algorithm developed by Newman (2003) and Newman and Girvan (2003) in order to discover community structure in networks9. This algorithm is very interesting, but calculating the betweenness central- ity of edges for the purpose of analysing large graphs is a very long-winded process. Consequently, it is of- ten replaced by Louvain's algorithm, which functions by aggregation in order to reveal communities. Some studies have sought rather to reveal the 'skeleton' of a graph. In a 1962 article, Claude Flament (1962) proposed a technique for pruning a graph that he called 'similarity analysis' and which amounts to removing from any triangle the edge that has the highest de- gree of betweenness. This produces a network that no longer has any cycles of length 3. The algorithm can then be continued by removing the edge that has the highest betweenness in the cycles of length 4, then length 5 and so on. In this way, the net- work's minimum spanning tree is obtained, from the point of view of the betweenness of the edges (Rosen- stiehl, 1967). Although Flament did not formally define betweenness centrality at this time, he had certainly intuited it. Other more recent works op- erate by counting typical motifs (Milo et al., 2002; Cunningham & al., 2012). The proportion of each type of motifs, most often a triad census (with 0, 1, 2 or 3 edges, directed or not, with various types of nodes...) provides a globalized indicator on the network, without however allowing to synthesize its overall structure which can be very different accord- ing to the cases. Security problems in networks may give rise to a desire either to protect the central nodes against attack, as in the case of the Internet or elec- trical or computer networks, or conversely to remove them in order to disconnect the network and break off communications, as when efforts are being made to combat the spread of infectious disease or control the contents of communications on the Internet. In chemistry, one may want to split complex molecules. In all these cases, it is interesting to know how to deconstruct a network in the most effective way. Five strategies can be found in the articles dealing with these questions10: RAa: Random attack: the nodes are removed randomly, one by one in succession; IDa: Initial degree: the nodes are removed one by one in the order of their initial degree centrality; IBa: Initial betweenness: the nodes are removed one by one in the order of their initial betweenness centrality; RDa: Recalculated degree: the nodes with maximum degree centrality are removed, with the degrees being recalculated after each removal; RBa: Recalculated betweenness: the nodes with maximum betweenness centrality are removed, with the betweenness centrality of the remaining nodes being recalculated after each removal. According to simulation studies and studies dealing with actual networks, the RB strat- egy seems to be the most effective way of undermining the functioning of a network.

4.2 Our choice of algorithm

We examined these various strategies (except for random attack, which cannot be reproduced). We also tested the recalculated closeness centrality algorithm. - Initial Degree (ID): Except for the isolated points on the initial graph, all the nodes have non-zero de- gree centrality. Consequently, a threshold has to be chosen in order to remove some of them. This choice is either arbitrary or reasoned and in all cases it re- mains relative. Even though certain thresholds may empirically produce some interesting results, this arbitrariness or dependence on the type of network limits the value of this strategy. Moreover, as mentioned above, degree centrality is focused on the node alone and does not differentiate on the basis of neighbourhood. - Initial betweenness: The most reasonable choice is to remove the nodes with maximum betweenness. If the network is formed from the outset from components connected to each other by a few intermediaries, the result will be close to that obtained using the recalculated betweenness method (and in the particular case of a network made up of com- ponents entirely unconnected to each other, it will actually be identical). However, in certain cases in which the nodes' betweenness centrality varies little, this may lead to a large number of nodes being re- moved indiscriminately. The threshold question also remains unresolved. There are links between close- ness centrality and betweenness centrality (Brandes, Borgatti & Freeman, 2016). We will not dwell on this because it is betweenness that concerns us here and also because using recalculated closeness to analyse a network does not offer a procedure with an automatic endpoint.

4.3 The recalculated betweenness algorithm (RB)

One technique commonly used in network analyses is based on the use of thresholds. It is technically much easier to put into practice, but we will see from examples that the algorithm we are proposing to use has certain advantages over a method based solely on initial thresholds. An initial illustration of our proposal can be found by taking a graph in the form of a simple chain (fig.5):

Fig.5. A chain of intermediaries

The betweenness centrality indexes are given between brackets. If we select the most central, the first will be 4, followed by 3 and 5. If the most central (4) is removed and the betweenness centralities are recalculated, we have:

Fig. 6. A chain of recalculated intermediaries after the removal of the most central one.

The most central now are 2 and 6, which turn out to be local intermediaries, whereas they were not the highest intermediaries at the overall level. Thus the iterative structural deconstruction approach has the property of revealing local intermediaries, which observation of the betweenness values does not do a priori. This is what we are looking for in the networks in our database. We included this algorithm in Appendix.

4.4 The process at work

Let us decompose the process gradually here, using Marc's network as already shown above. The process can be followed on the graphs below, on which the intermediaries are identified by diamonds and the other nodes by circles. The colours correspond to the stages of the process in which they are identified.

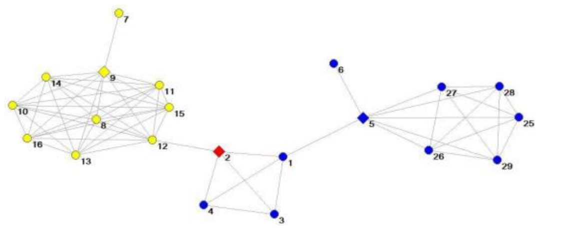
Fig.7. The deconstruction of Marc's network, stage 1 (initial)

The centralities during the initial stage 1 are, in descending order: 2 (0.526), 12 (0.521), 1 (0.479), 5 (0.468) and 9 (0.1). All the others are zero. So we begin by removing alter 2, who is David. He is both the one who was introduced to Marc by is childhood friend Stéphane and the one who introduced Karl to him. Thus he emerges as the most central intermediary between childhood and the most recent group of bikers.

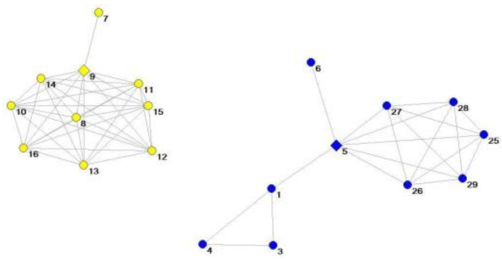
Fig.8. The deconstruction of Marc's network, stage 2

No alter with zero betweenness (final or isolated clique) emerges in this stage. Even though the left-hand component is now detached, it does not (yet) have zero betweenness because of the presence of node 7. So the betweenness centralities are recalculated. In stage 2, the values are as follows: 5 (0.134), 1 (0.082), 9 (0.047). It should be noted that alter 12, who was second during the initial stage, has 'lost' his betweenness, which was dependent on David's, which has been removed. This alter is Karl who, having lost the connection, now finds himself confined to his bikers' clique. Marc's mother, Dominique (5), is now 'promoted' to first place; her betweenness is raised by this removal and she also moves ahead of Stéphane (1). Thus in stage 3 this alter 5, Dominique, who acted as an intermediary between the family and friends via the oldest of them, Stéphane, is removed. The family clique on the right-hand side becomes isolated.

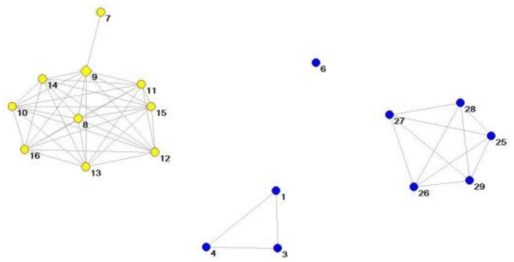

Fig.9. The deconstruction of Marc's network, stage 3

Now, as a result of this operation, node 1 has also lost its betweenness. Stéphane remains in a final clique. Only node 9 now has a non-zero between- ness of 0.052. This is a neighbour who knows Marc's employer (node 7). Here too, he acts as an intermediary between two universes, that of the bikers and the service station where Marc works. This neighbour is removed in order to end up with a 'final' network' of cliques and isolated individuals with zero betweenness.

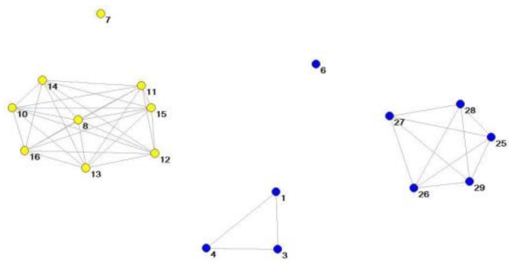

Fig.10. The deconstruction of Marc's network, stage 4

The network is now deconstructed, the process is finished. This sequence of operations can be described in the shape of a tree, with the nodes on the trunk representing the intermediaries and the leaves representing the isolated individuals or final cliques that are disconnected by their removal. To each of these elements can also be added the stage at which it is removed from the network. In the initial stage depicted here only Ego has been removed.

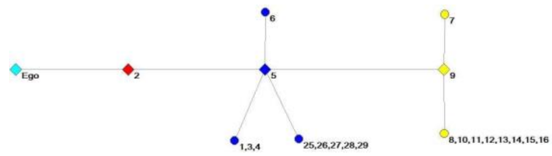

Fig.11. The tree of the deconstruction of Marc's network

4.5 Rapid deconstruction

In 2004, Agnès was living with Olivier and, as we saw above, her network was centred on him, with just a few childhood friends, work colleagues and a former partner not connected to him.

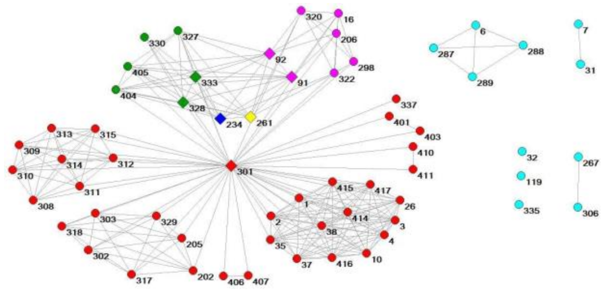

Fig.12. Agnès' network in 2004, with colours indicating the deconstruction stages

When Olivier (301), the most central alter, is removed, the three cliques and the couples at the bottom become detached and have then zero betweenness.

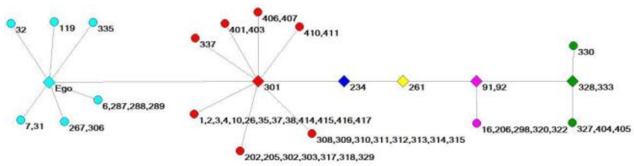

Fig.13. The tree of the deconstruction of Agnès' network

Here, in the initial stage, Ego has been removed along with the isolated individuals, dyads and small cliques whom, as we have seen, Agnès was not keen to connect to her spouse. It can be seen subse- quently that Olivier's removal has the greatest im- pact in terms of disconnected and final elements, of various sizes. The next two nodes (234 and 261) have high levels of betweenness but they do not disconnect anything since this part of the network is dense and there are other possible paths. Once Olivier is removed, they have high betweenness but do not articulate the various components of the network. They are a pair of friends who, with others, connect two groups of neighbours and friends from Agnès's home town. The blue clique is then removed because it has one more alter than the brown one; the betweenness of its nodes (91 and 92) is slightly higher than that of the brown nodes 328 and 333. These nodes did not have any visible betweenness before, so the process reveals local centralities. This deconstruction is massive in stage 2 and is completed in 5 stages. Thus large networks can be decomposed very rapidly. The largest networks are not necessarily the most stratified and therefore the algorithm being proposed here offers a perspective on personal networks that is not necessarily intuitive.

We will try to identify later which members of the network are intermediaries and which are in the final cliques. We will also consider the link between their status and their characteristics. The number of stages required to decompose the network completely is one of the indicators produced by this method. We use the term 'stratification' to denote this characteristic of the network on the grounds that each stage corresponds to a stratum of the network's structural organisation.

4.6 A slow deconstruction

In 1998, during the second wave of the survey, Florence was 21 and living with a partner. However, her network was not centred on her partner (this is more likely to be the case in the higher social categories, which is where she has her origins).

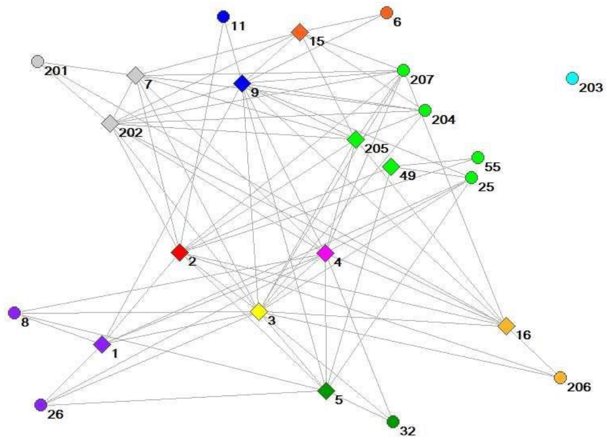

Fig.14. Florence's network in 1998, with colours indicating the deconstruction stages

This is one of the networks with the highest degree of stratification, as measured by the number of stages in our algorithm. This is evident from the graphic below, which comprises 11 stages. During stage 1, one isolated individual appears besides Ego.

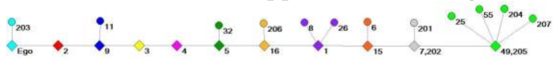

Fig.15. The tree of the deconstruction of Florence's network

During the second stage, as well as during the 4th and 5th stages, intermediaries are identified but no components are disconnected as a result. In each of the other stages, only isolated individuals are con- cerned, and the final stage disconnects four isolated individuals. Thus this is a process that inches its way along slowly, with very dispersed intermediaries and, surprising though it may seem in a dense network, articulators who disconnect only isolated individu- als, one by one. Deconstruction is certainly a process that differs from searching out the overall shape of a network or using a single, static indicator. It enables us gradually to identify the intermediaries by casting light on their roles within the network.

Thus besides the fact that, as we have seen, it is cited as the most effective by the authors of arti- cles on network attacks, the recalculated betweenness strategy has several advantages for network analysis:

- It does not require a threshold to be chosen for removal of the nodes; those that have the highest betweenness centrality are removed in succession and the betweennesses are recalculated after each removal.

- It provides an automatic endpoint criterion: when there are only independent cliques left, all the betweenness centralities are zero.

- The nodes with the highest betweenness centrality at one stage in the process and which, for that reason, are removed from the graph, are called 'intermediaries'. There may be nodes of equal betweenness that are removed at the same time; they are known as 'intermediary blocks'. An additional particular- ity of some nodes is that they connect parts of the graph that without them would be separated from each other11.These nodes or groups of nodes can be called 'articulators'.

- One of the results obtained is that the nodes can be divided into two distinct groups: on the one hand, the intermediaries and, on the other, those elements of the components that become disconnected when the articulators are removed (isolated nodes, isolated dyads or isolated cliques), which we describe as the 'final' nodes, dyads or cliques.

- With the nodes divided in this way, a tree summarising the calculation process can be constructed, the leaves of which are final components and the other nodes the removed intermediaries.

- The number of stages constitutes an indicator of network stratification, which we will call RB (recalculated betweenness) stratification.

- The size and order of exit of the final objects provide indications as to the alters concerned.

- The stage of the algorithm at which an intermediary comes into play is an interesting indicator, since those who appear at the beginning (i.e. when they have a high betweenness at the outset) gener- ally have an overall cohesive effect on the structure, whereas those that emerge at the end of the algorithm are often more local in character. Above all, the algo- rithm makes it possible to uncover these mid-ranking intermediaries (between 3 and 6). In all cases, they make the connections between small, separate worlds as well as, often, between separate life periods.

5 The intermediaries

5.1 Distribution of the alters among the categories derived from the deconstruction

This process of deconstruction also makes it possible to identify some of the alters' characteristics. Ta- ble 1 shows the way in which they are distributed among the various categories at the end of the net- work deconstruction process. We use the term ar- ticulators to denote those intermediaries whose re- moval disconnects other alters by adding at least one component to the network. They are either single alters or blocks of several alters with identical be- tweenness. Other intermediaries, such as alters 234 and 261 in Agnès's network, are not articulators. The non-intermediaries are the components with zero betweenness (isolated individuals, dyads or cliques); they were either present initially (rank 1) or revealed by the deconstruction ('final') and of a rank greater than 1.

Table 1. Distribution of types of alters after deconstruction

Type of alter after deconstruction	**N**	**%**
Single articulator	579	8.6
Articulator block	746	11.1
Non-articulating intermediaries	146	2.2
Total intermediaries	1471	21.9
Non-intermediary isolated individual	1229	18.3
Element in a non-intermediary dyad	1098	16.3
Element in a non-intermediary clique	2918	43.4
Total non-intermediaries	5245	78.1
Total	6716	100

As might be expected, the intermediaries are in the minority. Among them, in the vast majority of cases they are articulators. The cases in which the removal of the most central node in terms of betweenness does not disconnect any element in the network are relatively few in number.

[30] Gordon & Breach, New York. Sur S., Ganguly N.,Mukherjee A., 2016, Brokerage-based attack on real world temporal networks, Network Science 4(4),446-459.

[31] Yong Li, Wenguo Li, Yi Tan, Fang Liu, Yijia Cao, Kwang Y. Lee, (2017), Hierarchical Decomposition for Betweenness Centrality Measure of Complex Networks, Scientific Reports, 7,46491; doi:10.1038/srep46491. Zhe-Ming LU, Xin-Feng Li, (2016), Attack Vulnerability of Network Controlability, PLoS ONE 11(9): e0162289. doi: 10.137/journal.pone.0162289

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