Proton Holography

Discovering Odderon from Scaling Properties of Elastic Scattering

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Abstract. We investigate the scaling properties of elastic scattering data at ISR and LHC energies, and find that the significance of an Odderon observation is larger than the discovery threshold of 5σ. As an unexpected by-product of these investigations, for certain experimentally relevant cases, we also conjecture the possibility of proton holography with the help of elastic proton-proton scattering.

1 Introduction

This work summarizes two related topics: the discovery of a significant Odderon effect at LHC energies and the investigation of the possibility of four-momentum transfer t dependent phase measurement in elastic proton-proton (pp) and proton-antiproton ($p\bar{p}$) scattering.

The Odderon corresponds to a crossing-odd contribution to the scattering amplitude of elastic pp and $p\bar{p}$ scattering at asymptotically high energies, proposed by Lukaszuk and Nicolescu in 1973 [1]. In QCD, the quantum field theory of the strong interactions, such an Odderon exchange corresponds to the t-channel exchange of a color-neutral gluonic compound state consisting of an odd number of gluons, as noted by Bartels, Vacca and Lipatov in 1999 [2].

As the modulus square of the elastic scattering amplitude is proportional to the differential cross-section of elastic scattering, a phase reconstruction is equivalent to the possibility of proton holography. It is well known that such a t-dependent phase reconstruction from the measurements of modulus squared amplitudes, without further information, is simply impossible. We thus ask a different question here: what are those specific additional, and experimentally testable conditions, that actually allow for a t-dependent phase reconstruction in elastic pp collisions at LHC energies?

Holography of light by now is well developed technique that has several sub-topics and applications not only in science but also in arts, banking, programming, interferometry and security, including holograms on bank-notes,

vehicles, credit and identity cards as well as passports. The essential point of holography was highlighted in D. Gabor's Nobel Lecture [3]: Holography is based on the wave nature of light, and corresponds to phase level reconstruction. As every quantum field has a dual wave and quantum property, holography is possible not only using the wave properties of light quanta, but also that of other particles like electrons [4], atoms [5] as well as neutrons [6]. The key concept of holography is to produce a coherent source of these quanta, and have parts of this field diffractively scatter on some scattering center. Recording the resulting interference pattern corresponds to recording a holographic picture. Illuminating this picture with the original beam allows for reconstruction of the scattered wave including both the modulus and the phase of the amplitude. Actually, two different holographic images are created, the object beam that carries the original phase and modulus, and the conjugated object beam that reconstructs the modulus but conjugates the complex phase of the object beam.

The title of this work is proton holography, as we discuss here possibilities for a phase-level reconstruction using the elastically scattered protons at the TeV energy scale. Although such a title may look a bit fancy, it turns out that our idea is not unprecedented, as Ref. [7] has already considered four-momentum transfer dependent phase reconstruction in elastic scattering of protons at the ISR energies of $\sqrt{s} = 23.5 - 62.5$ GeV, but without introducing such a term.

A related topic is the search for a crossing-odd contribution in elastic proton-proton collisions at CERN LHC energies, the so called Odderon effect. As the Odderon effect is not significant at the ISR energy range but is found to be significant at LHC energies [8], the methodology of phase reconstruction at these two energies are also slightly different. For example, the phase-level reconstruction of

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Ref. [7] disregarded any Odderon contribution, so that method cannot be readily applied at TeV energies. Due to this reason, let us first briefly summarize some of our new results related to the Odderon discovery at the CERN LHC, and this way also prepare the ground for the idea of proton holography at high energies, as detailed in the second part of this work.

2 Formalism

Let us consider the elastic scattering of particles a and b with incoming four-momenta (p_1, p_2), and outgoing four-momenta (p_3, p_4), respectively. The Mandelstam variables s and t are defined as $s = (p_1 + p_2)^2$ and $t = (p_1 - p_3)^2$. The differential cross-section of such $(a, b) \rightarrow (a, b)$ scattering, $\frac{d\sigma(s,t)}{dt}$ can be expressed in terms of the scattering amplitude $T_{\text{el}}(s, t)$ as

$$\frac{d\sigma(s,t)}{dt} = \frac{1}{4\pi} |T_{\text{el}}(s,t)|^2 . \tag{1}$$

The elastic cross-section is given as an integral of the differential cross-section as

$$\sigma_{\text{el}}(s) = \int_0^\infty dt \; \frac{d\sigma(s,t)}{dt}. \tag{2}$$

At a given s, the t-dependent slope parameter $B(s, t)$ is the logarithmic slope of the differential cross-section:

$$B(s,t) = \frac{d}{dt} \ln \frac{d\sigma(s,t)}{dt} . \tag{3}$$

In the low-t region, corresponding to the diffractive cone, this function is frequently assumed or found within experimental errors to be a constant. To characterize this diffractive cone, a t-independent slope parameter $B(s)$ can be introduced as

$$B(s) \equiv B_0(s) = \lim_{t \to 0} B(s,t), \tag{4}$$

where the $t \to 0$ limit, the so called optical point is measured with a finite experimental resolution, so in general $B(s)$ is resolution, or $-t$ fit-range, dependent. The lowest values of $|-t|$ that we analyze in this work correspond to $-t \geq 0.00515$ GeV^2 at $\sqrt{s} = 7$ TeV, which is outside of the so-called Coulomb-Nuclear Interference (CNI) region, see Fig. 2 of Ref. [9]. This is why $B(s)$ is called hadronic or nuclear slope, as it is defined outside of the range of the CNI effects.

The optical point is also found by extrapolations from the measurements performed in the diffractive cone, in the $-t > 0$, experimentally accessible regions. The total cross-section is determined as

$$\sigma_{\text{tot}}(s) \equiv 2 \,\text{Im} \, T_{\text{el}}(t = 0, s). \tag{5}$$

In general, the (s, t) dependent ratio of the real to imaginary parts of the elastic amplitude is defined as

$$\rho(s,t) \equiv \frac{\text{Re} \, T_{\text{el}}(s,t)}{\text{Im} \, T_{\text{el}}(s,t)} . \tag{6}$$

The $t \to 0$ limit of this ratio is given by

$$\rho(s) \equiv \rho_0(s) = \lim_{t \to 0} \rho(s,t) . \tag{7}$$

Technically, ρ_0 is measured in the CNI region [9], with the help of the interference of the well understood Coulomb wave with the nuclear (or strong) amplitude. The $t \to 0$ limit is understood in terms of the finite experimental resolution and refers to ρ of the nuclear phase, without including the CNI effects. Then the differential cross section at the optical point can be written as

$$\left. \frac{d\sigma(s,t)}{dt} \right|_{t\to 0} = \frac{1 + \rho_0^2(s)}{16\pi} \sigma_{\text{tot}}^2(s) . \tag{8}$$

In the impact-parameter b-space, we have

$$t_{\text{el}}(s,b) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\mathbf{b}} T_{\text{el}}(s,t),$$

$$\boldsymbol{\Delta} \equiv |\boldsymbol{\Delta}|, \quad b \equiv |\mathbf{b}|, \quad \Delta = \sqrt{|t|}. \tag{9}$$

Here, the Fourier-transformed elastic amplitude $t_{el}(s, b)$ can be cast in the eikonal form as follows

$$t_{\text{el}}(s,b) = i \left[1 - e^{-\Omega(s,b)} \right], \tag{10}$$

where $\Omega(s, b)$ is the so-called opacity function (known also as the eikonal function), which is generically complex. The shadow profile function is then defined as

$$P(s,b) = 1 - \left| e^{-\Omega(s,b)} \right|^2 . \tag{11}$$

When the real part of the scattering amplitude is neglected, $P(b, s)$ is frequently denoted as $G_{\text{inel}}(s, b)$, see for example Refs. [10–14] for more details.

3 Odderon

The pp or $p\bar{p}$ elastic scattering amplitude can be written as a sum or a difference of crossing-even and crossing-odd contributions, respectively,

$$T_{\text{el}}^{pp}(s,t) = T_{\text{el}}^+(s,t) + T_{\text{el}}^-(s,t), \tag{12}$$

$$T_{\text{el}}^{p\bar{p}}(s,t) = T_{\text{el}}^+(s,t) - T_{\text{el}}^-(s,t), \tag{13}$$

$$T_{\text{el}}^+(s,t) = T_{\text{el}}^P(s,t) + T_{\text{el}}^f(s,t), \tag{14}$$

$$T_{\text{el}}^-(s,t) = T_{\text{el}}^O(s,t) + T_{\text{el}}^\omega(s,t). \tag{15}$$

The even-under-crossing part consists of the Pomeron and the f Reggeon trajectory, while the odd-under-crossing part contains the Odderon and a contribution from the ω Reggeon. A direct way to "see" the Odderon in the data is to compare the differential cross-section of in elastic pp and $p\bar{p}$ scattering at TeV energies [15, 16] since at sufficiently high $\sqrt{s}$ the Reggeon contributions decrease below the experimental errors. In this case, the Odderon and Pomeron contributions, $T_{\text{el}}^O(s, t)$ and $T_{\text{el}}^P(s, t)$, respectively, are found as follows

$$T_{\text{el}}^P(s,t) = \frac{T_{\text{el}}^{pp} + T_{\text{el}}^{p\bar{p}}}{2} \;\; \text{if} \; \sqrt{s} \geq 1 \, \text{TeV}, \tag{16}$$

$$T_{\text{el}}^O(s,t) = \frac{T_{\text{el}}^{pp} - T_{\text{el}}^{p\bar{p}}}{2} \;\; \text{if} \; \sqrt{s} \geq 1 \, \text{TeV}, \tag{17}$$

where we have suppressed the (s, t) dependence of the pp and $p\bar{p}$ scattering amplitudes, for the sake of brevity. Indeed, if the pp differential cross sections differ from that

amplitude cannot be equal to zero [8], i.e.

$$\frac{d\sigma^{pp}}{dt} \neq \frac{d\sigma^{p\bar{p}}}{dt} \quad \text{for} \quad \sqrt{s} \geq 1 \text{ TeV} \implies T_{\text{el}}^{O}(s,t) \neq 0, \tag{18}$$

which is considered to be the main Odderon signal. The Odderon contribution can be particularly strong around t_{dip}, as a pronounced diffractive dip (minimum) is seen in elastic pp collisions, while this structure is apparently missing in $p\bar{p}$ collisions in the TeV energy range [17, 18].

3.1 Scaling of the differential cross-section

The low-$|t|$ part of the measured distribution can be frequently approximated with an exponential shape, if the experimental errors are sufficiently large:

$$\frac{d\sigma}{dt}(s,t) = A(s) \exp[B(s)t]. \tag{19}$$

The normalization parameter is denoted by $A(s) = \frac{d\sigma}{dt}(s, t=0)$ and the nuclear slope parameter by $B(s)$ as discussed above. In the diffractive cone, $A(s) = B(s)\sigma_{\text{el}}(s)$, and the nearly exponential differential cross-sections can be scaled to a universal scaling function defined as [8]

$$H(x) \equiv \frac{1}{B(s)\sigma_{\text{el}}(s)} \frac{d\sigma}{dt}, \tag{20}$$

$$x = -tB(s). \tag{21}$$

At low-$|t|$ this scaling function is approximately written as $H(x) = \exp(-x)$. Thus, $H(x)$ scales out the trivial s dependencies of $B(s)$ and $\sigma_{\text{el}}(s)$ from the differential cross-sections in the diffractive cone. In the exponential approximation,

$$A(s) = B(s)\sigma_{\text{el}}(s) = \frac{1 + \rho_0^2(s)}{16\pi} \sigma_{\text{tot}}^2(s), \tag{22}$$

$$B(s) = \frac{1 + \rho_0^2(s)}{16\pi} \frac{\sigma_{\text{tot}}^2(s)}{\sigma_{\text{el}}(s)}, \tag{23}$$

thus $H(x)$ scales out the s-dependence, if it arises only due to the s-dependence of the total cross-section $\sigma_{\text{tot}}(s)$ and the real-to-imaginary ratio $\rho(s)$.

In Ref. [8] we have shown that our $H(x)$ scaling, defined as above, is valid not only in the diffractive cone but, surprisingly, also at the diffractive minimum and maximum of elastic proton-proton collisions: the $H(x, s)$ is s-independent in the ISR energy range of $\sqrt{s} = 23.5 - 62.5$ GeV. In addition, the $H(x)$ scaling was shown to be also valid, within statistical errors, in the LHC energy range of $\sqrt{s} = 2.76 - 7.0$ TeV, but with a scaling function that is significantly different from the one at ISR energies [8].

From the s-independence of the $H(x, s)$ at the lower LHC energies as well as from Eqs. (20,21) a new Odderon signature follows [8]: If the $H(x, s)$ scaling function for pp collisions differs from that of $p\bar{p}$ in a given s domain, where $H(x, s) \equiv H(x)$ is s-independent for pp collisions, then the differential cross-sections in that interval of s cannot be equal for pp and $p\bar{p}$ collisions, either. Hence, the

if $H(x, s_1)$ for $pp \quad \neq \quad H(x, s_2)$ for pp

$$\text{for} \quad 1 \lessgtr \sqrt{s_1}, \sqrt{s_2} \lessgtr 8 \text{ TeV}$$

$$\implies T_{\text{el}}^{O}(s,t) \neq 0. \tag{24}$$

The left panel of Fig. 1 indicates that for elastic proton-proton collisions, the $H(x, s)$ scaling function of the differential cross-section is independent, within statistical errors, of the colliding energy for $\sqrt{s} = 2.76$ and 7.0 TeV, at the confidence level of $CL = 99$ %. Similar results are obtained for the TOTEM preliminary large $-t$ differential cross-sections at $\sqrt{s} = 8$ TeV [19]. Given that in addition to the statistical errors, there are significant systematic errors present as well, this result indicates that the validity of the s-independence of the $H(x) \equiv H(x, s)$ scaling for pp collisions is greater than the $2.76 \leq \sqrt{s} \leq 8.0$ TeV region. Within *systematic* errors, indicated by vertical bars with widths proportional to the horizontal systematic error on $x = -tB$, the confidence level of the agreement of the $H(x, s)$ scaling function for the two datasets at $\sqrt{s} = 2.76$ and 7.0 TeV is 100 %. In Fig. 1, the regions with smaller and larger boxes correspond to two different datasets, measured in two different acceptance regions [20, 21].

Quantitatively, we have found a statistically significant Odderon signal in the comparison of the $H(x, s)$ scaling functions of elastic pp collisions at $\sqrt{s} = 7.0$ to that of $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. On the right panel of Fig. 1, we compare the $H(x, s)$ scaling functions of elastic pp collisions at $\sqrt{s} = 7.0$ TeV with that of the elastic $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. These scaling functions are statistically significantly different. The confidence level of their agreement is maximum $CL = 3.7 \times 10^{-8}$ %, corresponding to a statistically significant, of at least 6.26σ Odderon signal. This difference is larger, than the 5σ threshold, required for a discovery in particle physics. The advantages of our method, with respect to comparing the cross sections directly include the scaling out of the s-dependencies of $\sigma_{\text{tot}}(s)$, $\sigma_{\text{el}}(s)$, $B(s)$ and $\rho(s)$, as well as the normalization of the $H(x)$ scaling function that cancels the point-to-point correlated and t-independent normalization errors.

4 Proton holography

A new possibility for phase reconstruction in elastic proton-proton scattering is based on our detailed analysis of elastic pp collisions data in the dip region at ISR and LHC energies [8, 22–25]. To support this conjecture, we present some qualitative considerations as well as the results of a model-independent study, and also discuss the currently known limitations of this promising new method.

As obvious, the phase reconstruction from the measurement of the differential cross-section of elastic scattering, in general and without additional information, is not possible. For example, if the measurements are limited to the diffractive cone, outside of the CNI region, but stopping well before the diffractive minimum-maximum (dip and bump) structure, the phase reconstruction from a structureless, nearly exponential cone is clearly meaningless and impossible. These limiting cases were discussed
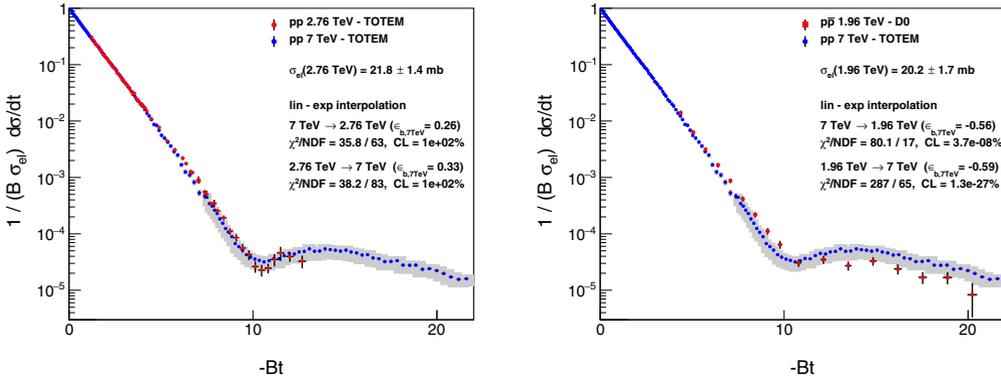
Figure 1. Left panel: For proton-proton elastic scattering, the $H(x, s) = \frac{1}{B\sigma_{el}}\frac{d\sigma}{dt}$ scaling function is energy independent in terms of $x = -tB$ at the TeV energy scale, or specifically, in the $\sqrt{s} = 2.76$ - 7 TeV energy range. TOTEM preliminary data at $\sqrt{s} = 8$ TeV [19] also follow this scaling law, while scaling violations are observed at $\sqrt{s} = 13$ TeV [8]. Right panel: A statistically significant difference is found between the $H(x, s)$ scaling functions for elastic pp collisions at $\sqrt{s} = 7.0$ TeV and that of $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. On both panels, vertical lines stand for the type A (point-to-point fluctuating) errors, while the horizontal lines indicate the point-to-point fluctuating uncertainty of the horizontal errors. Grey vertical bars stand for the type B (point-to-point changing, but overall correlated) errors and the width of this grey bars correspond to the type B errors in the horizontal (x) direction. Larger and smaller boxes correspond to two different measurements at $\sqrt{s} = 7$ TeV, detailed in Refs. [20, 21], respectively.

for example in Appendices C and D of Ref. [22]. However, the observation of a diffractive minimum, followed by a diffractive maximum corresponds to the observation of an interference pattern, one of the conditions of phase reconstruction and holography. Such an interference pattern is clearly visible as a function of the deflection angle in Fig. 1 of Ref. [26], originally from Ref. [27].

Recently, we have discussed model-independent, whole $-t$ range fits to the differential cross-section in Appendix A and B of Ref. [22] for pp and for $p\bar{p}$ elastic collisions, respectively. A closer inspection of these fits indicate that at the ISR energy range, the total cross-section is reproduced reasonably well, while the ρ_0 measurements are reproduced only in one case, at $\sqrt{s} = 44.7$ GeV, within two standard deviations. Given that we did *not* utilize any data in the CNI region, the bad reproduction of several of the measured ρ_0 values were not surprising. As discussed in Ref. [22], in the case of the very well measured TOTEM data at $\sqrt{s} = 13$ TeV, we were able to reproduce within experimental errors also the measured value of ρ_0, and we have obtained similarly good quality preliminary results at $\sqrt{s} = 7$ TeV as well.

We started to wonder, whether or not such a good quality ρ_0 determination from outside the CNI region is a coincidence only: it is possible that there is a deeper reason for it. As a first step, we have shown in Ref. [22], that *within* the technique of the model-independent Lévy series expansion, the reconstruction of the t-dependent phase or equivalently the determination of the $-t$ dependent $\rho(t)$ function is unique, similarly to how the coefficients of a Taylor-series can be uniquely determined if the measurement of the Taylor-expanded function is precise enough. Of course, behind this reconstruction lies a powerful physical assumption about the Lévy processes driving the elastic scattering in QCD which dictates a very specific form

of the elastic amplitude, in the form of generalised Lévy series. However, other methods may result in other $\phi(t)$ functions. We are currently working on these subtleties of the $-t$ dependent phase reconstruction by comparing the results of Ref. [22] with other, model-dependent efforts.

In this work, we thus start to explore some of the basic details of t-dependent phase reconstruction. We ask the following question: what are those special experimentally testable, but mathematically well defined conditions that are sufficient for an experimentally validated phase reconstruction in elastic pp or $p\bar{p}$ scattering? Let us recall two examples here about how such a phase reconstruction might work in elastic pp scattering.

The first example deals with phase reconstruction at the ISR energies. Such a reconstruction of the nuclear phase from the ISR data was performed in Ref. [7] utilizing the Phillips-Barger model [28]. This model was shown to be well suited to describe the ISR data in the $\sqrt{s} = 23.5$ - 62.1 GeV region with statistically acceptable, good quality fits in the $1.05 \leq -t \leq 2.5$ GeV^2 interval. Table 3 of Ref. [7] reported the values of $|\epsilon|$, the modulus square of the phase difference from π, with values small but close to zero in the range of (0.06 ± 0.06) – (0.34 ± 0.10). These values suggested that the imaginary part of the scattering amplitude approximately vanishes near the diffractive minimum, and the real part is dominant at the dip region, but due to the $\cos(\phi) = \cos(\pi \pm \epsilon) \approx 1 + 0.5\epsilon^2$ type of Taylor expansion, the sign of the phase difference, ϵ cannot be determined from the experimental data. Assuming that a crossing-odd contribution is negligible at the ISR energies, a more detailed, t-dependent reconstruction of $\rho(t) = \mathcal{R}eT/\mathcal{I}mT$ is reported in Fig. 22 of Ref. [7].

The Reggeized Phillips-Barger model [28] gave good quality fits to the ISR and LHC data in larger $-t$ intervals as well, starting from $-t \geq 0.3$ GeV^2 and in 2011 it was

used to study the Odderon effect in the elastic pp scattering data at LHC energies [15]. However, this model at small values of $-t \leq 0.3$ GeV^2 did not give a statistically acceptable data description, hence this model cannot be used to connect the phase measurement at the dip region of $-t \approx 0.5$ GeV^2 with CNI measurements at the optical point $t \approx 0$ GeV^2, so this model cannot be used to resolve the Cul-de-Sac of elastic scattering [14]. However, the model-independent Lévy series [22] is able to do this job. As far as we know, this is the only method so far, that is able to describe the elastic pp collisions at the currently highest energy of $\sqrt{s} = 13$ TeV [27] with a statistically acceptable, 2% confidence level. Within this model-independent approach the t-dependent phase reconstruction appears to be unique [22]. Increasing the order of the series, the domain of the convergence starts from $t \approx 0$ and is increasing gradually. The convergence region for the phase of the fourth order Lévy series is $0 \leq -t \lessapprox 0.5$ and 0.9 GeV^2 at $\sqrt{s} = 7$ and 13 TeV, respectively, as shown on Fig. 2.

The left panel of Fig. 2 indicates that for proton-proton elastic scattering at $\sqrt{s} = 13.0$ TeV, the nuclear phase reconstruction based on a 4th-order Levy expansion converges up to $-t \lessapprox 0.9$ GeV^2. The right panel of the same figure indicates similar results but at $\sqrt{s} = 7.0$ TeV. These data are less precise as compared to the measurements at $\sqrt{s} = 13.0$ TeV, correspondingly the domain of convergence of the phase reconstruction is smaller. Generically, the greater the experimental precision, the larger the domain of convergence of this method. These plots suggest that the reconstruction of the t-dependent phase is possible with the help of this Lévy series technique, providing a new kind of model-independent proton holography. This is our second example, detailed below.

One of the most intriguing properties of elastic pp collisions at the ISR energy range of $\sqrt{s} = 23.5 - 62.1$ GeV and at the LHC energy range of $\sqrt{s} = 2.76 - 13.0$ TeV is the existence of a unique, single diffractive minimum, and the existence of a nearly exponential diffractive cone not only at very low $|t|$ but also a similar, secondary diffractive cone that follows after the diffractive maximum [25]. From measurements of the CNI region, reviewed recently for example in Fig. 15 of Ref. [29], we also know that in each of these measurements, $\rho_0 \leq 0.15$ that implies that close to $t \approx 0$, the real part of the scattering amplitude is, as compared to the imaginary part, relatively small.

By means of the optical theorem, a total cross section measurement uniquely determines the imaginary part of the elastic amplitude at $t = 0$, as indicated by Eq. (5). This implies that not only the imaginary but also the ρ_0^2 or the squared value of real part of the forward scattering amplitude can also be uniquely determined at the optical point, $t = 0$. Indeed, using the diffractive cone approximation and Eqs. (22) and (23), from the values of $A(s)$, $B(s)$ and $\sigma_{\text{tot}}(s)$, the values of $\sigma_{\text{el}}(s)$ and $\rho_0^2(s)$ are determined uniquely. From such a determination of $\rho_0^2(s)$, the value of the phase of the scattering amplitude at the optical point, $\phi(t = 0)$ is fixed to two possible values by measurements. However, to select one of these values unambiguously, further information about the CNI interference has to be utilized. This dual degeneracy is related to the fact

that the sign of ρ_0 cannot be determined from measurements of ρ_0^2, such that both possible values, corresponding to $\pm\sqrt{\rho_0^2(s)}$, are allowed. These two different allowed phases correspond to the object and the conjugated object wave reconstruction in holography.

One can determine the nuclear phase at the dip t_{dip}, too, where the differential cross-section of elastic proton-proton collisions has a diffractive minimum. At this point, the imaginary part of the elastic scattering amplitude approximately vanishes, and the differential cross-section is dominated in the dip region by the real part of the forward scattering amplitude. Thus, the nuclear phase is approximately an integer multiple of π close to the dip region. This is a degeneracy similar to the ρ_0 sign problem, mentioned in the previous paragraph, and to the object beam and the conjugated object beam reconstruction in holography, mentioned in the Introduction. This problem has been called the Cul-de-Sac of elastic scattering in Ref. [14]: at the diffractive minimum, only the modulus but not the sign of the real part can be uniquely determined, hence the nuclear phase is either 0 or π, up to small corrections.

The fact that at $\sqrt{s} = 13$ TeV, the phase at the dip connects analytically to the phase measured in the CNI region indicates, that with the help of the Lévy expansion method, the nuclear phase of the elastic scattering amplitude can be realistically determined, at least in certain specific cases. A posteriori, our method is validated by the excellent reproduction of the ρ_0 value measured in the CNI region in elastic pp collisions at $\sqrt{s} = 13$ TeV by the TOTEM Collaboration [29]. Fits detailed in Appendices A and B of Ref. [22] indicate that indeed the 4th and 3rd order Lévy expansions reproduce well measured values of ρ_0 not only at $\sqrt{s} = 13$ TeV but at lower LHC energies as well, if the fit quality is satisfactory. However, one should put a pull on ρ_0 in the definition of χ^2 as the sign problem at the dip cannot be resolved otherwise. The two different signs of the phase near the dip can lead to two different analytic continuation of the phase to $t = 0$ but only one of these continuations lead to the correct value of ρ_0.

5 Summary

We have described a new Odderon signal [8], the validity of the $H(x)$ scaling for pp collisions and its violation in $p\bar{p}$ collisions in the few TeV energy range shown and quantified in the right panel of Fig. 1. The statistical significance of this Odderon signal is found to be at least 6.26σ.

We have also discussed the related topic of proton holography or the t-dependent reconstruction of the nuclear phase. If we assume that this nuclear phase $\phi(t)$ is an analytic and continuous function of t, and the differential cross-section of elastic scattering data is determined with great precision, the two different approximate values of the phases at the dip, 0 and π can be analytically and continuously extrapolated to $t \to 0$. Furthermore, our analysis with Lévy series performed in Ref. [22] suggested that if these two extrapolated values are within the errors of the extrapolations significantly different from one another, and only one of them is consistent with the determination of
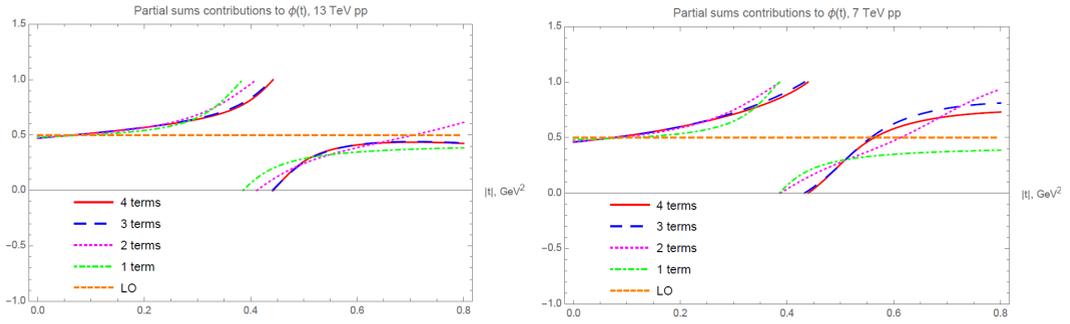
Figure 2. Left: For *pp* elastic scattering at $\sqrt{s}$ = 13.0 TeV, the nuclear phase reconstruction based on a 4th order Levy expansion converges up to $-t \lessapprox 0.9$ GeV^{-2}. Right: The phase reconstruction at $\sqrt{s}$ = 7.0 TeV converges up to $-t \lessapprox 0.5$ GeV^{-2}, as detailed in [24].

the ρ_0 using the CNI methods, then the *t*-dependence of the nuclear phase can be reconstructed not only at a given value of $t = 0$ or $t = t_{\text{dip}}$ but in the entire *t* region, where such an extrapolation is possible and convergent.

The search for a sufficient and necessary condition that would make the *t*-dependent phase reconstruction unique not only within, but also outside the Lévy series method is still ongoing at the time of closing this manuscript.

Acknowledgments

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