

Modified Statistical Analysis of SNe1a Data

Lisa Goh^{1,*} and Cindy Ng^{1,**}

¹Department of Physics, National University of Singapore, 2 Science Drive 3 Singapore 117542

Abstract. We review an improved maximum likelihood analysis of the Type 1a Supernova (SNe1a) data. We calculate the profile likelihood in the Ω_m - Ω_Λ parameter space by conducting a parameter sweep across the 8 SNe1a parameters, using a Markov Chain Monte Carlo (MCMC) optimization algorithm. This improved analysis, which does not assume arbitrary values for the uncertainties, has the advantage of being bias-free as compared to the original analysis. We use the Joint Lightcurve Analysis (JLA) dataset containing 740 SNe1a data samples for our study, and compare among 5 different models: the Λ CDM model, the flat w CDM model, its non-flat generalization, as well as two dynamical $w(z)$ parametrizations. We find that the Λ CDM model is favoured over the other models, and the best fit values based on this model are $\Omega_m=0.40$ and $\Omega_\Lambda=0.55$. Interestingly, in most of the contour plots we obtain, the line of no acceleration is crossed at $2\sim 3\sigma$ confidence levels, which is similar to the results published by Nielsen et al, the original authors who introduced the improved maximum likelihood analysis. When we generalize the w CDM model to the dynamical $w(z)$ parametrizations, the evidence for cosmic acceleration becomes even weaker. This raises the question of how secure we can be of an accelerating expansion of the universe.

1 Introduction

In 2011, Perlmutter, Schmidt and Riess were awarded the Nobel Prize in Physics for their discovery of the accelerating expansion of the Universe [1][2], by studying distant Type 1a Supernovae (SNe1a). They noticed that these supernovae were dimmer than expected of a constantly expanding Universe, and after some data analysis, concluded that the cosmological term Λ in Einstein's field equations had to be non-zero, or there existed an additional dark energy term.

Eight years on, surveys such as the Sloan Digital Sky Survey [3], the Supernova Legacy Survey [4], the Hubble Space Telescope [5], ESSENCE [6], LOSS [7], Harvard-Smithsonian Centre for Astrophysics CfA1-4 survey [8], and the Carnegie Supernova Project [9] have collected large amounts of SNe1a data in the past two decades covering a wide range of redshifts from $0.01 < z < 0.1$. In the last one year alone, there has also been new surveys conducted, such as the PAN-STARRS 1[10] and Dark Energy Survey[11], which has brought the total number of supernova samples to an unprecedented number of over 1000 samples. With an increasing number of datasets, we can place further constraints on

*e-mail: e0008130@u.nus.edu

**e-mail: cindy.ng@nus.edu.sg

the exact quantities of dark energy and matter in the Universe, consequently allowing us to improve or debunk our current theories and models.

However, data will only be useful if we can analyse them accurately in order to obtain meaningful results, and draw correct conclusions. Many different statistical data analysis methods have been proposed as means to analyse SNeIa data, based on both Bayesian [12][13] and frequentist [14] approaches. Particularly in a paper that we will be looking at in more detail, the methods they have used led them to the conclusion that a constant expansion cannot be ruled out. Their results have been verified by some [13][15], which is interesting to note. Therefore this paper aims to study their method more in-depth and put forth some modifications to it, by exploiting the advantages of both Bayesian and frequentist statistics. We will then use our new method on an expanded range of cosmological models to test their goodness of fit, and perhaps give additional insight towards the probability of a constant rate of expansion.

This paper has been arranged as such: Section 2 introduces the cosmological theories behind Universe expansion, Section 3 will explain and justify our proposed methodology, in Section 4 we present our results, Section 5 will provide a discussion of our results, and finally we give a conclusion in Section 6.

2 Theory

The main source of data being used to investigate the expansion of the Universe are Type Ia supernovae, also known as SNeIa.

SNeIa are characterised by their lack of hydrogen lines, unlike Type II Supernovae. Since all SNeIa occur at the same mass through the same mechanisms, their peak luminosities are roughly constant across all SNeIa, thus making them good standard candles with which to measure distances in the Universe. Using the equations

$$\mu = m - M \tag{2.1a}$$

$$\mu = 25 + 5 \log(d_L) \tag{2.1b}$$

where m is the apparent magnitude and M is the absolute magnitude (≈ -19), the distance modulus μ and luminosity distance d_L can be known.

The graph of luminosity against time (known as the lightcurve) of every SNeIa show very similar shapes, therefore researchers make use of the lightcurve data in order to obtain distance estimates of μ .

2.1 Models of the Universe

The models of the Universe which we will be testing our data against are the concordance model, the Λ CDM model, and other variations of it, namely the w CDM model and its parametrizations.

2.1.1 Λ CDM Model

The current concordance model of the Universe, developed as a response to the discovery of accelerating expansion, is known as the Λ CDM, or Λ Cold Dark Matter model. Here,

Λ represents dark energy. It has a negative pressure $p = -\rho c^2$ which, when substituted into the (ii) components of the momentum-energy tensor $T_{\mu\nu}$, (where $i = 1, 2$ and 3), gives an accelerating expansion. Therefore it is now widely accepted to exist in our Universe, although the exact way in which it manifests itself is still shrouded in mystery.

Based on this model, the expression for the evolution of H as a function of redshift z , known as the Hubble function, is given by the relation

$$H(z)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda] \quad (2.2)$$

where $E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}$, and Ω_m , Ω_Λ and Ω_k are the mass density, dark energy density and curvature density parameter respectively.

2.1.2 w CDM Model

An extension of the Λ CDM model is the w CDM model, where we are concerned with the EoS parameter of dark energy, w . In the Λ CDM model, it is assumed to take the value of -1 . We model this evolution with 2 different parametrizations, and test them against our data. Most w CDM models assume a flat Universe with $\Omega_k = 0$, however we shall generalize this in order to find the best fit value for Ω_k .

The first parametrization (hereafter referred to as parametrization A) is to treat w as a scalar value unchanging with redshift. The equation for $H(z)$ can thus be written as

$$H(z)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda(1+z)^{3(1+w)}] \quad (2.3)$$

Parametrization B defines the evolution of w as a linear function of redshift [16]:

$$w(z) = w_0 + w_1 z \quad (2.4a)$$

$$H(z)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda(1+z)^{3(1+w_0-w_1)} e^{3w_1 z}] \quad (2.4b)$$

Parametrization C defines the evolution of w as a first order Maclaurin expansion of redshift [16], and is also known as the Chevallier-Porslarski-Linder parametrization [17]:

$$w(z) = w_0 + w_1 \frac{z}{1+z} \quad (2.5a)$$

$$H(z)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda(1+z)^{3(1+w_0+w_1)} e^{3w_1[1/(1+z)-1]}] \quad (2.5b)$$

We find the best fit values for w , w_0 and w_1 for each parametrization.

We compare the data to the model by calculating μ as given by equations (2.1a) and (2.1b). The equation for d_L , the luminosity distance, is given by

$$d_L = \begin{cases} (1+z) \frac{d_H}{\sqrt{\Omega_k}} \sinh\left(\sqrt{\Omega_k} \int_0^{z_0} \frac{dz}{E(z)}\right), & \text{if } \Omega_k > 0 \\ (1+z) d_H \int_0^{z_0} \frac{dz}{E(z)}, & \text{if } \Omega_k = 0 \\ (1+z) \frac{d_H}{\sqrt{|\Omega_k|}} \sin\left(\sqrt{|\Omega_k|} \int_0^{z_0} \frac{dz}{E(z)}\right), & \text{if } \Omega_k < 0 \end{cases} \quad (2.6)$$

where $d_H = c/H_0$ and $E(z) = H(z)/H_0$.

3 Methodology

3.1 SALT2 Lightcurve Fitting

We will be using the modified equation for calculating μ , based on Guy et al. [18] to address the fact that the brighter the SNe1a, the wider its lightcurve, and the bluer it will appear to be. Therefore

$$\mu_B = m_B^* - M + \alpha \times x_1 - \beta \times c \quad (3.1)$$

where m_B^* is the new notation for the apparent magnitude, and the values of α and β are constant throughout the dataset being studied. Each SNe1a will be stretched along the time axis and colour corrected by a certain amount to fit the template model. This method has been adopted by many survey teams when presenting lightcurves of new SNe1a data, and is the method we will be adopting to analyze our SNe1a dataset.

The dataset that we will analyze is the Joint Lightcurve Analysis (JLA) dataset published by Betoule et al [14] (hereafter B14). It combines data from the Sloan Digital Sky Survey (SDSS) and the Supernova Legacy Survey (SNLS) to give 740 SNe1a data samples.

They present their errors in the form of a 3×740 by 3×740 covariance matrix for the parameters m_B^* , x_1 and c . The total covariance matrix \mathbf{C} is a sum of 8 covariance matrices that address two main sources of uncertainties: the systematic uncertainty and statistical uncertainty. It is therefore given by

$$\mathbf{C} = \mathbf{C}_{\text{cal}} + \mathbf{C}_{\text{model}} + \mathbf{C}_{\text{host}} + \mathbf{C}_{\text{bias}} + \mathbf{C}_{\text{pecvel}} + \mathbf{C}_{\text{non1a}} + \mathbf{C}_{\text{dust}} + \mathbf{C}_{\text{stat}} \quad (3.2)$$

where \mathbf{C}_{cal} is uncertainty matrix associated with the calibration, $\mathbf{C}_{\text{model}}$ is the light-curve model uncertainty, \mathbf{C}_{host} is the mass step uncertainty, \mathbf{C}_{bias} is the bias correction uncertainty, $\mathbf{C}_{\text{pecvel}}$ is the peculiar velocity uncertainty, $\mathbf{C}_{\text{non1a}}$ is the contamination due to non-type 1a SN, and \mathbf{C}_{dust} is the uncertainty due to dust extinction in the Milky Way.

3.2 The Maximum Likelihood Estimation Method

The maximum likelihood estimation method was introduced by Nielsen et al.[12] (hereafter N16), which is based on calculating the likelihood ratio instead of the χ^2 value. Therefore the best fit value is the value that gives a maximum likelihood value, and not a minimum χ^2 . Furthermore, they do not just treat Ω_m and Ω_Λ as the only parameters to find the best fit values of, but also do a sweep across the set of 8 parameters $\{M_0, x_{1_0}, c_0, \sigma_{M_0}, \sigma_{x_{1_0}}, \sigma_{c_0}, \alpha, \beta\}$. Here, we give an introduction of their method.

3.2.1 Bayesian Statistics

In Bayesian statistics, the probability density is given by [19]

$$P(A|B) \propto P(B|A)P(A) \quad (3.3)$$

where $P(A)$ is the prior distribution, $P(A|B)$ is the posterior distribution and the value of $P(B|A)$ is the likelihood ratio. In this case, A is the model and B is the data. Therefore we are calculating the probability of the model being a good fit, given the data.

The method used in N16 is based on an extension of Bayesian statistics, known as the Hierarchical Bayesian Model. This model should be used, instead of regular Bayesian

statistics, when there are multiple parameters with prior distributions that depend on other parameters [20]. This is indeed true for the data we have, and we shall see that the set of parameters $\{M_0, x_{10}, c_0, \sigma_{M_0}, \sigma_{x_{10}}, \sigma_{c_0}, \alpha, \beta\}$ depend on one another. In this case, the Hierarchical Bayesian probability density is given by

$$P(\gamma, \alpha|\beta) \propto P(\beta|\alpha, \gamma)P(\alpha|\gamma)P(\gamma) \tag{3.4}$$

Here, $P(\gamma, \alpha|\beta)$ is the posterior distribution, $P(\gamma)$ is the prior distribution and $P(\beta|\alpha, \gamma)P(\alpha|\gamma)$ is the new likelihood ratio. In the context of SNeIa data analysis, we shall see what α, β and γ represent, and why we need to include one extra parameter to draw more accurate conclusions based on the data.

3.2.2 Calculating Likelihood L

The values of m_B^*, x_1 and c obtained are not the true values due to various sources of noise and error, both systematic and statistical. We shall denote these observed values as \hat{m}_B^*, \hat{x}_1 and \hat{c} , and the true values without a hat. Therefore, adopting the form of (3.4), we can express the total probability density as

$$P[\theta, (M, x_1, c)|(\hat{m}_B^*, \hat{x}_1, \hat{c})] \propto P[(\hat{m}_B^*, \hat{x}_1, \hat{c})|(M, x_1, c), \theta]P[(M, x_1, c)|\theta]P(\theta) \tag{3.5}$$

where the likelihood ratio is $L = P[(\hat{m}_B^*, \hat{x}_1, \hat{c})|(M, x_1, c), \theta]P[(M, x_1, c)|\theta]$, and θ denotes the model. In terms of (3.4), θ is γ , and the data we obtain, $(\hat{m}_B^*, \hat{x}_1, \hat{c})$, is β . We also wish to infer the true parameters from the observed parameters, therefore (M, x_1, c) is α , where M is the absolute magnitude which is not taken to be a constant value for every SNeIa here. The likelihood can be written in terms of an integral

$$L = \int P[(\hat{m}_B^*, \hat{x}_1, \hat{c})|(M, x_1, c), \theta]P[(M, x_1, c)|\theta]dMdx_1dc \tag{3.6}$$

The first factor of the integrand addresses the discrepancy between the obtained values and the true values, and the second factor addresses the difference between the true values and the model. We shall now derive an equation to calculate, and subsequently maximize, the value of L .

When we plot a histogram of \hat{x}_1 and \hat{c} , we see that they roughly follow a Gaussian distribution.

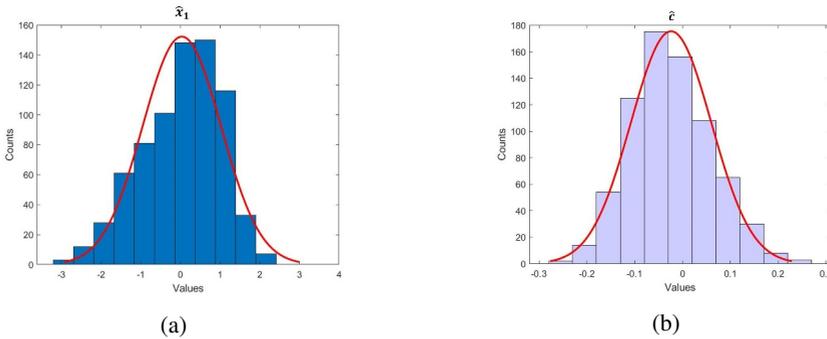


Figure 1: Histograms of \hat{x}_1 and \hat{c} , with a Gaussian distribution overlaid. We can see that they roughly follow a Gaussian distribution, therefore we assume that the true values also follow a Gaussian distribution.

Assuming a Gaussian distribution of M as well, we model the probability distribution of M , x_1 and c as a Gaussian function:

$$P(M) = (2\pi\sigma_{M_0}^2)^{-1/2} \exp\left(-[(M - M_0)/\sigma_{M_0}]^2/2\right) \quad (3.7a)$$

$$P(x_1) = (2\pi\sigma_{x_{10}}^2)^{-1/2} \exp\left(-[(x_1 - x_{10})/\sigma_{x_{10}}]^2/2\right) \quad (3.7b)$$

$$P(c) = (2\pi\sigma_{c_0}^2)^{-1/2} \exp\left(-[(c - c_0)/\sigma_{c_0}]^2/2\right) \quad (3.7c)$$

where M_0 , x_{10} and c_0 are their mean values and σ_{M_0} , $\sigma_{x_{10}}$ and σ_{c_0} are their variances. The total probability density is a product of these three terms: $P(M, x_1, c|\theta) = P(M)P(x_1)P(c)$. For N SNeIa data points, we combine the three parameters to form a vector $Y = \{M_1, x_{11}, c_1, \dots, M_N, x_{1N}, c_N\}$. The total probability density is then

$$P(Y|\theta) = |2\pi\Sigma_l|^{-1/2} \exp\left(-(Y - Y_0)\Sigma_l^{-1}(Y - Y_0)^T/2\right) \quad (3.8)$$

where $Y_0 = \{M_0, x_{10}, c_0, \dots\}$ is a $3N$ vector of the mean values, and $\Sigma_l = \text{diag}(\sigma_{M_0}^2, \sigma_{x_{10}}^2, \sigma_{c_0}^2)$ is the $3N \times 3N$ diagonal matrix of the variances. The absolute symbol denotes the determinant of the matrix. (3.8) corresponds to the second term that we will substitute into (3.6) in order to calculate L .

Now, we deduce the equation for the first term in (3.6). We define the $3N$ vector of observed values $\hat{X} = \{\hat{m}_{B1}^*, \hat{x}_{11}, \hat{c}_1, \dots, \hat{m}_{BN}^*, \hat{x}_{1N}, \hat{c}_N\}$ and the $3N$ vector of true values $X = \{m_{B1}^*, x_{11}, c_1, \dots, m_{BN}^*, x_{1N}, c_N\}$. The probability density of the observed data X given some true parameters \hat{X} , $P(\hat{X}|X, \theta)$, is

$$P(\hat{X}|X, \theta) = |2\pi\Sigma_d|^{-1/2} \exp\left(-(\hat{X} - X)\Sigma_d^{-1}(\hat{X} - X)^T/2\right) \quad (3.9)$$

where Σ_d is the $3N \times 3N$ covariance matrix which is referred to as \mathbf{C} in B14. However, N16 did not include the mass step correction C_{host} , as it does not make a significant difference to the results, which was also demonstrated by [13]. To convert m_B^* to M , we define the $3N$ vector $\hat{Z} = \{\hat{m}_{B1}^* - \mu_1, x_{11}, c_1, \dots, \hat{m}_{BN}^* - \mu_N, x_{1N}, c_N\}$, where μ is the theoretical value of the distance modulus based on the model, and the $3N \times 3N$ block diagonal matrix $A = \{1, 0, 0; -\alpha, 1, 0; \beta, 0, 1; \dots\}$, to get the relation $\hat{X} - X = (\hat{Z}A^{-1} - Y)A$. Then, $P(\hat{X}|X, \theta) = P(\hat{Z}|Y, \theta)$. This is the first term in (3.6). Combining everything,

$$\begin{aligned} L &= \int P(\hat{Z}|Y, \theta)P(Y|\theta)dY \\ &= |2\pi\Sigma_d|^{-1/2}|2\pi\Sigma_l|^{-1/2} \int dY \exp\left(-(Y - Y_0)\Sigma_l^{-1}(Y - Y_0)^T/2\right) \exp\left(-(Y - \hat{Z}A^{-1})A\Sigma_d^{-1}A^T(Y - \hat{Z}A^{-1})^T/2\right) \\ &= |2\pi(\Sigma_d + A^T\Sigma_lA)|^{-1/2} \exp\left[-(\hat{Z} - Y_0A)(\Sigma_d + A^T\Sigma_lA)^{-1}(\hat{Z} - Y_0A)^T/2\right] \end{aligned} \quad (3.10)$$

Just like in R98, we can construct contour plots in the $\Omega_m - \Omega_\Lambda$ parameter space, this time by finding the set of values for the 8 parameters $\{M_0, x_{10}, c_0, \sigma_{M_0}, \sigma_{x_{10}}, \sigma_{c_0}, \alpha, \beta\}$, that give a maximum likelihood for each pair of $(\Omega_m, \Omega_\Lambda)$. This is known as the profile likelihood L_P . Denoting ϕ as the 8 parameters, which we treat as the nuisance parameters as we are not plotting them in the parameter space, and θ as the interesting parameters Ω_Λ and Ω_m ,

$$L_P = \max_\phi L(\theta, \phi) \quad (3.11)$$

The likelihood value L is related to χ^2 by the formula $L = \exp(-\chi^2/2)$ —a maximum likelihood corresponds to a minimum χ^2 value. Therefore in terms of χ^2 , (3.10) becomes

$$\chi^2 = 2\pi|(\Sigma_d + A^T\Sigma_lA)| - (\hat{Z} - Y_0A)(\Sigma_d + A^T\Sigma_lA)^{-1}(\hat{Z} - Y_0A)^T \quad (3.12)$$

which is a much easier equation to work with. Therefore, we calculate the χ^2 values using (3.12) instead for every $(\Omega_m, \Omega_\Lambda)$, and plot the contour lines of 1σ , 2σ and 3σ .

3.2.3 Comparison to the Least χ^2 Method

The Maximum Likelihood Estimation (MLE) method does not only vary Ω_m and Ω_Λ , but also the other 8 parameters $\{M_0, x_{1_0}, c_0, \sigma_{M_0}, \sigma_{x_{1_0}}, \sigma_{c_0}, \alpha, \beta\}$. An advantage the MLE method has over the χ^2 method is that it does not add an arbitrary uncertainty σ_{μ_0} (see [1]), which N16 argues can be adjusted to create biased results towards a preferred model, in this case the Λ CDM model. Therefore the χ^2 method only tests for a goodness of fit to the assumed model, but does not check if it is indeed the correct model. On the other hand, the calculation of L in the MLE method solely depends on the data obtained without adding uncertainties whose value differs for each SNeIa data sample.

From the results obtained in N16, the contour plot significantly crosses the line of no acceleration, therefore they argue that a non-accelerating expansion cannot be ruled out. Other papers have referenced their method with modifications [21] or additional constraints [15] to come to the conclusion that current evidence still favour an accelerating expansion. Therefore it is interesting to expand this new method to explore other Universe models or datasets to see what results we might obtain from it, and to see if our Universe is indeed expanding at an accelerating rate, and by how much.

3.3 Markov Chain Monte Carlo Optimization

The method we will be using to find the best fit values of $\{M_0, x_{1_0}, c_0, \sigma_{M_0}, \sigma_{x_{1_0}}, \sigma_{c_0}, \alpha, \beta\}$ is the Markov Chain Monte Carlo (MCMC) method. This method, based on Bayesian statistics, is widely used in many computational fields to approximate the distribution of multiple parameters—here, we aim to obtain the distributions of all the 8 parameters and find the set of 8 values which gives the maximum likelihood value. We adapt an MCMC code by Pitkin [22] and write our own code to calculate the likelihood ratio and the theoretical value of μ based on the model. The likelihood ratio calculation is based on the MLE method outlined in Section 3.2.2.

Pitkin implements an affine invariant ensemble method based on Goodman and Weare [23], a more sophisticated version of the Metropolis-Hastings method that is faster and can handle highly non-symmetric distributions.

4 Results

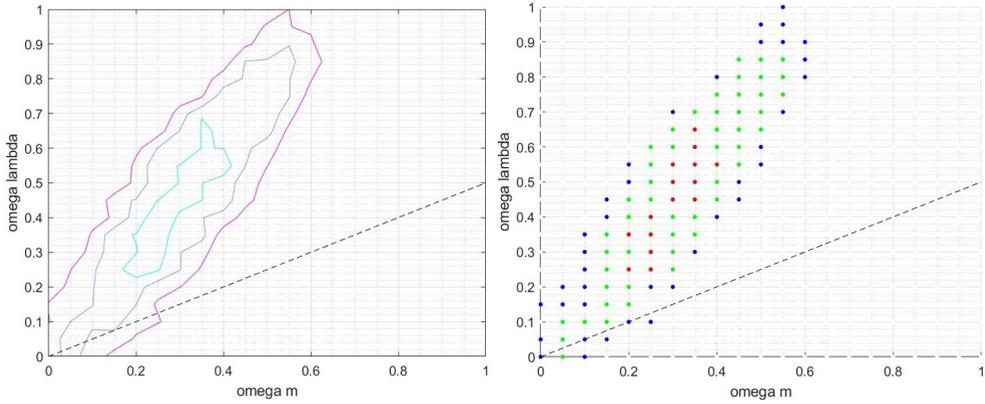


Figure 2: Contour plot (left) and scatter plot (right) of Ω_m against Ω_Λ for the Λ CDM model, using the MLE and MCMC method, with 1σ , 2σ and 3σ lines drawn. The dotted line represents the line of no acceleration. The best fit values of Ω_m and Ω_Λ are 0.40 and 0.55 respectively, with a minimum χ^2 value of -231.209 .

For the Λ CDM model, we run through Ω_m and Ω_Λ from the values of 0 to 1, with a step size of 0.05. The plot we obtain is comparable to N16 where the 1σ and 2σ lines cross the line of no acceleration. Unlike N16, the contours are not smooth, which is due to the fact that an MCMC method was used— [13] also obtained ragged contours when they implemented their own MCMC algorithm.

Next, we test the flat w CDM model, following [15]. We do a parameter sweep across Ω_m and w for $\Omega_k = 0$, ie a flat case.

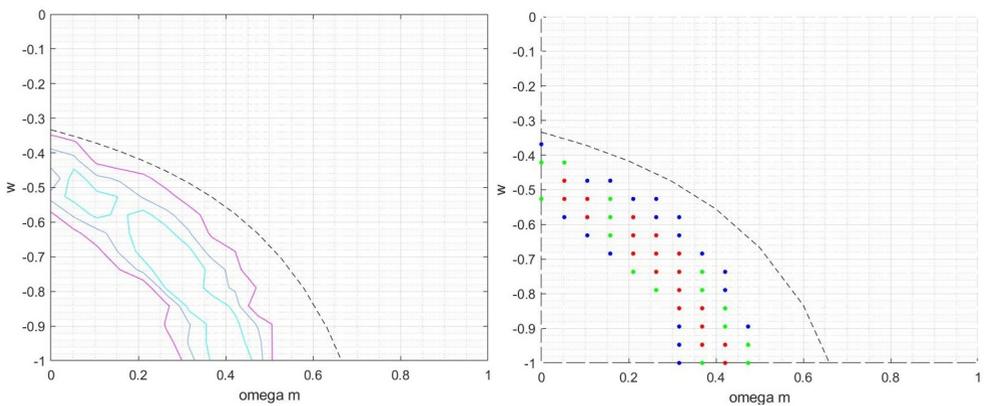


Figure 3: Contour plot (left) and scatter plot (right) of Ω_m against w for $\Omega_k = 0$ with 1σ , 2σ and 3σ lines. The dotted line is the line of no acceleration. We see that in our plot, the contour lines do not touch the line of no acceleration. The minimum χ^2 value is -229.6220 , for $\Omega_m = 0.4211$ and $w = -1$.

Now, we generalize to a non-flat wCDM model where we do a parameter sweep across Ω_m , Ω_Λ and w .

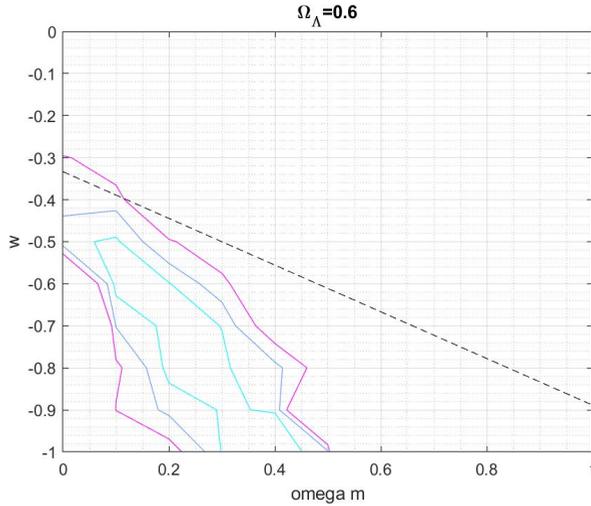


Figure 4: Plot of w against Ω_m for $\Omega_\Lambda = 0.6$, with 1σ , 2σ and 3σ contour lines corresponding to a difference in χ^2 values for 3 parameters, since we run through 3 parameters instead of just 2. The best fit value is $\Omega_m = 0.4, w = -1$ and $\Omega_\Lambda = 0.6$ with a χ^2 of -229.8145 .

Following [16], we run through the values of $-2 < w_0, 0, -2 < w_1 < 8$ and Ω_m for a flat wCDM parametrization B model. We present plot of w_0 against Ω_m for interesting values of w_1 .

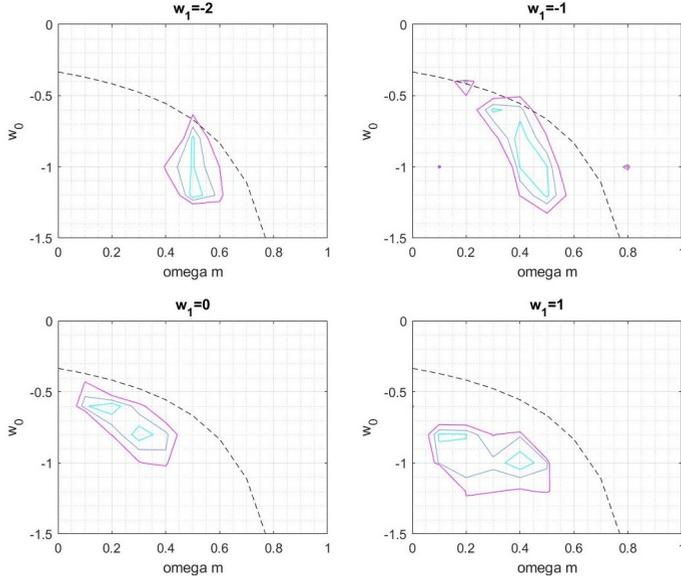


Figure 5: Plots of w_0 against Ω_m for values of $-2 < w_1 < 1$, with 1σ , 2σ and 3σ contours. The best fit values are $\Omega_m = 0.4$, $w_0 = -0.8$, $w_1 = -1$, with a χ^2 value of -229.4125 .

We run through the same range of w_0 and w_1 as parametrization B for parametrization C and present our results, again with plots of Ω_m against w_0 for interesting values of w_1 .

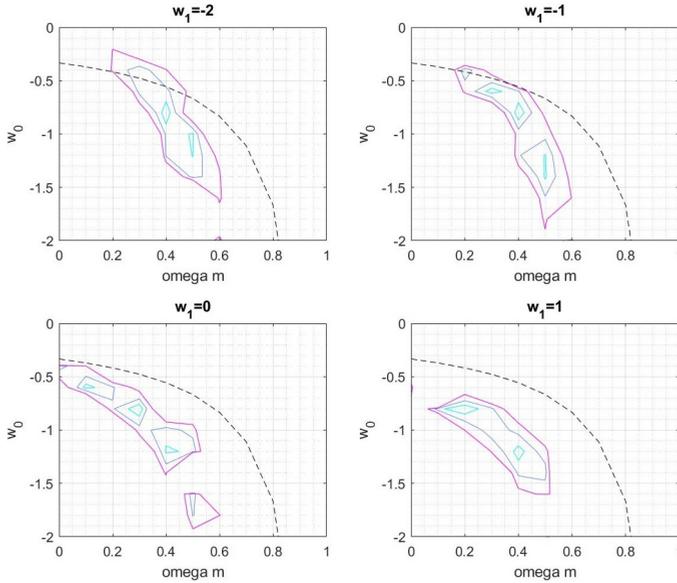


Figure 6: Plots of w_0 against Ω_m for values of $-2 < w_1 < 1$, with 1σ , 2σ and 3σ contours. The best fit values are $\Omega_m = 0.4$, $w_0 = -0.8$, $w_1 = -1$, with a χ^2 value of -227.4537 .

5 Discussion

5.1 Λ CDM Model

Our results for the Λ CDM model differ slightly from N16 as the contour lines cross the line of no acceleration to a greater extent. Our minimum χ^2 value of -231.209 is also lower than the value published by N16 (-214.97). The table below shows how our best-fit Ω_m and Ω_Λ values compare to other works:

Table 1: Comparison of our best fit values of Ω_m and Ω_Λ against literature values

	Ω_m	Ω_Λ
Our work	0.40	0.55
N16	0.341	0.569
Sharriff et al	0.340	0.524
R98 (MLCS)	0.2551	0.7143

We see that our best-fit values are within 1σ of N16 and Sharriff et al's results, further lending clout to the accuracy of our results. This shows that our MCMC code is sufficiently accurate in finding the maximum likelihood and optimizing over the 8 parameter space.

5.2 wCDM Model

5.2.1 Parametrization A

We present the best fit values obtained for both the flat and non-flat wCDM models for parametrization A.

Table 2: Best fit values of $\Omega_m, \Omega_\Lambda, \Omega_k$ and w for flat and non-flat models of Parametrization A

Model	Ω_m	Ω_Λ	Ω_k	w
Flat	0.4211	0.5789	0	-1
Non-flat	0.4	0.6	0	-1

We see that the best fit values for the flat and non-flat models are in agreement with each other. Moreover, for both cases, the contour plot is much wider than that obtained by Haridasu et al, especially in the non-flat case, where the 3σ contour crosses the line of no acceleration. This differs from their claim that the results for an accelerating expansion is reinforced in the wCDM model, particularly when the constraint for a flat Universe is removed since the contours are wider than in the flat case. Therefore the claim for accelerating expansion should not be made purely based on assuming a flat Universe

5.2.2 Parametrizations B and C

We present the best fit values for Parametrizations B and C below:

In the best fit plots (ie. $w_1 = -1$) for both parametrizations, we see that the contour lines also cross the line of no acceleration to an even greater extent than in parametrization A ($\sim 2\sigma$ level). By generalizing the evolution of w , the evidence for constant acceleration becomes stronger. Perhaps if we extend these parametrizations to include the non-flat case, the data will become even more consistent with a constant acceleration model.

Table 3: Best fit values of Ω_m , w_0 and w_1 for Parametrizations B and C

Parametrization	Ω_m	w_0	w_1
B	0.4	-0.8	-1
C	0.4	-0.8	-1

5.3 Model Comparison

To assess which model gives the best fit of our data, we calculate the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), two values that are often calculated to test the goodness of fit of different statistical models. The formulae to calculate AIC and BIC are given by [24] [25]:

$$\text{AIC} = 2k - 2 \ln(L) \quad (5.1a)$$

$$\text{BIC} = \ln(n)k - 2 \ln(L) \quad (5.1b)$$

where k is the number of parameters estimated, n is the number of data samples (in this case, the number of SNeIa data samples), and L is the maximum likelihood. The best fit model is the model with the lowest AIC and BIC values. However, the AIC and BIC test does not give an absolute benchmark on the goodness of fit of a model, only a comparison between models.

We find that the Λ CDM model has the lowest AIC and BIC values, and here, we present the ΔAIC and ΔBIC values for the other cosmological models as compared to the Λ CDM model: Based on the AIC and BIC values, the Λ CDM model is still the best fit

Table 4: ΔAIC and ΔBIC values for the five different models tested

Model	ΔAIC	ΔBIC
Λ CDM	0	0
Flat wCDM	1.5870	1.5870
Non-flat wCDM (Param. A)	3.3945	8.0012
Flat wCDM (Param. B)	3.7965	8.4032
Flat wCDM (Param. C)	5.7553	16.9686

cosmological model given the data, followed by the flat wCDM model. Moreover, the results from their generalizations serve to further validate their best fit value results, and the contour plots obtained using the MLE method provide interesting insights regarding the extent to which an accelerating expansion is certain. In almost all of our results, an accelerating expansion is only consistent at $\sim 2\sigma$ confidence levels. As such, constant expansion of the Universe cannot be ruled out, especially in the case of generalized cosmological models.

6 Conclusion

In this paper, we have used a modified statistical method to analyze data from the JLA SNeIa data set against 5 different cosmological models. This statistical method is based on a Maximum Likelihood Estimation and an MCMC optimization procedure to obtain the best fit values of 8 SNeIa parameters, along with Ω_m , Ω_Λ , w , w_0 and w_1 . This method has the advantage of being bias-free as compared to the conventional least χ^2 method, as arbitrary

values of uncertainty σ can be added to each data point in the χ^2 calculation to make the data fit a desired model.

For the Λ CDM model, we obtain best fit values of $\Omega_m = 0.40$, $\Omega_\Lambda = 0.55$, which is within 1σ of N16. When we do a model comparison based on calculated AIC and BIC values, we see that the Λ CDM model is still favoured over the wCDM model and its parametrizations.

With the MLE and MCMC method, our contour plots for all 5 models cross the line of no acceleration, which is a different result from that obtained by R98 during their discovery of the Universe's accelerating expansion, and is similar to the contour plot published by N16. Moreover, when we generalize the Λ CDM model to the wCDM model and its parametrizations, we see that constant expansion becomes increasingly favoured (the contours cross the line of no acceleration at smaller σ values). The conclusion of an accelerating expansion purely based on the study of SNe1a data does not seem to be as secure as before, thus more work needs to be done for us to better understand the Universe we live in.

References

- [1] A.G. Riess, A.V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P.M. Garnavich, R.L. Gilliland, C.J. Hogan, S. Jha, R.P. Kirshner et al., *The Astronomical Journal* **116**, 1009 (1998)
- [2] S. Perlmutter, G. Aldering, G. Goldhaber, R.A. Knop, P. Nugent, P.G. Castro, S. Deustua, S. Fabbro, A. Goobar, D.E. Groom et al., *The Astrophysical Journal* **517**, 565 (1999)
- [3] J.A. Frieman, B. Bassett, A. Becker, C. Choi, D. Cinabro, F. DeJongh, D.L. Depoy, B. Dilday, M. Doi, P.M. Garnavich et al., *Astronomical Journal* **135**, 338 (2008), **0708.2749**
- [4] A. Conley, J. Guy, M. Sullivan, N. Regnault, P. Astier, C. Balland, S. Basa, R.G. Carlberg, D. Fouchez, D. Hardin et al., *Astrophys. J. Suppl.* **192**, 1 (2011), **1104.1443**
- [5] A.G. Riess et al., *Astrophys. J.* **659**, 98 (2007), **astro-ph/0611572**
- [6] G. Miknaitis et al., *Astrophys. J.* **666**, 674 (2007), **astro-ph/0701043**
- [7] M. Ganeshalingam, W. Li, A.V. Filippenko, *Monthly Notices of the Royal Astronomical Society* **433**, 2240 (2013), <http://oup.prod.sis.lan/mnras/article-pdf/433/3/2240/13761647/stt893.pdf>
- [8] M. Hicken, P. Challis, S. Jha, R.P. Kirshner, T. Matheson, M. Modjaz, A. Rest, W.M. Wood-Vasey, G. Bakos, E.J. Barton et al., *Astrophys. J.* **700**, 331 (2009), **0901.4787**
- [9] C. Contreras, M. Hamuy, M.M. Phillips, G. Folatelli, N.B. Suntzeff, S.E. Persson, M. Stritzinger, L. Boldt, S. González, W. Krzeminski et al., *Astronomical Journal* **139**, 519 (2010), **0910.3330**
- [10] D.M. Scolnic et al., *Astrophys. J.* **859**, 101 (2018), **1710.00845**
- [11] D. Brout et al. (DES) (2018), **1811.02377**
- [12] J.T. Nielsen, A. Guffanti, S. Sarkar, *Sci. Rep.* **6**, 35596 (2016), **1506.01354**
- [13] H. Shariff, X. Jiao, R. Trotta, D.A. van Dyk, *Astrophys. J.* **827**, 1 (2016), **1510.05954**
- [14] M. Betoule et al. (SDSS), *Astron. Astrophys.* **568**, A22 (2014), **1401.4064**
- [15] B.S. Haridasu, V.V. Luković, R. D'Agostino, N. Vittorio, *Astron. Astrophys.* **600**, L1 (2017), **1702.08244**
- [16] S. Nesseris, L. Perivolaropoulos, *Phys. Rev.* **D72**, 123519 (2005), **astro-ph/0511040**
- [17] Y.Y. Xu, X. Zhang, *The European Physical Journal C* **76**, 588 (2016)
- [18] J. Guy et al. (SNLS), *Astron. Astrophys.* **466**, 11 (2007), **astro-ph/0701828**
- [19] M.G. Kendall, A. Stuart, J.K. Ord, eds., *Kendall's Advanced Theory of Statistics* (Oxford University Press, Inc., New York, NY, USA, 1987), ISBN 0-195-20561-8
- [20] G.E.P. Box, G.C. Tiao, *Ann. Math. Statist.* **36**, 1468 (1965)
- [21] D. Rubin, B. Hayden, *The Astrophysical Journal* **833**, L30 (2016)
- [22] M. Pitkin, *Yet another matlab mcmc code* (2015), <http://www.astro.gla.ac.uk/~matthew/blog/?p=813>
- [23] J. Goodman, J. Weare, *Commun. Appl. Math. Comput. Sci.* **5**, 65 (2010)
- [24] H. Akaike, *IEEE Transactions on Automatic Control* **19**, 716 (1974)
- [25] G. Schwarz, *Ann. Statist.* **6**, 461 (1978)