EVALUATION OF APPROXIMATIONS IN REACTOR PULSE

Ingrid Vavtar, Luka Snoj

Jozef Stefan Institute, Reactor Physics Department (F8)
Jamova cesta 39, 1000 Ljubljana, Slovenia

ingrid.vavtar@ijs.si, luka.snoj@ijs.si

ABSTRACT

An evaluation of validity of theoretical pulse models (Fuchs-Hansen and Nordheim-Fuchs model) in reactor pulse modelling was performed by developing so called Improved Pulse Model. The effect of each of the five assumptions on the most important pulse physical parameters, maximal power, total released energy and full width at half maximum was studied. In the Improved Pulse Model the assumptions are disposed out with the improvements, where to account the delayed neutrons the six point kinetic equations are solved, the temperature dependences of the temperature reactivity coefficient of fuel and specific heat are taken into account, also the final ejected time of transient control rod from reactor core, whose value of reactivity varies in height and the heat dissipation from the fuel are considered. It is found that the theoretical models predict a higher maximum power, lower total released energy and full width at half maximum than the Improved Pulse Model.

KEYWORDS: pulse experiment, TRIGA reactor, Improved Pulse Model, Fuchs-Hansen model, pulse physical parameters

1. INTRODUCTION

Some reactors and critical assemblies can operate in pulse mode. The simplest and very common approach to model the reactor pulse is the Fuchs-Hansen (FH) model and the Nordheim-Fuchs (NF) model. Both theoretical models are derived from the point kinetics equations on the basis of five assumptions; system is adiabatic, effective temperature reactivity coefficient of fuel is a constant value (i.e. does not depend on temperature), delayed neutron production can be neglected, power of the reactor before the pulse is low or equal zero and that reactivity change is instantaneous. It is important to note that both models predict same pulse physical parameters; maximum power (P_{max}), the total released energy (E_{tot}) and full width at half maximum (FWHM). Hence in the paper we will refer to one model only. In order to asses validity of each approximation, we developed an Improved Pulse Model (IPM) that takes into account delayed neutrons by solving six-group point kinetics equations, the temperature dependence of fuel temperature coefficient of reactivity and the specific heat, a finite time of reactivity insertion and heat removal from the fuel. We study the effect of each of the assumptions on the most important physical parameters of a pulse, i.e. P_{max}, E_{tot} and FWHM.

The paper is structured as follows, in section 1 is presented an IPM, where the structure of the model is presented and the method to account the improvements. In section 2 the comparison between the theoretical FH model and IPM model. A comparison of power versus time and of pulse physical parameters is made, where in all cases also the contributions of improvements in IMP model are shown.

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1. IMPROVED PULSE MODEL

In order to improve existing theoretical models (Fuchs-Hansen model (FH) [1] and Nordheim-Fuchs model (NH) [2]) an algorithm for the Improved Pulse Model (IPM) was developed by taking into account all effects as presented below. Such equations can not be solved analytically as in the case of FH and NH models, hence we used numerical solver (NDSolve using runge kutta method) in the Wolfram Mathematica [3]. The NDSolve solver allows you to specify the desired precision of the result and then the size of each interval is determined to satisfy this precision condition. The goal at the IPM is to asses the validity contribution of individual assumptions in the FH model and evaluate their individual contributions to physical parameters of a reactor pulse. There are five assumptions used by theoretical models:

1. negligible contribution of delayed neutrons,
2. the effective temperature reactivity coefficient of fuel γ is constant and does not depend on temperature,
3. adiabatic system,
4. inserted reactivity is step function,
5. power before pulse experiment is low or equal zero.

The algorithm solves the six group point kinetics equations:

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta(t)}{\Lambda(t)} P(t) + \sum_j \lambda_j C_j(t) + Q(t),$$
$$\frac{dC_j(t)}{dt} = \frac{\beta_j(t)}{\Lambda(t)} P(t) - \lambda_j C_j(t) \qquad j = 1,2,\dots 6, \tag{1.1}$$

where P is power, ρ reactivity, β effective delayed neutron fraction, β_j delayed neutron fractions of the j-th group of the precursors of the delayed neutrons, Λ mean neutron generation time, λ_j the decay constant of the precursors of delayed neutrons, C_j concentration of precursors of delayed neutrons and Q external neutron source [4]. In the point kinetic equations six groups of delayed neutrons are considered resulting in the elimination of the assumption that the delayed neutrons are irrelevant. In order to solve the point kinetics equations, the definitions of prompt reactivity ρ' is defined as:

$$\rho' = \rho_0 - \beta, \tag{1.2}$$

where ρ_0 is inserted reactivity. The reactivity changes according to the equation:

$$\rho(t) - \beta = \rho' - \gamma E(t), \tag{1.3}$$

where γ is effective temperature reactivity coefficient of fuel and E_{tot} total energy released during the pulse:

$$E_{tot} = \int_0^t P(t')dt'. \tag{1.4}$$

The mean neutron generation time Λ is defined as:

$$\Lambda = \frac{l}{k}, \tag{1.5}$$

where k is the multiplication factor and l is prompt neutron lifetime and equation which connects inserted reactivity and multiplication factor:

$$\rho \equiv \frac{k-1}{k}. \tag{1.6}$$

The definition of the effective temperature reactivity coefficient of the fuel γ is:

$$\gamma = \frac{\alpha}{mC} = 2.1 \cdot 10^{-9}, \tag{1.7}$$

where α represents temperature reactivity coefficient of fuel, m mass of fuel in the reactor core and C is specific heat [5]. The algorithm considers the temperature dependence of the temperature reactivity coefficient of fuel α as shown in the Figure 1, where it takes into account the curve of fresh fuel (burnup is 0 %) as it is found that the fuel burnup may be neglected [6]. The temperature dependence of the specific heat is also considered, where the temperature dependent specific heat C for standard fuel element (12 w/o) is equal [7]:

$$C(T) = \left(333 + 0.678 \cdot (T[K] - 274.15)\right)\frac{J}{kgK}. \tag{1.8}$$

This eliminates the second assumption that the effective temperature reactivity coefficient of fuel is a constant value and does not depend on temperature. For proper model comparison it is necessary to make sure that the average values of the temperature reactivity coefficient of fuel and specific heat in IPM are the same as the values used in the theoretical models.

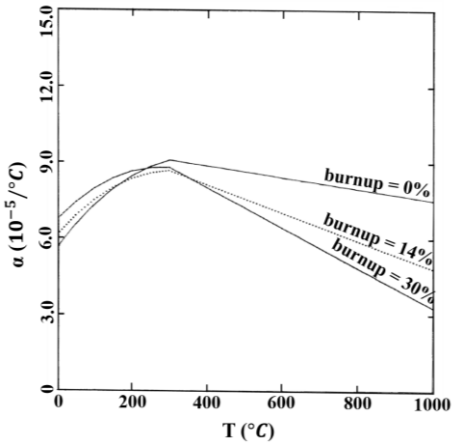


Figure 1: Temperature reactivity coefficient of fuel (absolute value) depending on the temperature and burnup of fuel for standard fuel element (12 w/o) in TRIGA reactor [8] [9].

In order to take into account the heat dissipation from the fuel the equation for calculating the released energy during the pulse experiment $E(t)$ (1.4) is transformed as:

$$E(t) = \int_0^t \left(P(t') - P_H(t')\right)dt', \tag{1.9}$$

where P_H presents the heat dissipation from the reactor fuel. As the fuel heats up during the pulse experiment, the heat dissipation occurs due to the released energy. Two models of heat dissipation are considered for the purpose of verifying the effect of heat dissipation from fuel ($P_H(t)$) [6]. In the first case it is assumed that a constant proportion of the fuel is removed or in other words, constant percentage of instantaneous power is removed. In the second case a Newton heat dissipation is assumed, where heat removal is proportional to the difference between the fuel temperature and the temperature of water surrounding the fuel. In the final version of the IPM a Newton heat dissipation is used. For maximum impact of heat dissipation on pulse power vs. time, the maximal thermal conductivity is wanted, which is obtained if the maximum temperature difference is considered and that the heat is translated through a small thickness. For this purpose is assumed that the temperature of the fuel is constant throughout the radius of the fuel rod and that the heat is removed/transferred through a stainless steel cladding (the gap between the fuel and the cladding is neglected). The equation of cooling power due to heat dissipation follows as:

$$P_H(t) = \lambda_{NJ}\frac{A}{d}\Delta T(t)\,, \tag{1.10}$$

where λ_{NJ} is the thermal conductivity of stainless steel, A surface of the fuel element cladding and d is thickness of the cladding. Introducing the new label:

$$H = \lambda_{NJ}\frac{A}{d}, \tag{1.11}$$

where for 1 fuel element in the reactor core the value is $H_1 = 1396\,W/K$, and for 59 fuel elements it is equal: $H_{59} \approx 0.1\frac{MW}{K} = H$. The relation between time and temperature is represented by the following equation:

$$E(t) = mC(T)\Delta T, \tag{1.12}$$

by which the temperature T at each instant t is determined.

With the IPM the assumption of instantaneous inserted reactivity is eliminated, since the finite removal time of the transient control rod is considered. The transient control rod is being accelerated in the first 30 ms after ejection from the reactor core, then moved with a constant speed for approximately 40 ms and at last slowed down for about 10 ms [10]. Therefore, two movements of the transient control rod are analysed, one with constant velocity motion and one with uniformly accelerated motion, which are also limit cases of real motion [6]. It is found that the two mentioned models differ in P_{max}, E_{tot} and FWHM by less than 5 % therefore in the final version of the IPM the constant velocity motion of transient control rod from reactor core is considered. Constant velocity (5.3 m/s) is defined as the ratio of the length of the entire transient control rod (0.425 m) and the travel time from the reactor core (80 ms). By considering the finite time of travel of the transient control rod from the reactor core, an additional improvement is taken into account by the change in the value of the transient control rod by height or steps.

The last, fifth, assumption is valid since the reactor is subcritical or at low power before the pulse experiment and also the neutron source is not strong. Therefore, in the IPM the power before pulse experiment is low, 0.001 W.

In addition to the data of delayed neutron fractions of the j-th group of the precursors of the delayed neutrons, which are listed in Table 1, data of effective delayed neutron fraction $\beta = 0.007$ [11][12][13][14] and prompt neutron lifetime $l = 4 \cdot 10^{-5} s$ [11][12][13][14] are also required.

Table 1: Data of the neutron fractions of the precursors of delayed neutrons and data of decay constant of the precursors of delayed neutrons for six group point kinetic equations. [15]

β_j	λ_j
0.00023097	0.0124
0.00153278	0.0305
0.00137180	0.1115
0.00276451	0.3010
0.00080489	1.1380
0.00029396	3.0100

The algorithm solves the described equations and reads the values of the temperature reactivity coefficient of fuel and value of transient control rod from graphs.

2. COMPARISON OF IMPROVED PULSE MODEL WITH THEORETICAL PREDICTION

An IPM is compared with theoretical model. The theoretical model takes into account the FH model, the reader is reminded that the pulse physical parameters (P_{max}, E_{tot}, FWHM) for both theoretical models, FN and NH, are the same, however, they differ in the predictions of time dependence of power and energy.

Figure 2 shows a comparison of the IPM (red line) and the theoretical FH model (blue line). The graph also shows individual improvements, where only basic model and addition of one individual improvement are considered. On the graphs label *BM* shows the basic model of IPM, whose equations are identical to the FH model (on graph black and blue dashed lines). Label α presents basic model with addition of improvements of temperature dependence of temperature reactivity coefficient of fuel, *C* presents basic model with addition of temperature dependent specific heat, label ρ shows basic model with finite velocity of transient control rod from reactor core, to which the value of reactivity differ in height, label *HD* take into account basic model with heat dissipation from reactor fuel and label *DN* present basic model with addition of contribution due to delayed neutrons. It can be observed that the models, IPM and FH, reach P_{max} at different time, which is due to the consideration of improvements in the IPM, where the biggest contribution to the time delay is finite time of the transient control rod withdrawal from reactor core, to which the value of negative reactivity differ in steps (on graph contribution marked with gray). This improvement in IPM considers the gradual addition of reactivity, unlike the FH model where the total reactivity is added immediately. The graph also shows how certain contributions affect the value of P_{max} (higher or lower).

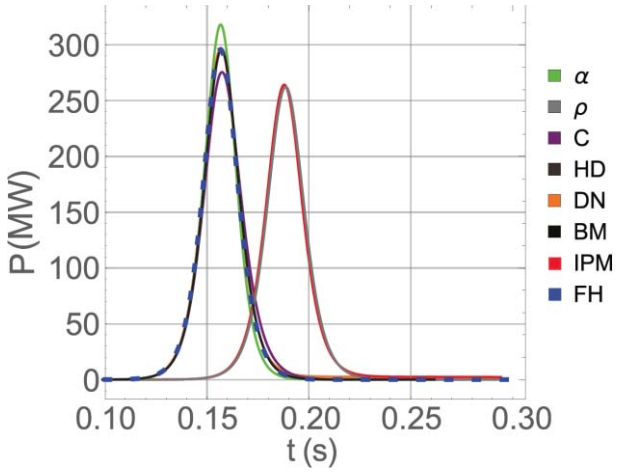


Figure 2: Comparison of power from IPM (red) and FH (blue) for 2 $ inserted reactivity. Individual improvements of IPM are also shown (basic model and basic model with each improvement).

Two phenomena are contributing to the negative temperature reactivity coefficient of fuel, first one is the Doppler effect, since due to increasing power the fuel heating increases the absorption of neutrons in uranium (^{238}U) and the second one is the thermal spectrum hardening effect. The contribution of the Doppler effect in the standard fuel is on average around 20 % in the temperature range between 294 and 800 K while

the rest is contributed by the spectrum hardening which is affected by specific resonance structure of thermal scattering cross section of hydrogen in Zr lattice.

On Figure 3 the contributions of individual improvements of IPM are shown, with all maxima shifted in time to the time at which the theoretical FH model reaches peak power. The contributions of the improvements are stacked one above the other. Graphically, the sum of all contributions are equal to the IPM model results. It can be observed that in addition to delayed neutrons, the asymmetry of the pulse power is also influenced by the temperature dependence of the temperature reactivity coefficient of fuel and the temperature dependence of the fuel specific heat. Due to asymmetric power vs. time higher E_{tot} and FWHM are obtained in comparison with theoretical FH model.

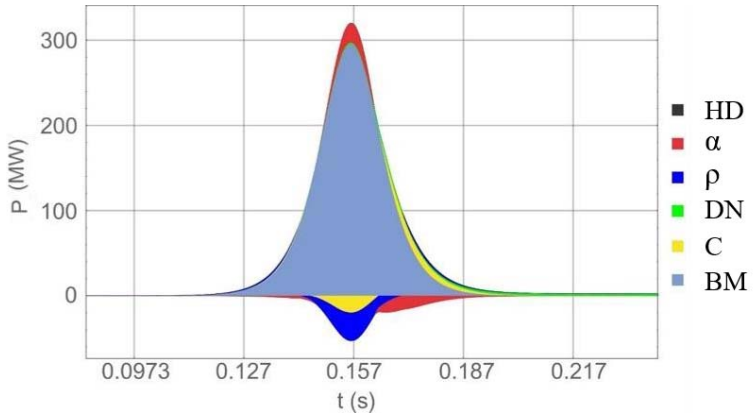
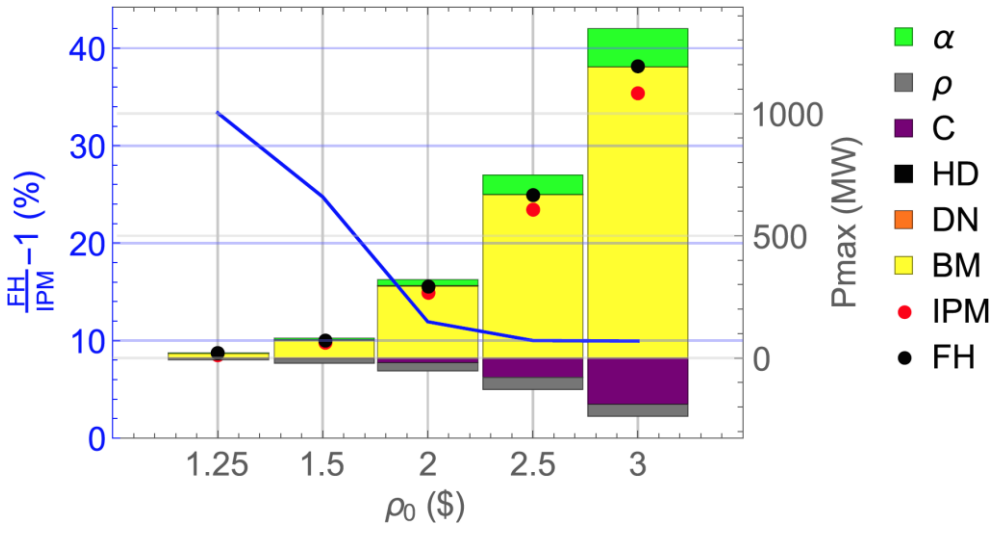


Figure 3: Individual contributions of improvements of IPM for 2 $ inserted reactivity. All maxima power displaced to the same time.

In order to compare P_{max}, E and FWHM, depending on the inserted reactivity ρ_0 of both models (IPM and FH), the results are shown graphs on Figure 4. The values obtained with the IPM are shown with red dots, while the values of the FH model with black dots. Like in Figure 3 the contributions of individual improvements of the IPM are also shown. The relative deviation of FH and IPM is shown with blue curve (linked to the left axis).



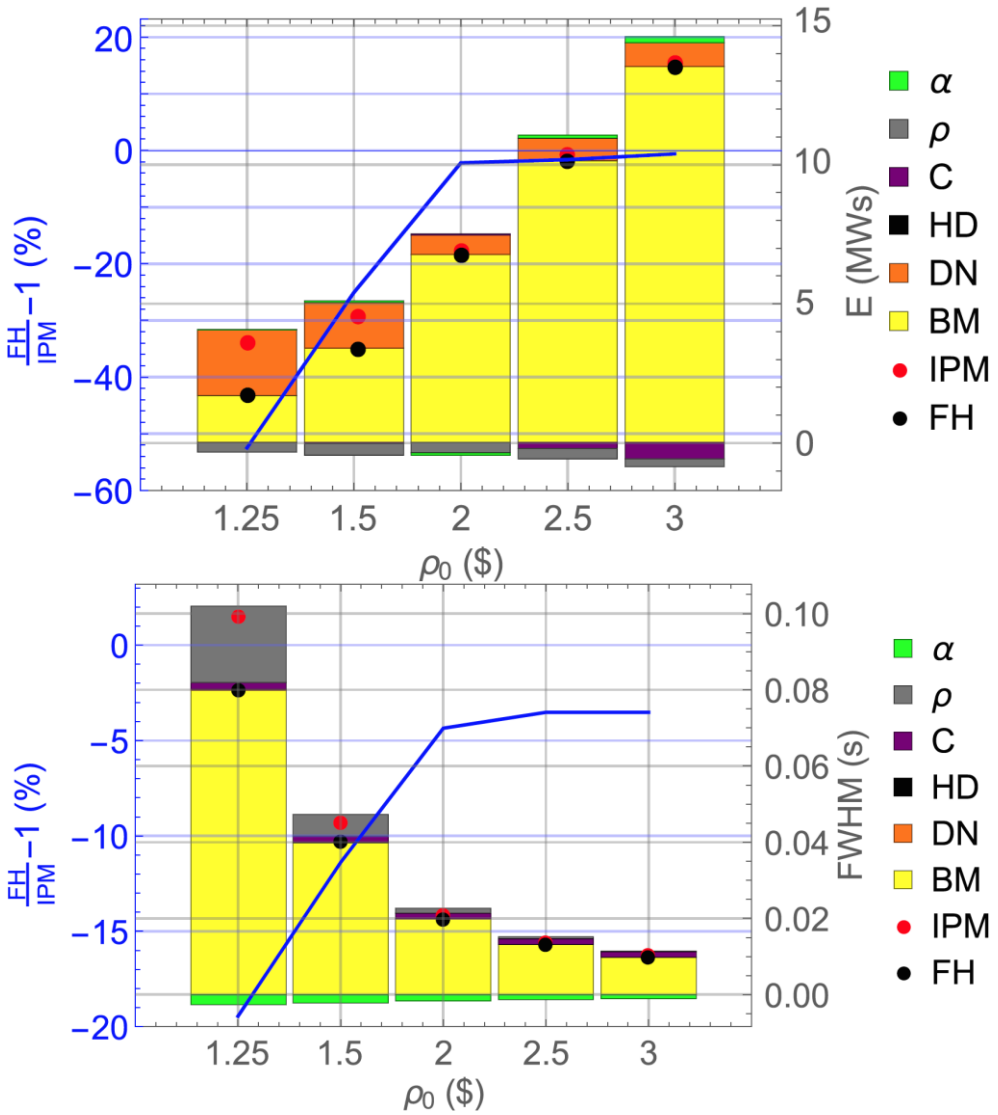


Figure 4: Comparison of the IPM and FH for P_{max} (top), E_{tot} (center) and FWHM (bottom) versus inserted reactivity. Individual contributions of improvements of IPM and relative deviations of FH from IPM are also shown.

Both models, IPM and FH, consequently NF, predict comparable values of P_{max}, E_{tot} and FWHM (Figure 4). As expected it can be seen from the graphs (blue line) of the P_{max}, E_{tot} and FWHM that the difference in results is greater at the lower inserted reactivities. It can be observed that the theoretical FH model predicts higher P_{max} than the IPM, but the IPM predicts higher E_{tot}, which is mainly due to the consideration of the delayed neutrons. The effect is more prononced at low pulses as expected.

3. CONCLUSIONS

An Improved Pulse Model is developed to enable assessment of validity of the assumptions of FH and NH theoretical models. Each contribution of improvements has a different effect on the physical parameters of pulse. It is found that the theoretical models predict a higher P_{max} than the IPM, but the difference between predictions decreases with increasing inserted reactivity. At inserted reactivity of 1.25 $ the difference in P_{max} is approximately 25 % at inserted reactivity of 3 $ the difference is reduced to 10 %. The IPM predicts

greater E_{tot}, which is mainly due to asymmetric power function of time. In an IPM the asymmetric power function occurs due to the consideration of delayed neutrons and the temperature dependent temperature reactivity coefficient of fuel and specific heat. Also in this case the difference decrease due to the increasing in inserted reactivity, since the delayed neutrons are more important at lower reactivity. At inserted reactivity of 1.25 $ the difference is significant and is approximately 110 %, while at inserted reactivity of 3 $ the difference is only about 1 %. The IPM also predicts higher value of FWHM than the theoretical models, where the difference between the inserted reactivity of 1.25 $ and 3 $ decreases from about 25 % to approximately 4 %. The P_{max} at inserted reactivity of 2 $ is achieved approximately 0.5 s earlier with the theoretical model (FH) as with an IPM, which is mainly due to the consideration of final travel time of the transient rod from the reactor core. The IPM is applied to the Jozef Stefan Institute (JSI) TRIGA Mark II reactor and in future the calculated physical parameters will be compared to the measured values from our database of more than 300 pulses available at: http://trigapulse.ijs.si/ [16]. However it is important to note that by changing physical parameters of the fuel, the model can be easily applied to any other pulse system as well. The theoretical models are also used to evaluate uncertainties in pulse physical parameters due to uncertaionties in physical parameters of the reactor [17].

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