U(2)$ flavor symmetry

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Abstract. We analyzed how to test flavor and helicity structures of the corresponding amplitudes in view of future data, motivated by the recent hints of lepton flavor universality violation observed in semileptonic B decays. The general assumption that non-standard effects are controlled by a $U(2)^5$ flavor symmetry, minimally broken as in the Standard Model Yukawa sector, leads to stringent predictions on leptonic and semileptonic B decays. Future measurements will allow to prove or falsify this general hypothesis independently of its dynamical origin.

1 Introduction

The current data collected by LHCb, BaBar and Belle experiments exhibit intriguing hints of violations of Lepton Flavor Universality (LFU) both in charged-current [2–6] and neutral-current [7–12] semileptonic B decays, and it gets attention as B-anomalies. The features of the hypothetical NP should have dominant couplings to third generation fermions and smaller couplings to second generation fermions. This non-trivial flavor structure resembles the observed flavor hierarchies in the Standard Model (SM) Yukawa couplings, and the possibility of a common explanation for these phenomena is opened.

In the context of the recent anomalies, we adopt the general assumptions that the NP effects are controlled by $U(2)^5$ flavor symmetry, which is a useful organizing principle to address the flavor hierarchies in the SM [13–15]. The paradigm of $U(2)^5$ flavor symmetry turns out to be successful in addressing B-anomalies with satisfying all existing bounds (e.g. Ref [16]). It is a global symmetry that the SM Lagrangian satisfies in good approximation; in the limit where we neglect all entries in the Yukawa couplings but for third generation masse, and given as

$$U(2)^5 \equiv U(2)_q \times U(2)_\ell \times U(2)_u \times U(2)_d \times U(2)_e, \quad (1)$$

where the first two SM fermion families transform as doublets of the $U(2)$. A minimal set of $U(2)^5$ breaking terms (*spurions*) which lets us reproduce all the observable SM flavor parameters without tuning and with minimal size for the breaking terms, is

$$\begin{aligned} V_q &\sim (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}), & V_\ell &\sim (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \\ \Delta_{u(d)} &\sim (\mathbf{2}, \mathbf{1}, \bar{\mathbf{2}}(\mathbf{1}), \mathbf{1}(\bar{\mathbf{2}}), \mathbf{1}), & \Delta_e &\sim (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \bar{\mathbf{2}}). \end{aligned} \quad (2)$$

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In terms of these spurions, the 3×3 Yukawa matrices can be decomposed as

$$Y_{u(d)} = y_{t(b)} \begin{pmatrix} \Delta_{u(d)} & x_{t(b)} V_q \\ 0 & 1 \end{pmatrix}, \quad Y_e = y_\tau \begin{pmatrix} \Delta_e & x_\tau V_\ell \\ 0 & 1 \end{pmatrix}, \quad (3)$$

where $x_{t,b,\tau}$ and $y_{t,b,\tau}$ are free complex parameters, expected to be of $\mathcal{O}(1)$. By the requirement of no tuning in the $\mathcal{O}(1)$ parameters, the order of spurion $|V_q| = \mathcal{O}(10^{-1})$ is implied, which shows a good fit of the anomalies in semileptonic B decays as discussed below. In this work, we present a systematic investigation of the consequences of this symmetry hypothesis in (semi)leptonic B decays, in model independent manner.

2 The EFT for semileptonic B decays based on the $U(2)^5$ flavor symmetry

Assuming no new degrees of freedom below the electroweak scale, we can describe NP effects in full generality employing the so-called SMEFT (SM effective Lagrangian), and we write the Lagrangian as

$$\mathcal{L}_{\text{EFT}} = -\frac{1}{v^2} \sum_{k, [ij\alpha\beta]} C_k^{[ij\alpha\beta]} O_k^{[ij\alpha\beta]} + \text{h.c.}, \quad (4)$$

where $v \approx 246$ GeV is the SM Higgs vev, $\{\alpha, \beta\}$ are lepton-flavor indices, and $\{i, j\}$ are quark-flavor indices.

Under the $U(2)$ flavor symmetry, the right handed light fermion operators are suppressed and this feature reduces the number of relevant semileptonic operators. Also, we do not consider $O_{qe} = (\bar{q}_L^i \gamma^\mu q_L^j)(\bar{e}_R^\alpha \gamma_\mu e_R^\beta)$ for simplicity because it contributes at tree-level only to $b \rightarrow s\tau\bar{\tau}$, which is poorly constrained currently. Now, the relevant operators

in the Warsaw basis [17] is given as

$$\begin{aligned} \mathcal{O}_{\ell q}^{(1)} &= (\bar{\ell}_L^\alpha \gamma^\mu \ell_L^\beta)(\bar{q}_L^i \gamma_\mu q_L^j), \\ \mathcal{O}_{\ell q}^{(3)} &= (\bar{\ell}_L^\alpha \gamma^\mu \tau^I \ell_L^\beta)(\bar{q}_L^i \gamma_\mu \tau^I q_L^j), \\ \mathcal{O}_{\ell edq} &= (\bar{\ell}_L^\alpha e_R^\beta)(\bar{d}_R^j q_L^j), \end{aligned} \quad (5)$$

and we are left with the following effective Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{EFT}} = & -\frac{1}{v^2} \left[C_{V_1} \Lambda_{V_1}^{[ija\beta]} \mathcal{O}_{\ell q}^{(1)} + C_{V_3} \Lambda_{V_3}^{[ija\beta]} \mathcal{O}_{\ell q}^{(3)} \right. \\ & \left. + (2C_S \Lambda_S^{[ija\beta]} \mathcal{O}_{\ell edq} + \text{h.c.}) \right], \end{aligned} \quad (6)$$

where $C_{V_i,S}$ control the overall strength of the NP effects and $\Lambda_{V_i,S}$ are tensors that parametrize the flavor structure. They are normalized by setting $\Lambda_{V_i,S}^{[3333]} = 1$, which is the only term surviving in the exact $U(2)^5$ limit. Note that the $U(2)^5$ assumption matches the U_1 vector leptoquark, which is the best solution for B-anomalies so far, transforming as $(\mathbf{3}, \mathbf{1})_{2/3}$ under the SM gauge group. The EFT in (6) nicely matches the structure generated by integrating out a U_1 vector leptoquark. The relation $C_V \equiv C_{V_1} = C_{V_3}$ is predicted under U_1 leptoquark scenario.

The flavor structure Λ_S is factorizes to

$$\Lambda_S^{[ija\beta]} = (\Gamma_L^\dagger)^{\alpha j} \times \Gamma_R^{i\beta}, \quad (7)$$

where, in the interaction basis,

$$\Gamma_L^{i\alpha} = \begin{pmatrix} x_{q\ell} V_q^i (V_\ell^\alpha)^* & x_q V_q^i \\ x_\ell (V_\ell^\alpha)^* & 1 \end{pmatrix}, \quad \Gamma_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (8)$$

Here $x_{q,\ell,q\ell}$ are $O(1)$ coefficients and we have neglected higher-order terms in $V_{q,\ell}$. Moving to the mass-eigenstate basis of down quarks and charged leptons, where

$$q_L^i = \begin{pmatrix} V_{ji}^* u_L^j \\ d_L^i \end{pmatrix}, \quad \ell_L^\alpha = \begin{pmatrix} V_L^\alpha \\ e_L^\alpha \end{pmatrix}, \quad (9)$$

we get $\Gamma_L \rightarrow \hat{\Gamma}_L \equiv \Gamma_L^\dagger \Gamma_L L_e$ and $\Gamma_R \rightarrow \hat{\Gamma}_R \equiv R_d^\dagger \Gamma_R R_e$, where L_f and R_f diagonalize the Yukawa matrices as $L_f^\dagger Y_f R_f = \text{diag}(Y_f)$, with $f = u, d, e$. The new matrices $\hat{\Gamma}_{L,R}$ can be written as

$$\begin{aligned} \hat{\Gamma}_L &= e^{i\phi_q} \begin{pmatrix} \Delta_{q\ell}^{de} & \Delta_{q\ell}^{d\mu} & \lambda_q^d \\ \Delta_{q\ell}^{se} & \Delta_{q\ell}^{s\mu} & \lambda_q^s \\ \lambda_\ell^e & \lambda_\ell^\mu & x_{q\ell}^{b\tau} \end{pmatrix} \approx e^{i\phi_q} \begin{pmatrix} 0 & 0 & \lambda_q^d \\ 0 & \Delta_{q\ell}^{s\mu} & \lambda_q^s \\ \lambda_\ell^e & \lambda_\ell^\mu & 1 \end{pmatrix}, \\ \hat{\Gamma}_R &\approx e^{i\phi_q} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{m_s}{m_b} s_b \\ 0 & -\frac{m_\mu}{m_\tau} s_\tau & 1 \end{pmatrix}. \end{aligned} \quad (10)$$

The (complex) parameters $x_{q\ell}^{b\tau}$, λ_q^i , λ_ℓ^α , and $\Delta_{q\ell}^{ai}$ are a combination of the spurions in (8) and the rotation terms from $L_{d,e}$, that satisfy

$$\begin{aligned} \lambda_q^s &= O(|V_q|), & \lambda_\ell^\mu &= O(|V_\ell|), \\ x_{q\ell}^{b\tau} &= O(1), & \Delta_{q\ell}^{s\mu} &= O(\lambda_q^s \lambda_\ell^\mu), \\ \frac{\lambda_q^d}{\lambda_q^s} &= \frac{\Delta_{q\ell}^{d\alpha}}{\Delta_{q\ell}^{s\alpha}} = \frac{V_{td}^*}{V_{ts}^*}, & \frac{\lambda_\ell^e}{\lambda_\ell^\mu} &= \frac{\Delta_{q\ell}^{ie}}{\Delta_{q\ell}^{i\mu}} = s_e. \end{aligned} \quad (11)$$

The r.h.s. of the first line of (10) is at lowest order in the spurion ($|V_{q,\ell}|$) expansion. The structure of $\Lambda_V^{[ija\beta]}$ is written in same manner.

From the flavor structure shown above, it is found that following special features are predicted by $U(2)^5$.

- The NP effect in neutral-current $b \rightarrow s\mu\mu$ is smaller than one in charged current $b \rightarrow c\tau\nu$, which is compatible with the situation of B-anomalies.
- The NP strengt in $b \rightarrow c(s)$ is equivalent to one in $b \rightarrow u(d)$ (see eq.(16)):

$$\frac{b \rightarrow c\ell\nu}{b \rightarrow u\ell\nu} = \frac{b \rightarrow c\ell\nu}{b \rightarrow u\ell\nu} \Big|_{\text{SM}} \quad (12)$$

$$\frac{b \rightarrow s\ell\ell}{b \rightarrow d\ell\ell} = \frac{b \rightarrow s\ell\ell}{b \rightarrow d\ell\ell} \Big|_{\text{SM}} \quad (13)$$

- Scalar operators with light fermions suppressed by factor m_s/m_b and m_μ/m_τ .

3 Observables in Charged-current

In this section, we discuss the NP effects on the charged-current. In the $b \rightarrow c\tau\bar{\nu}$ case, we conveniently re-define them as

$$\begin{aligned} C_{V(S)}^c &= C_{V(S)} \left[1 + \lambda_q^s \left(\frac{V_{cs}}{V_{cb}} + \frac{V_{cd}}{V_{cb}} \frac{V_{td}^*}{V_{ts}^*} \right) \right] \\ &= C_{V(S)} \left(1 - \lambda_q^s \frac{V_{tb}^*}{V_{ts}^*} \right), \end{aligned} \quad (14)$$

where, in the last line, we have used CKM unitarity. When defining $C_{V(S)}^c$, we have factorized the CKM factor V_{cb} , such the that the left-handed part of the interactions is modified as

$$\mathcal{A}^{\text{SM}} \rightarrow (1 + C_V^c) \mathcal{A}^{\text{SM}}. \quad (15)$$

In the absence of the simplifying hypothesis $\Gamma_{L^3} = \Gamma_L$, one would need to redefine C_V^c replacing λ_q^s with $\tilde{\lambda}_q^s$. Employing this hypothesis, as in the leptoquark case, the ratio $C_S^c/C_V^c = C_S/C_V$ is flavor blind and depends only on the helicity structure of the NP amplitude.

Current measurements of the LFU ratios R_D and R_{D^*} , where $R_H = \Gamma(\bar{B} \rightarrow H\tau\bar{\nu})/\Gamma(\bar{B} \rightarrow H\ell\bar{\nu})$, lead to the constraints on C_S^c and C_V^c . In fig.1, chi-square fit results (dashed contour lines) for $b \rightarrow c\tau\nu_\tau$ together with $b \rightarrow u\tau\nu_\tau$ are shown. Here, we use the results in [18, 19] for the $\bar{B} \rightarrow D^{(*)}\ell\bar{\nu}$ form factors and decay rates. For comparison, the directions corresponding to a pure left-handed ($\beta_R = 0$) or a vector-like interaction ($\beta_R = -1$) for the U_1 leptoquark, where β_R is right-handed d_R - e_R coupling mediated by U_1 , are also indicated. It is found that the fit results taking into account only the information from $b \rightarrow c\tau\nu_\tau$ transitions (R_D and R_{D^*}) are deviated from 3σ from the SM predictions (zero point), and the $U(2)$ prediction for $b \rightarrow u$ transition ($B \rightarrow \tau\nu$) is compatible with them.

The predictiveness of $U(2)^5$ is also found in the predictions for the polarizations in $B \rightarrow D^{(*)}\tau\nu_\tau$. The τ polarizations $P_\tau^D, P_\tau^{D^*}$ and the D^* polarization $F_L^{D^*}$ are sensitive

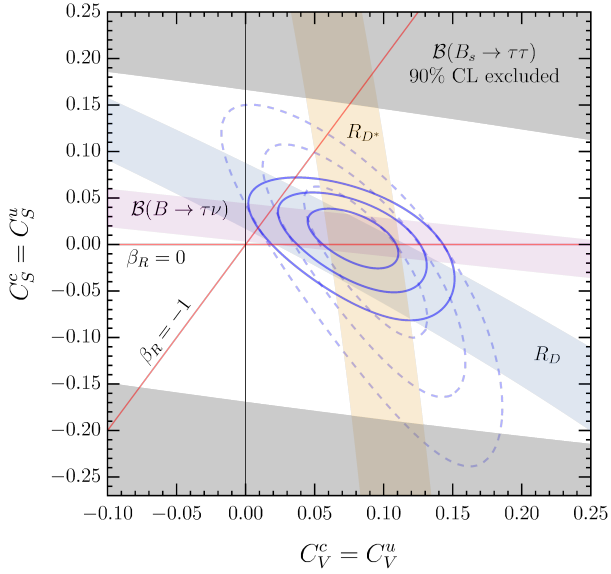


Figure 1. Best fit regions in the (C_V^c, C_V^u) plane. The three contours corresponds to 1, 2, and 3 σ intervals. The dashed blues lines take into account only the information from R_D and R_{D^*} (for which we use the HFLAV average [20]), whereas the continuous lines also include constraints from $b \rightarrow u$ observables. The colored bands correspond to the 1 σ regions defined by each observable. The gray bands show the 90% CL exclusion region from $\mathcal{B}(B_s \rightarrow \tau\bar{\tau})$.

to the NP models (e.g. Ref [21]) and expected to be measured at the Belle II experiment in the near future. In fig.2, the sharp correlations between R_D, R_{D^*} , and the τ polarizations $P_\tau^D, P_\tau^{D^*}$ and the D^* polarization $F_L^{D^*}$ are shown, which can be tested by Belle II in the foreseeable future.

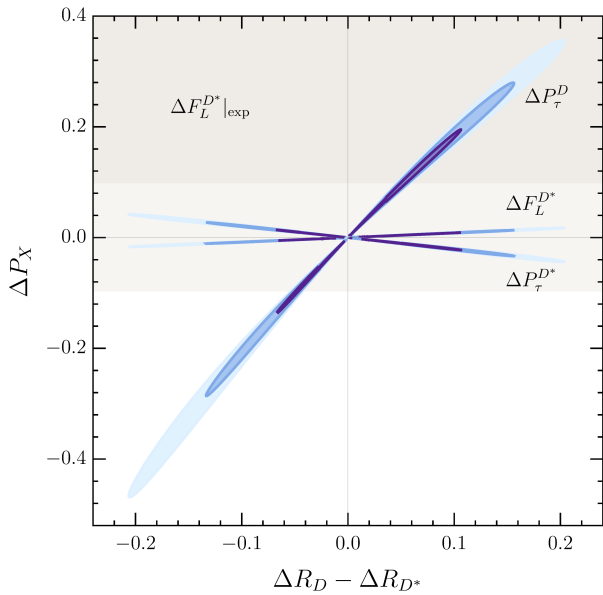


Figure 2. Deviations of the polarization asymmetries compared to the SM as a function of $\Delta R_D - \Delta R_{D^*}$, where $\Delta X \equiv X/X^{\text{SM}} - 1$. The predictions are obtained using the fit in Fig. 1 (continuous lines). In gray, the experimental value of $\Delta F_L^{D^*}$ at 1 σ and 2 σ .

Next, we discuss the $b \rightarrow c\tau\bar{\nu}$ transitions. The analog of $C_{V(S)}^c$ for $b \rightarrow u$ transitions are the effective couplings

$$C_{V(S)}^u = C_{V(S)} \left[1 + \lambda_q^s \left(\frac{V_{us}}{V_{ub}} + \frac{V_{ud}}{V_{ub}} \frac{V_{td}^*}{V_{ts}^*} \right) \right] = C_{V(S)}^c, \quad (16)$$

where the result in the second line follows from CKM unitarity. The prediction of same size NP effects, relative to the SM, in $b \rightarrow u$ and $b \rightarrow c$ transitions is a distinctive feature of the minimally-broken $U(2)^5$ hypothesis. It predicts

$$\frac{\mathcal{B}(\bar{B}_u \rightarrow \tau\bar{\nu})}{\mathcal{B}(\bar{B}_u \rightarrow \tau\bar{\nu})_{\text{SM}}} \approx \frac{\mathcal{B}(\bar{B}_c \rightarrow \tau\bar{\nu})}{\mathcal{B}(\bar{B}_c \rightarrow \tau\bar{\nu})_{\text{SM}}}, \quad (17)$$

where the difference among the two modes arises by sub-leading spectator mass effects in the chirality-enhancement factors. Also, in the future, very interesting constraints are expected from $\bar{B} \rightarrow \pi\tau\bar{\nu}$. This process also has specific relation with $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}$, and we get following approximate relation

$$\frac{R_\pi}{R_\pi^{\text{SM}}} \approx 0.75 \frac{R_D}{R_D^{\text{SM}}} + 0.25 \frac{R_{D^*}}{R_{D^*}^{\text{SM}}}, \quad (18)$$

where $R_\pi \equiv \mathcal{B}(\bar{B} \rightarrow \pi\tau\bar{\nu})/\mathcal{B}(\bar{B} \rightarrow \pi\ell\bar{\nu})$ and we use the hadronic parameters in [22, 23]. This relation would allow a non-trivial test of the $U(2)^5$ structure of the interactions. In Fig. 3 we show the predictions for $\mathcal{B}(\bar{B}_u \rightarrow \tau\bar{\nu})$, $\mathcal{B}(\bar{B} \rightarrow \pi\tau\bar{\nu})$, and $\mathcal{B}(\bar{B}_c \rightarrow \tau\bar{\nu})$, as a function of $\Delta R_D - \Delta R_{D^*}$.

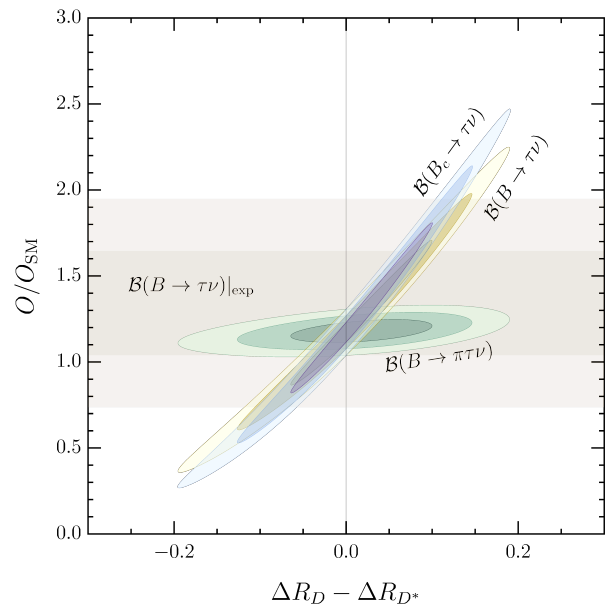


Figure 3. Predictions for $\mathcal{B}(\bar{B}_c \rightarrow \tau\bar{\nu})$, $\mathcal{B}(\bar{B}_u \rightarrow \tau\bar{\nu})$ and $\mathcal{B}(\bar{B} \rightarrow \pi\tau\bar{\nu})$, all normalized to the corresponding SM expectations, as a function of $\Delta R_D - \Delta R_{D^*}$. In gray, the experimental value of $\mathcal{B}(\bar{B}_u \rightarrow \tau\bar{\nu})$ at 1 σ and 2 σ .

4 Observables in Neutral current

The $b \rightarrow s$ semileptonic transitions have a rich phenomenology and have been extensively discussed in the

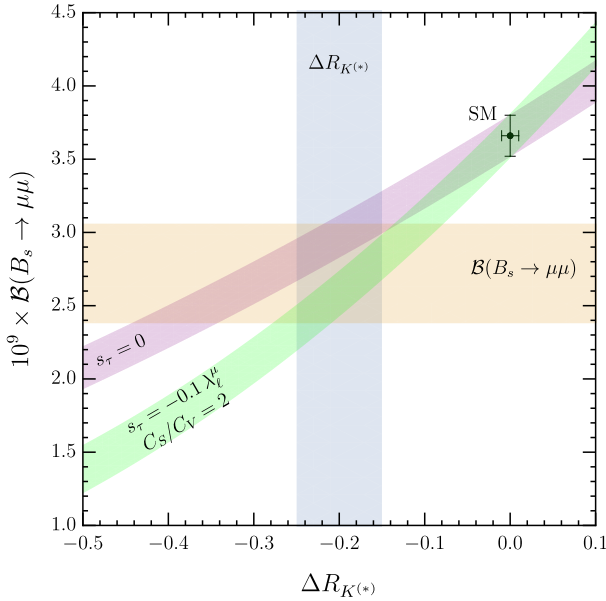


Figure 4. Predictions for $\mathcal{B}(B_s \rightarrow \mu\bar{\mu})$ as a function of $\Delta R_{K^{(*)}}$. The purple and green bands correspond to two different benchmark parameter values. The combination of ATLAS, CMS and LHCb measurements of $\mathcal{B}(B_s \rightarrow \mu\bar{\mu})$, and the combined $R_{K^{(*)}}$ measurement are also shown.

recent literature. Contrary to the charged-current case, here model-dependent assumptions, play a more important role.

One of most relevant observables are the LFU ratios $R_{K^{(*)}} = \Gamma(B \rightarrow K^{(*)}\mu\bar{\mu})/\Gamma(B \rightarrow K^{(*)}e\bar{e})$, which are particularly interesting due to their robust theoretical predictions. In our setup, one gets [24, 25]

$$R_K \approx R_{K^*} \approx 1 + 0.47 \Delta C_9^\mu. \quad (19)$$

The prediction $R_K \approx R_{K^*}$, is a direct consequence of our flavor symmetry assumptions and is independent of the initial set of dimension-six SMEFT operators. In addition to (19), we expect

$$R_\phi(B_s) \approx R_{\pi K}(B) \approx R(\Lambda_b)_\Lambda \approx R(\Lambda_b)_{pK} \approx \dots \approx R_K. \quad (20)$$

Current experimental data hint to sizable NP effects in R_K and R_{K^*} consistent with $R_K \approx R_{K^*}$. This numerical value R_K and R_{K^*} provides an important constraint on the size of the leptonic spurion (λ_ℓ^μ): since $\Delta_{q\ell}^{SM} = O(\lambda_q^s \lambda_\ell^\mu)$, setting $\lambda_q^s = O(10^{-1})$ and $C_V = O(10^{-2})$, as suggested by the $R_{D^{(*)}}$ fit, the value of $R_{K^{(*)}}$ implies $\lambda_\ell^\mu = O(10^{-1})$.

Among $b \rightarrow s\mu\bar{\mu}$ transitions, a special role is played by $B_s \rightarrow \mu\bar{\mu}$, where the chiral enhancement of the scalar amplitude allows us to probe the helicity structure of the NP interaction. In Fig. 4, we show the predictions for this observable as a function of $\Delta R_{K^{(*)}}$ for $s_\tau = 0$ (purple band), where s_τ is mixing parameter in the rotation matrix, and for $s_\tau = -0.1$ λ_ℓ^μ setting $C_S/C_V = 2$ (green band). As can be seen, the current experimental values show tension with the SM [26] and are consistent with $U(2)^5$ prediction.

5 Conclusions

Motivated by the recent hints of lepton flavor universality violation observed in semileptonic B decays (B-anomalies), we adopt the general assumptions that the NP effects are controlled by a $U(2)^5$ flavor symmetry, minimally broken as in the SM Yukawa sector, and analyzed how to test flavor and helicity structures of the corresponding amplitudes in view of future data. It is found that stringent predictions on leptonic and semileptonic B decays in charged-current and neutral-current are led, and the current data consistent with a $U(2)^5$ flavour symmetry. A $U(2)^5$ flavor symmetry is very predictive, and future measurements by LHCb and Belle II will provide an invaluable help in clarifying the origin of this intriguing phenomenon.

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References

- [1] J. Fuentes-Martín, G. Isidori, J. Pagès, K. Yamamoto, Phys. Lett. **B800**, 135080 (2020), 1909.02519
- [2] R. Aaij et al. (LHCb), Phys. Rev. Lett. **115**, 111803 (2015), [Erratum: Phys. Rev. Lett. 115, no. 15, 159901 (2015)], 1506.08614
- [3] J.P. Lees et al. (BaBar), Phys. Rev. **D88**, 072012 (2013), 1303.0571
- [4] S. Hirose et al. (Belle), Phys. Rev. Lett. **118**, 211801 (2017), 1612.00529
- [5] R. Aaij et al. (LHCb), Phys. Rev. **D97**, 072013 (2018), 1711.02505
- [6] A. Abdesselam et al. (Belle) (2019), 1904.08794
- [7] R. Aaij et al. (LHCb), Phys. Rev. Lett. **113**, 151601 (2014), 1406.6482
- [8] R. Aaij et al. (LHCb), JHEP **08**, 055 (2017), 1705.05802
- [9] R. Aaij et al. (LHCb), Phys. Rev. Lett. **122**, 191801 (2019), 1903.09252
- [10] J.T. Wei et al. (Belle), Phys. Rev. Lett. **103**, 171801 (2009), 0904.0770
- [11] A. Abdesselam et al. (Belle) (2019), 1904.02440
- [12] B. Aubert et al. (BaBar), Phys. Rev. **D73**, 092001 (2006), hep-ex/0604007
- [13] R. Barbieri, G. Isidori, J. Jones-Perez, P. Lodone, D.M. Straub, Eur. Phys. J. **C71**, 1725 (2011), 1105.2296
- [14] G. Blankenburg, G. Isidori, J. Jones-Perez, Eur. Phys. J. **C72**, 2126 (2012), 1204.0688
- [15] R. Barbieri, D. Buttazzo, F. Sala, D.M. Straub, JHEP **07**, 181 (2012), 1203.4218

- [16] R. Barbieri, G. Isidori, A. Pattori, F. Senia, Eur. Phys. J. **C76**, 67 (2016), 1512.01560
- [17] B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek, JHEP **10**, 085 (2010), 1008.4884
- [18] F.U. Bernlochner, Z. Ligeti, M. Papucci, D.J. Robinson, Phys. Rev. **D95**, 115008 (2017), [erratum: Phys. Rev.D97,no.5,059902(2018)], 1703.05330
- [19] M. Tanaka, R. Watanabe, Phys. Rev. **D87**, 034028 (2013), 1212.1878
- [20] <https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html>
- [21] S. Iguro, T. Kitahara, Y. Omura, R. Watanabe, K. Yamamoto, JHEP **02**, 194 (2019), 1811.08899
- [22] M. Tanaka, R. Watanabe, PTEP **2017**, 013B05 (2017), 1608.05207
- [23] J.A. Bailey et al. (Fermilab Lattice, MILC), Phys. Rev. **D92**, 014024 (2015), 1503.07839
- [24] A. Celis, J. Fuentes-Martin, A. Vicente, J. Virto, Phys. Rev. **D96**, 035026 (2017), 1704.05672
- [25] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias, J. Virto, JHEP **01**, 093 (2018), 1704.05340
- [26] M. Beneke, C. Bobeth, R. Szafron (2019), 1908.07011