Incorporating a two-step mechanism into calculations of (p,t) reactions used to populate compound nucleus spin-parity distributions in support of surrogate measurements

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Abstract.

The surrogate reaction method may be used to determine the cross section for neutron-induced reactions not accessible through standard experimental techniques by creating the same compound nucleus which the desired reaction would pass through, but via a different entrance channel. A variety of direct reactions have been employed in order to generate the required compound nuclei for surrogate studies.

In this work, a previously developed (p,t) reaction model has been extended to incorporate a two-step reaction mechanism, which takes the form of sequential neutron transfer. This updated model is applied to the ^{92}Zr(p,t)^{90}Zr reaction and is found to modify the strengths of the previously predicted populated levels.
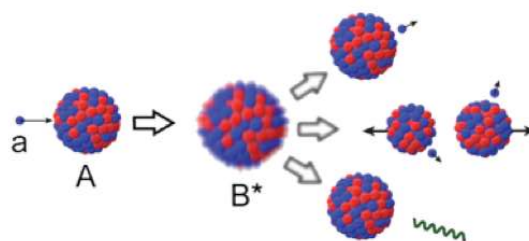
It is planned that this improved (p,t) model will be used to attempt to constrain cross section predictions for a number of (n,γ) reactions in future, as well as provide a possible comparison against other surrogate studies utilising different direct reactions such as (p,d).

1 Introduction

For many isotopes of interest it is unfortunately not possible to conduct conventional neutron-induced cross section measurements. There are various reasons why it may not be possible to measure a certain nuclear reaction directly. Most common amongst these reasons is too short a lifetime to produce a target for the nucleus in question.

The majority of reactions of interest to the nuclear industry, and many relevant to astrophysics, involve the collision of an incident neutron with a target nucleus. Aside from elastic scattering, the reactions which may occur due to an incident neutron typically take place through an intermediate compound nucleus state, as shown in Figure 1.

Here the incident neutron is absorbed by, and shares its energy with, the target nucleus and forms an excited compound nucleus. This compound nucleus is unstable and will decay after some time to form the final products of the reaction. The basis of the surrogate reaction method is to exploit the Bohr assumption, that the mode of decay of a compound nucleus is independent of the type of reaction from which it formed [1]. It is assumed that only the spin distribution of the states, in both energy and angular momentum, populated in the compound nucleus plays a role in determining the statistical likelihood of decays via each possible channel. In the surrogate reaction method, a suitable surrogate nucleus and reaction are sought, such that the same compound nucleus, and if possible its spin distribution, will be formed as is expected in the reaction of interest. The surrogate process is illustrated in Figure 2.
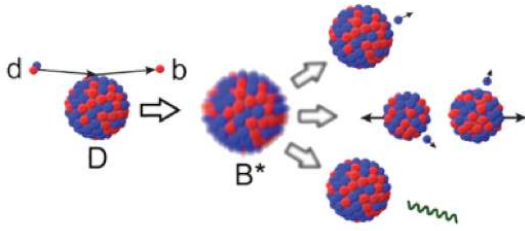

Figure 1. Illustration of a nuclear reaction proceeding through the compound nucleus stage. Here the projectile *a* collides with *A* to form B^*, which then decays to a number of possible products. Image taken from Ref. [3].

In the desired reaction case, one needs to determine the cross section $\sigma_{\alpha\chi}(E_a)$ for a reaction with incident channel $(a + A)$ at an incident energy E_a and exit channel χ. The cross section $\sigma_{\alpha\chi}(E_a)$ is often split into two components, as shown in Equation 1;

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_{\alpha}^{B^*}(E_{ex}, J, \pi) G_{\chi}^{B^*}(E_{ex}, J, \pi) . \quad (1)$$

Here $\sigma_{\alpha}^{B^*}(E_{ex}, J, \pi)$ is the cross section for $(a+A)$ forming the compound nucleus B^* with an excitation energy E_{ex} in the state J^π and $G_{\chi}^{B^*}(E_{ex}, J, \pi)$ is the probability, or branching ratio, of B^* decaying to channel χ.

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Figure 2. Illustration of the surrogate reaction method. Here, d interacts with D to form the required compound nucleus B^*. d may be scattered inelastically or may gain or lose nucleons to form b.

Figure 3. Schematic showing the translation of quantum numbers used in the two-neutron basis into those required for calculations in the di-neutron pseudo-particle basis.

In a standard measurement $\sigma_{\alpha\chi}(E_a)$ is measured directly. In the surrogate approach however, $G^{B^*}_\chi(E_{ex}, J, \pi)$ is determined by indirect measurement and $\sigma^{B^*}_\alpha(E_{ex}, J, \pi)$ calculated, generally via the optical model.

A number of different direct reactions have been studied by various research groups, e.g. (p,p′), (p,d) [4], (^{3}He,α) [7] and (d,p) [6]. Recent studies have shown that the cross section for the (n,γ) reaction can be successfully constrained for some isotopes using the surrogates approach [9].

The (p,t) reaction has also been investigated as a potential surrogate reaction [4], but not to the same level of scrutiny as the previously listed reactions. A (p,t) reaction model was previously developed by this author and was applied to (p,t) data captured as part of a separate (p,d) surrogate reaction study [8]. The remainder of this paper describes extensions made to this previously developed (p,t) model to incorporate two-step transfer mechanisms.
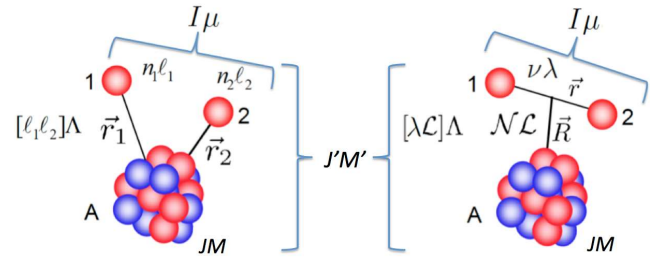
2 (p,t) model

For the (p,t) model developed previously we made two assumptions for the reaction:

- The target nucleus is even-even, i.e. it possesses even numbers of both protons and neutrons, is spherical, and in its groundstate has a spin and parity assignment of $J^\pi = 0^+$.

- The two neutrons are transferred simultaneously, in one-step, as a single spin-singlet 'di-neutron' quasi-particle, with zero intrinsic angular momentum.

It is the validity of the latter of these two assumptions that we will investigate in this study.

In order to perform calculations for the transfer of a di-neutron, the quantum numbers of the two tranferred neutrons in their single particle orbitals must first be translated into those of an equivalent single particle. The translated quantum numbers are illustrated in Figure 3.

Calculations of cross sections for the direct (p,t) transfer reaction on a mass $A + 2$ target nucleus, populating specific J^π, A-body final states, involve a number of components, see e.g. Glendenning [10]. The expression used for the cross section in the developed model is

$$\frac{d\sigma}{d\Omega}(0^+ \to J^\pi) = |\sum_{N\Lambda S} G_{N\Lambda S J}B_{N\Lambda S J}(\vec{k_p}\vec{k_t})|^2 \ . \quad (2)$$
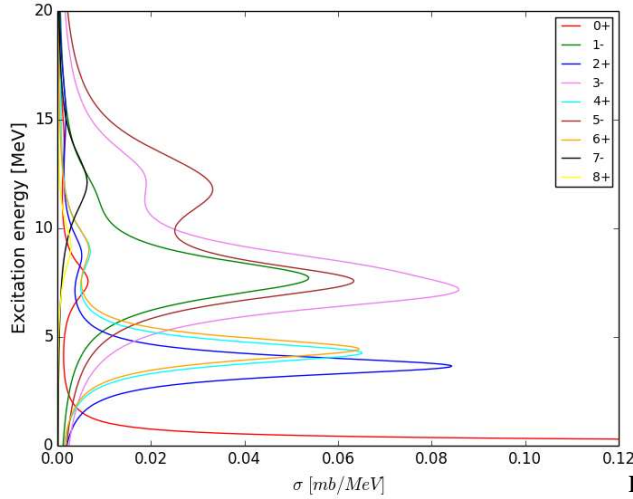
Here $B_{N\Lambda S J}$ is the (p,t) transition amplitude calculated via the Distorted Wave Born Approximation (DWBA) method and $G_{N\Lambda S J}$ is broadly analogous to the spectroscopic amplitude of single-nucleon transfer reactions. $G_{N\Lambda S J}$ is constructed from a number of nuclear strucutral factors;

$$G_{\gamma N\Lambda S J} = \sum_\gamma g_\gamma \beta_{\gamma N\Lambda S J}\Omega_\nu \langle \nu\lambda, NL; \Lambda|n_1 l_1, n_2 l_2; \Lambda \rangle \ . \quad (3)$$

Here, $g_\gamma = \sqrt{2/(1 + \delta_{ij})}$ is a symmetry factor dependent on the (like or unlike) pair of orbitals i and j occupied by the pair of transferred neutrons. $\langle \nu\lambda, NL; \Lambda|n_1 l_1, n_2 l_2; \Lambda \rangle$ is a *Moshinsky bracket* [11] which gives the overlap between the wavefunctions of the two neutron single particle states with their required wavefunction within the di-neutron. $\beta_{\gamma N\Lambda S J}$ is a parentage coefficient which measures the component of the target 0^+ $A + 2$ mass nucleus which is constructed from a specific J^π A mass core coupled to the two transferred neutrons. Ω_ν is the overlap of the two-neutron relative motion wave functions between the intitial and final states. Ω_ν is generally taken to be 1.

As only 0^+ target nuclei are considered in this model, the total and orbital angular momentum transferred are equal, i.e. ($\Lambda =)L = J$, and only *natural* parity states with $\pi = (-1)^J$ are populated.

The developed model was applied to the ^{92}Zr(p,t)^{90}Zr reaction, for which data had been taken as part of a separate (p,d) surrogate reaction study [5]. These data was taken using the STARLiTeR detector [17] fielded at the Texas A&M K150 Superconducting Cyclotron.

Figure 4. Predicted excited states of ^{90}Zr calculated to be populated via the ^{92}Zr(p,t)^{90}Zr reaction.

Figure 5. Comparison of the observed triton spectrum against the theoretical predictions integrated over the entire detector surface.

The DWBA portions of the calculation were performed using the code TWOFNR [18]. A calculation was performed for the transfer of each possible pair of neutrons in the ^{92}Zr target nucleus, assuming standard shell model filling rules. A Hartree-Fock calculation was performed using the DENS code [13] with the Skx Skyrme potential in order to calculate energy levels of the orbitals of the target nucleus. These energies were then scaled using the experimental S_{2n} value, which is assumed to correspond to the removal of two neutrons from the outermost orbital. The excitation energy of the residual nucleus due to the removal of the neutron pair was then determined by summing the energies of the two orbitals the neutrons were removed from. Degenerate excited levels are split using the phenomenological method suggested by Casten [2].

The model assumes integer occupation of the shell model orbitals in the target nucleus and so a spreading width is also applied to the calculated excited levels in order to account for the expected *fragmentation* due to the reality of fractional occupancy of levels [14]. The form selected is, aside from an additional factor of $\frac{1}{2}$, that of Brown and Rho [12] and which has previously been applied to single particle transfers;
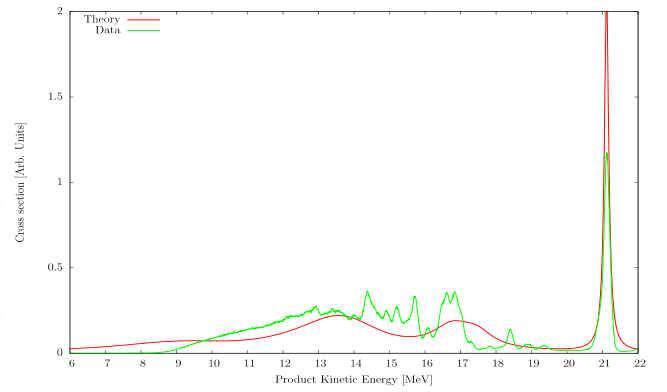
$$\Gamma(E_{ex}) = \frac{1}{2}\left[\frac{\epsilon_0(E_{ex})^2}{(E_{ex})^2 + E_0^2} + \frac{\epsilon_1(E_{ex})^2}{(E_{ex})^2 + E_1^2}\right], \quad (4)$$

where E_{ex} is the excitation energy of the residual state and $\epsilon_0, \epsilon_1, E_0$ and E_1 are constants.

Figure 4 shows the distribution of states predicted to be populated in the residual ^{90}Zr nucleus. This distribution of excited states is converted into a spectrum of outgoing tritons and compared against the experimental data in Figure 5. As may be seen, the match between the experimental data and the theoretical model is reasonable.

3 Extension to two-step mechanism

Although the one-step transfer of a di-neutron quasi-particle has been applied to many systems with a great deal of success, a number of authors have noted deficiencies in this model, particularly that it does not represent the correct physics of the reaction. Various authors including Igarashi et al [15], Potel et al [16], etc, suggest a two-step sequential transfer process is required.

For the present study we have applied a sequential neutron transfer process to the ^{92}Zr(p,t)^{90}Zr groundstate to groundstate transition which will involve the transfer of a pair of $d_{\frac{5}{2}}$ neutrons. Multi-step processes due to collective excitations have been ignored for the time being for this system.

A more general expression for the (p,t) cross section is given by

$$\frac{d\sigma}{d\Omega}(J_0^{\pi_0} \to J^\pi) = \frac{\mu_p\mu_t}{(2\pi\hbar)^2}\frac{k_t}{k_p}\frac{1}{(2S_p+1)(2J_0+1)}$$
$$\times \sum_{\sigma_p\sigma_t MM_0} |T_{JM\sigma_t, J_0M_0\sigma_p}(\vec{k_t}, \vec{k_p})|^2. \quad (5)$$
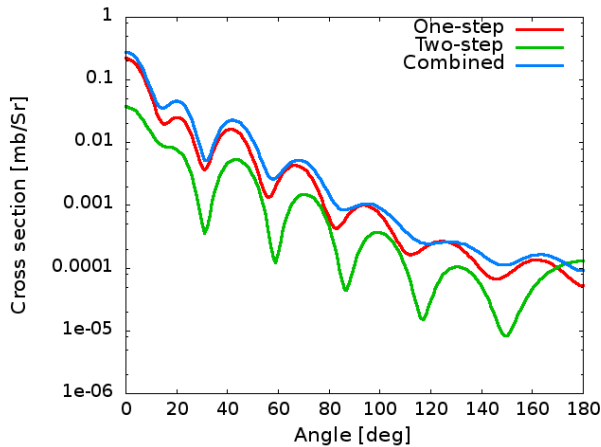
For the case of sequential neutron transfers, the *T-matrix* element will be constructed from three separate components

$$T = T^{\text{one-step}} + T^{\text{two-step}} - T_{\text{NO}}^{\text{two-step}}, \quad (6)$$

where the final term is a required *non-orthogonality* correction.

$$T^{\text{two-step}} - T_{\text{NO}}^{\text{two-step}} =$$
$$\left\langle \Phi(A)\phi_t\chi_t^{(-)}|V_{d-n}|\Phi(A+1)\phi_d\right\rangle$$
$$\times\ G_d^+ \left\langle\Phi(A+1)\phi_d|V_{p-n}|\Phi(A+2)\phi_p\chi_p^{(+)}\right\rangle$$
$$-\left\langle\Phi(A)\phi_t\chi_t^{(-)}|\Phi(A+1)\phi_d\right\rangle$$
$$\times\left\langle\Phi(A+1)\phi_d|V_{p-n}|\Phi(A+2)\phi_p\chi_p^{(+)}\right\rangle, \quad (7)$$

where $\Phi()$ is a wavefunction describing the collective behaviour of the nucleons within the nucleus, ϕ is the wavefunction describing the behaviour of the nucleons within the light ion, V is the interaction strength between the light

Figure 6. Comparison of the predicted triton angular distribution for the one-step, two-step, and combined model calculations.

ion and an individual neutron [19], χ is a spinor for the light ion, and G_d^+ is a Green's function which describes the propagation of the (unbound) deuteron in the intermediate state and has the form;

$$G_d^+(R, R') = -\frac{1}{k_d} F(R_<) H^+(R_>) \tag{8}$$

where F is a regular Coulomb function and H^+ is a Hankel function. G_d^+ is also distorted by an apropriate deuteron OMP to account for possible compound nucleus reactions which may occur between the deuteron and (A-1) nucleus.

Each step of the calculation requires an amplitude of the overlap between the wavefunctions of the nuclei in each partition. We will continue to assume (perhaps naïvely) integer occupancy of the valence orbitals and so the same parentage factors determined in the original model may be used. Unfortunately the version of TWOFNR available for this study lacked the functionality to perform two-step transfers and so the FRESCO code [20] was used instead.

4 Results and analysis

The results of calculations performed using FRESCO for the ^{92}Zr(p,t)^{90}Zr reaction to the 0^+ groundstate are shown in Figure 6. Calculations have been performed for the case of a simultaneous transfer of the neutron pair, sequential transfer of the pair, and also the combined case where both channels are possible. It may be seen that the strengths of the simultaneous and sequential channels do not add to the combined strength as it is their amplitudes that are summed and so may interfere either constructively or destructively. It may also be seen that, for this transition, the two-step process, although weak compared to the one-step

contribution, is required for completeness, particularly at backwards angles.

Similar results are also seen for the ^{92}Zr(p,t)^{90}Zr reaction populating 2^+, 4^+ and 6^+ states. Further transitions have yet to be studied.

5 Summary and Conclusions

A two-step mechanism has been added to a previously developed (p,t) reaction model. This two-step process proceeds via the sequential transfer of the neutron pair. Preliminary results show that this two-step process is weaker than that of simultaneous transfer, but is still significant.

Acknowledgments

References

[1] J.M. Blatt & V.F. Weisskopf, *Theoretical nuclear physics,* (John Wiley & Sons, 1952)
[2] R.F Casten, *Nuclear Structure from a Simple Perspective,* (Oxford University Press, 2000)
[3] J.E. Escher et al, Rev. Mod. Phys. **84**, 353 (2012)
[4] R.O. Hughes et al, Phys. Rev. C **90**, 014304 (2014)
[5] R.O. Hughes et al, Phys. Rev. C **93**, 024315 (2016).
[6] A. Ratkiewicz et al, Phys. Rev. Lett. **122**, 052502 (2019).
[7] G. Boutoux et al, Phys. Rev. B **712**(4-5), 319 (2012).
[8] J. Benstead et al, EPJ Web of Conferences **122**, 12002 (2016).
[9] J.E. Escher et al, Phys. Rev. Lett. **121**, 052501 (2018).
[10] N.K. Glendenning, Phys. Rev. B **137**, 102 (1965).
[11] M. Moshinsky, Nucl. Phys. **13**, 104 (1959).
[12] G.E. Brown & M. Rho, Nucl. Phys. A **372**(3), 397 (1981).
[13] B.A. Brown et al, MSU-NSCL Report, **1289** (2004).
[14] M. Matoba., Nucl. Phys. A **581**, 21 (1995).
[15] M. Igarashi., *PhD Thesis*, Department of Physics, Tokyo Medical College (1988).
[16] G. Potel et al, Phys. Rev. Lett. **107**, 092501 (2011).
[17] S.R. Lesher et al, Nucl. Instr. and Meth. in Phys. Res. Section A. **621**, 286 (2010).
[18] J.A. Tostevin, University of Surrey version of code TWOFNR of M. Toyama, M. Igarashi and N. Kishida (2012).
[19] D. Feng et al, Phys. Rev. Lett. **44**, 1037 (1980).
[20] I.J. Thompson, FRESCO homepage, http://www.fresco.org.uk (2019).