

TWO PARAMETER-RETRIEVAL ALGORITHMS OF AIRCRAFT WAKE VORTEX WITH DOPPLER LIDAR IN CLEAR AIR

Chun Shen, Jianbing Li*, Hang Gao

State Key Laboratory of Complex Electromagnetic Effects on Electronics and Information System,
 National University of Defense Technology, Changsha 410073, China

*Email: jianbingli@nudt.edu.cn

ABSTRACT

Aircraft wake is a pair of strong counter-rotating vortices generated behind an aircraft, which might be very hazardous to a following aircraft and the detection of which has attracted much attention in aviation safety field. This conference paper introduces two parameter-retrieval algorithms, i.e., Optimization method and Max-min method. They have been integrated into a toolbox and can retrieve the parameters of wake vortex efficiently and robustly.

1. INTRODUCTION

Aircraft wake vortex is a strong counter-rotating turbulence generated by a flying aircraft. It might be very dangerous for a following aircraft because it may cause the following aircraft to roll out of control. To avoid this hazard, a special flight separation rule was proposed by ICAO, but it is conservative and has limited the flight capacity of airports. Therefore, it is very important to detect the wake vortex in real time to make a compromise between the flight safety and capacity. The Single European Sky Air Traffic Management Research (SESAR)^[1] plan in EU and the Next Generation Air Transportation System (NGATS)^[2,3] plan in US have also delivered a lot of projects on this issue.

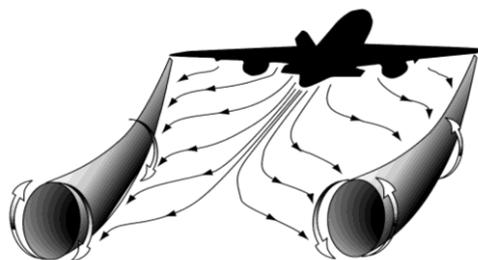


Fig. 1 Formation of Wake Vortex

Basically, there are two types of sensors for wake vortex detection: Radar and Lidar^[4]. For wake vortex generated in clean air, the proposed sensor is Lidar, and the representative parameter-retrieval

algorithms include the TV method^[5] and the radial velocity method^[6-8]. We have been studying on the radar characteristics and detection of wake vortex for many years and we recently proposed two methods to retrieve the parameters of aircraft wake vortex, say the Optimization method and the Max-Min method. In this conference paper, we briefly introduce these two methods, and whose performances are verified with simulated and field detection data.

2. TWO NEW ALGORITHMS

2.1 Optimization Method

For Lidar detection of aircraft wake vortex in clear air, the particles to be sensed are the aerosols, which are generally with weak inertia. In this sense, the obtained Doppler velocity is assumed to be approximately the same to that of the ambient velocity. According to this principle, a movement equation^[9] is established to connect the Doppler velocity and the parameters of wake vortex, and an optimization method has been proposed to solve the equation well.

The radial velocity of the i^{th} detection cell (equal to and opposite to the doppler velocity, $-V_D^{(i)}$) represents the projection of the background velocity ($V^{\text{flow},(i)}$) on the lidar line of sight:

$$-V_D^{(i)} = \mathbf{V}^{\text{flow},(i)} \Big|_{\mathbf{r}} = \mathbf{V}^{\text{wind},(i)} \Big|_{\mathbf{r}} + \mathbf{V}^{\text{wake},(i)} \Big|_{\mathbf{r}} \quad (1)$$

where $\mathbf{V}^{\text{wind},(i)}$ and $\mathbf{V}^{\text{wake},(i)}$ are the background wind velocity and wake vortex velocity at the detection cell, $\mathbf{X} \Big|_{\mathbf{r}} = \mathbf{k} \cdot \mathbf{X}$ are the velocity vector \mathbf{X} project on the lidar beam, and \mathbf{k} is the direction vector of the beam with an elevation Angle of α : $\mathbf{k} = [\cos \alpha \quad \sin \alpha]$

According to equation (1), the equation of wake vortex characteristic parameter-retrieval is composed of background wind velocity, wake velocity and doppler velocity:

$$\begin{aligned} \mathcal{F}^{(i)}(\Gamma_L, \Gamma_R, O_L, O_R; V_D^{(i)}, x_i, y_i) &= V^{\text{wind},(i)} \Big|_r + \\ V^{\text{wake},(i)}(\Gamma_L, \Gamma_R, O_L, O_R) \Big|_r + V_D^{(i)} &= 0 \end{aligned} \quad (2)$$

The equation contains six unknowns (O_L, O_R respectively represent left vortex-core and right vortex-core position), so when we use M detection cells $[x_i, y_i, V_D^{(i)}]$ ($M \geq 6$), the equation can be solved:

$$[\hat{\Gamma}_L, \hat{\Gamma}_R, \hat{O}_L, \hat{O}_R] = \underset{\Gamma_L, \Gamma_R, O_L, O_R, r_c}{\text{argmin}} \sum_{i=1}^M \left| \mathcal{F}^{(i)}(\Gamma_L, \Gamma_R, O_L, O_R; V_D^{(i)}, x_i, y_i) \right|^2 \quad (3)$$

The strengths of this method include:

1. The ground effect of wake vortex has been considered by introducing two imaging vortices in the velocity model.
2. The wake vortex moves while the Lidar beam scans up and down alternately, which leads to distortion of Doppler velocity on the RHI plane. This distortion has been fixed by taking into account the acceleration of wake vortex.
3. The fitting algorithm has been introduced to predict the vortex core positions, which could be very helpful to get robust and accurate retrieval results.

2.2 Max-min Method

The difference between the maximum and minimum doppler velocities at different elevation and the same radial distance R_k is called "doppler velocity range"^[10]

$$\Delta V(R_k) = \max_{\varphi \in [\varphi_{\min}, \varphi_{\max}]} \{V_D(R_k, \varphi)\} - \min_{\varphi \in [\varphi_{\min}, \varphi_{\max}]} \{V_D(R_k, \varphi)\} \quad (4)$$

where $V_D(R_k, \varphi)$ represents the doppler velocity.

The "velocity range" at the center of the vortex-core should be significantly larger than that at other radial distances. Therefore, the "velocity range" at different radial distances is fitted as a double Gaussian curve, and two peak points corresponding to the left and right vortex cores can be obtained. The radial distance from the peak point to the lidar is the estimated value of left or right vortex radial distance (R_{c1} or R_{c2}).

The Doppler velocity along the elevation direction is used to determine the left and right vortices' elevation angles (φ_{c1} and φ_{c2}). If the maximum and minimum Doppler velocities are found at $(R_{ci}, \varphi_{ci, \max})$ and $(R_{ci}, \varphi_{ci, \min})$ ($i = 1, 2$), the elevation angle of a vortex-core can be approximated as $\varphi_{ci} = \frac{1}{2}(\varphi_{ci, \max} + \varphi_{ci, \min})$ ($i = 1, 2$).

From the Burnham-Hallock velocity model, the tangential velocity of a single vortex whose core located at (R_i, φ_{ci}) is:

$$V^\alpha = \frac{\Gamma_i}{2\pi R} \cdot \frac{R^2}{R^2 + r_c^2} \quad (5)$$

where R is the distance from the detection cell to the vortex-core, Γ_i is the circulation, r_c is the vortex-core radius.

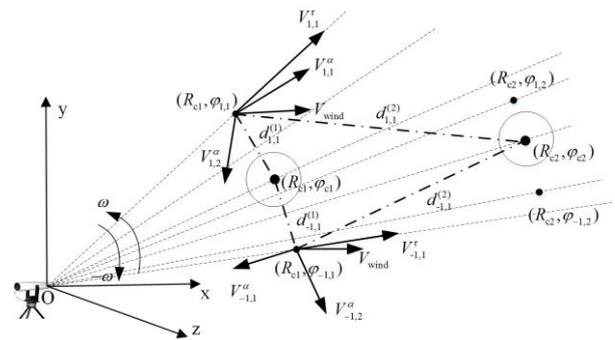


Fig. 2 Velocity Component of The Detection Cell

Fig 2 shows velocity component of the detection cell on the radial distance of the left vortex, $V_{1,1}^r$ and $V_{-1,1}^r$ respectively represent velocity above and below the vortex core:

$$\begin{aligned} V_{1,1}^r &= \frac{\Gamma_1}{2\pi} \cdot \frac{R_{c1}}{d_{1,1}^2 + r_c^2} \cdot \sin(\varphi_1 - \varphi_{c1}) \\ &- \frac{\Gamma_2}{2\pi} \cdot \frac{R_{c2}}{d_{1,2}^2 + r_c^2} \cdot \sin(\varphi_1 - \varphi_{c2}) + V_{\text{wind}} \cdot \cos \varphi_1 \end{aligned} \quad (6)$$

$$\begin{aligned} V_{-1,1}^r &= \frac{\Gamma_1}{2\pi} \cdot \frac{R_{c1}}{d_{-1,1}^2 + r_c^2} \cdot \sin(\varphi_{-1} - \varphi_{c1}) \\ &- \frac{\Gamma_2}{2\pi} \cdot \frac{R_{c2}}{d_{-1,2}^2 + r_c^2} \cdot \sin(\varphi_{-1} - \varphi_{c2}) + V_{\text{wind}} \cdot \cos \varphi_{-1} \end{aligned} \quad (7)$$

where d_{1i}, d_{-1i} ($i = 1, 2$) are the distances from the detection cell (above and below the vortex core) to the i^{th} vortex core. φ_1 (φ_{-1}) is the elevation

angle of the extreme Doppler velocities above (below) the vortex core. Normally the Lidar set at a long distance to the wake vortex, we can assume $V_{wind} \cdot \cos \varphi_1 \approx V_{wind} \cdot \cos \varphi_{-1}$, thus the effect of background wind is mitigated by the subtraction of the two velocities:

$$\Delta V_1 = V_1^r - V_{-1}^r \approx [\Psi_1 \quad \Psi_2] \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \quad (8)$$

where

$$\Psi_1 = \frac{R_{c1}}{2\pi} \cdot \left[\frac{\sin(\varphi_1 - \varphi_{c1})}{d_{11}^2 + r_c^2} + \frac{\sin(\varphi_{c1} - \varphi_{-1})}{d_{-11}^2 + r_c^2} \right]$$

$$\Psi_2 = -\frac{R_{c2}}{2\pi} \cdot \left[\frac{\sin(\varphi_1 - \varphi_{c2})}{d_{12}^2 + r_c^2} + \frac{\sin(\varphi_{c2} - \varphi_{-1})}{d_{-12}^2 + r_c^2} \right]$$

with $r_c = 0.052b_0$ being the vortex core size, and b_0 being the separation between the two vortex cores.

$[\Gamma_1, \Gamma_2]$ can be easily solved by equation (8).

3. RESULTS

3.1 Wake-Vortex Parameter-Retrieval System

We have developed a comprehensive wake vortex simulation and parameter-retrieval system as shown in Fig. 3.

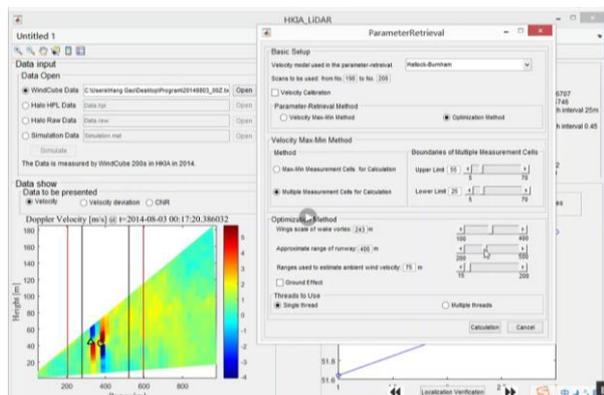


Fig. 3 Wake Vortex Parameter-Retrieval System

The system can well simulate the dynamics, scattering, echo signal, Doppler spectra, and etc. It can handle the Lidar detection data from a number of different Lidars, for example the Halo Streamline, Wind Cube 400S, WindTracer, etc. The two algorithms have been integrated into the processing system to retrieve the wake vortex parameters from simulated data or field detection data, and good performances have been well verified.

3.2 Retrieval Result of Circulation

While calculating on the same computer, the time costs of the RV method, Optimization method, Max-min method and TV method are several minutes, 5.6s, 3.5s and 1.9s. For the purpose of real time, this conference paper does not show the result of RV method.

Fig 4 shows the circulation result of the three methods. It can be found that Max-min method and Optimization method can get more robust result than TV method. The TV method could overestimate the circulation, which is generally caused by locating the tangential velocity improperly. The Max-min method and Optimization methods have a slight underestimation, which is caused by spatial averaging error of the simulated data.

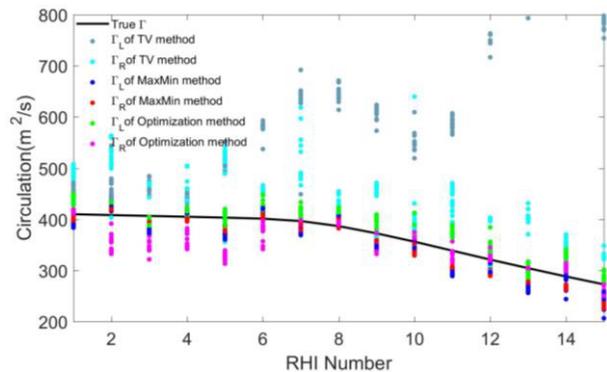


Fig. 4 Circulation Result on 15 Simulated RHIs

4. CONCLUSION

This paper introduces two parameter-retrieval methods we recently developed, i.e., the optimization method and the Max-Min velocity method. They have been integrated into a software toolbox and the good performances have been well verified. The two methods can be very helpful for real time detection and prediction of wake vortex in air traffic control.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (No. 61771479) and distinguished young scholar fund of Hunan Province (No.2018JJ1030).

REFERENCES

- [1] Sesar Consortium. European Air Traffic Management Master Plan [R]. SESAR Consortium, 2009.
- [2] J. Burden, P. A. Curry, D. Roby, F. Love. Introduction to the Next Generation Automatic Test System (NGATS) [R]. IEEE Autotestcon, 2005.
- [3] P. A. Curry, J. Burden, G. A. Lundy. Next Generation Automatic Test System (NGATS) Update [R]. IEEE Autotestcon, 2006.
- [4] S. Hannon, J. Alexthomson. Aircraft Wake Vortex Detection and Measurement with Pulsed Solid-State Coherent Laser Radar [J]. *Optica Acta International Journal of Optics*, 1994, 41(11): 2175-2196.
- [5] F. Holzäpfel, T. Gerz, F. Köpp. Strategies for Circulation Evaluation of Aircraft Wake Vortices Measured by Lidar [J]. *Journal of Atmospheric and Oceanic Technology*, 2003, 20: 1183-1195.
- [6] I. N. Smalikho, V. A. Banakh, F. Holzäpfel, S. Rahm. Method of Radial Velocities for the Estimation of Aircraft Wake Vortex Parameters from Data Measured by Coherent Doppler Lidar [J]. *Optics Express*, 2015, 23(19): A1194-A1207.
- [7] I. N. Smalikho, V. A. Banakh. Estimation of Aircraft Wake Vortex Parameters from Data Measured by a Stream Line Lidar [C]// 21st International Symposium on Atmospheric and Ocean Optics: Atmospheric Physics, 2015.
- [8] I. N. Smalikho, V. A. Banakh. Estimation of Aircraft Wake Vortex Parameters from Data Measured with a 1.5-Um Coherent Doppler Lidar [J]. *Optics Letters*, 2015, 40(14): 3408-3411.
- [9] H. Gao, J. B. Li, P. W. Chan, K. K. Hon, X. S. Wang. Parameter-Retrieval of Dry-Air Wake Vortices with a Scanning Doppler Lidar [J]. *Optics Express*, 2018, 26(13):16377.
- [10] H. Gao, J. B. Li, P. W. Chan, K. K. Hon. Parameter-Retrieval of Aircraft Wake Vortex Based on Its Max-Min Distribution of Doppler Velocities Measured by a Lidar [C]. IET International Radar Conference, Oct. 2018, Nanjing, China.