

Using Prompt Neutron Decay Constant Measurements to Obtain Additional Kinetics Information

George McKenzie¹, Travis Grove¹, and Jesson Hutchinson¹

¹Los Alamos National Laboratory
PO Box 1663 Los Alamos, NM 87545

gmckenzie@lanl.gov, tgrove@lanl.gov, jesson@lanl.gov

ABSTRACT

Prompt neutron decay constant measurements are typically used to infer the prompt neutron lifetime in a chain-reacting system, but these measurements include much more information about the kinetics of the system. This work focuses specifically on the Rossi- α method of measuring the prompt neutron decay constant. The Rossi- α system of equations consists of five unknown parameters. With five unknowns and three equations, the system of equations solved by the Rossi- α measurement is under-determined. It has been postulated that a well posed problem can be formulated using independent measurement configurations on a single system. The external source strength, the effective delayed neutron fractions, and Diven's parameter do not typically change for different configurations of a single experiment. If good detector geometry is chosen, the detector efficiency does not change for multiple measurements of a single experiment using Rossi- α measurements. By making these assumptions, there are only two unique variables. This knowledge can be used to solve for most of the unique parameters using two independent, unique Rossi- α measurements.

KEYWORDS: neutron noise, prompt neutron decay constants, kinetics parameters, Rossi- α

1. INTRODUCTION

Neutron noise measurements are very powerful techniques to determine kinetics information from neutron chain-reacting systems. Techniques like Feynman Variance-to-Mean are used to calculate system multiplication. Techniques like the Rossi- α method are used to measure the system prompt neutron decay constant. The prompt neutron decay constant is the rate at which the prompt neutron population in a chain-reacting system changes. Below prompt critical this rate is a decay. This work investigates the feasibility of using the Rossi- α technique to determine more kinetics information about an experiment through application of a numerical solution for a simplified problem and comparison to experimental data.

The prompt neutron decay constant, often referred to as the Rossi- α or α -eigenvalue, is a dynamic variable of a chain-reacting nuclear fission system. The prompt neutron decay constant of a system is the rate at which the prompt neutron population changes. Like the Inhour relation, α -eigenvalue methods are another useful method of calibrating reactivity [1–4], as long as the α -eigenvalue, the value of α at delayed critical, is defined.

2. Theory of the Rossi- α Method

The prompt neutron decay constant, α , depends on both the prompt multiplication factor, k_p , and the neutron lifetime, l as shown by Eq. 1 [5]. Specifically, the prompt neutron lifetime mentioned in this context is the average length of time a prompt neutron exists in a system before a terminating event. Termination can be caused by leakage from the system, non-fission capture, or fission capture.

$$\alpha = \frac{k_p - 1}{l} \quad (1)$$

The Rossi- α technique relies on an understanding of the distinction between accidental and correlated neutron pairs. Much like the distinction between prompt and delayed neutrons, it is necessary to divide detected neutrons into two subcategories. In a single fission chain, the correlated and accidental pairs relate to the prompt and delayed neutron groups. Correlated neutron pairs refer to the prompt neutrons generated from a common fission ancestor. Accidental neutron pairs are neutrons originating from a random source such as the background, delayed emission, or other sources. When multiple fission chains are being analyzed, neutrons originating in a different fission chain are considered as accidental neutron pairs.

2.1. Traditional Rossi- α Derivation

Measurements of the α -eigenvalue rely on a certain set of assumptions to be valid. The first and most important assumption is that the system is in a fundamental mode without significant fission chain overlap [2,7]. The second assumption is that the measurement is taken at a point that is symmetric with respect to source and detector geometry, so that spatial correlations do not need to be considered [8]. The third and final assumption is that the system is not heavily reflected [1].

The Rossi- α method is an α -eigenvalue technique that measures the correlation in neutron counts to determine α . Before the Rossi- α method was developed prompt neutron decay constants were measured using fairly unreliable accelerators. Reliability issues fueled the desire to find another way. The theory behind Rossi- α consists of a model of the time behavior of the prompt neutron population. The model for the Rossi- α method was developed heuristically by Richard Feynman [9] based on an idea from Bruno Rossi who was a detector builder at Los Alamos National Laboratory. Rossi proposed that active fission systems are self-modulated; meaning that the emission rate of delayed neutrons is sufficiently slow that neutrons produced directly from two separated fission events are discernible [10]. Many of the scientific minds at Los Alamos wrote off Rossi's idea because of his position, but Feynman heard Rossi's proposal and immediately began deriving the method that is still used today to measure prompt neutron decay constants.

Experiments using α -eigenvalue methods are performed subprompt critical. Measurements of α between delayed and prompt critical are often difficult because the power level and subsequently the neutron population of the neutron multiplying system are increasing. The increasing neutron population poses two issues to Rossi- α measurements. The first issue is the saturation of the detection system. The second issue is the increasing overlap of fission chains.

Using the statistics of the likelihood of this sequence of events, a distribution of the promptly born neutron population as a function of time is created. This process is completed over and

over again until the distribution smooths and the distribution approximates a smooth, continuous function. When a system is subprompt, the prompt neutron population decays as a function of time because the likelihood of prompt neutron in any given chain surviving is low. Thus, all chains decay back to some constant background determined by the random neutron population at the time of the measurement. This background is related to the strength of the interrogating source, and the multiplication of the system. The multiplication of the system is important because delayed neutrons are born randomly in time, and are therefore treated as a random source of neutrons. This distribution is described by Eq. 2 [1]. When measuring systems above delayed critical, the delayed neutrons create an exponentially increasing neutron source in the system. This does not affect the measurement so long as the neutron population does not significantly increase over the data collection period.

$$p(t)\Delta = F\epsilon\Delta + \epsilon \frac{D_\nu k_p^2}{2(1 - k_p)l} e^{\alpha t} \Delta \quad (2)$$

Where, the Δ in Eq. 2 is the time window in which the correlated neutron is measured within. The F is the system fission rate, ϵ is the detector efficiency per fission, and D_ν is Diven's parameter.

Often for simplicity, the total probability to detect a neutron event in some Δ after detecting the initial event is written in the general form shown in Eq. 3. Equation 3 is fit to experimental data during analysis. Where the parameters A and B are represented by Eqs. 4 and 5 [1], and α was previously defined in Eq. 1.

$$P(t) = A + B e^{\alpha t} \quad (3)$$

$$A = F\epsilon \quad (4)$$

$$B = \frac{\epsilon D_\nu k_p^2}{2(1 - k_p)l} \quad (5)$$

2.2. Simplify Equations for Numerical Analysis

For the three equations to have a solvable solution, it is ideal to have at least one common variable in Eqs. 1, 4, and 5. Through examination of the equations, it becomes apparent that all three terms have a relationship to the system multiplication. Equations 1 and 5 relate to the multiplication of prompt neutrons alone while Eq. 4 depends on the multiplication of both prompt and delayed neutrons. The fission rate is really a combination of the source strength and the system multiplication as shown by Eq. 6.

$$F = \Psi M \quad (6)$$

In Eq. 6, Ψ is the neutron source strength in fissions per second, and M is the system total multiplication. The total multiplication is used in this case because the detection system is assumed to be inherently part of the experiment (i.e. inside the experiment). This would include all measurements where the detection system is inside the assembly and therefore influencing the measured result. For external measurements, leakage multiplication may be the more appropriate term. In these cases, a different analysis method should be derived or incorrect values of k_p would be obtained.

$$M = \frac{1}{1 - k_{eff}} \quad (7)$$

The total multiplication can be described as a function of the prompt neutron multiplication factor and the effective delayed neutron fraction as shown by Eq. 8.

$$M = \frac{1 - \beta_{eff}}{1 - \beta_{eff} - k_p} \quad (8)$$

Although this process adds an additional parameter, this substitution introduces flexibility in simplifying the system of equations. Combining Eqs. 6 and 8, fission source rate can be described as a function of the prompt neutron multiplication, the effective delayed neutron fraction, and the external source strength as shown by Eq. 9.

$$F = \Psi \frac{1 - \beta_{eff}}{1 - \beta_{eff} - k_p} \quad (9)$$

This change only really affects Eq. 4 which now becomes Eq. 10.

$$A = \frac{\Psi \epsilon (1 - \beta_{eff})}{1 - \beta_{eff} - k_p} \quad (10)$$

2.3. Method to Obtain Additional Kinetics Parameters

This work examines the use of several unique measurements to determine more kinetics information from Rossi- α measurements than only the prompt neutron decay constant. This methodology relies on the assumption that relevant neutronic parameters remain constant between measurements of unique configurations of the same experiment. Each measurement produces a fit with three parameters A , B , and α . Equations 10, 5, and 1, which compose A , B , and α respectively, contain six variables: Ψ , ϵ , β_{eff} , k_p , D_ν , and l .

It is assumed that the effective delayed neutron fraction β_{eff} and Diven's parameter D_ν do not appreciably change between configurations of the same experiment. If the system consists of highly enriched uranium and utilizes a neutron source, then the source strength is also static between different configurations. Additionally, if the neutron detectors are set-up in such a way that their geometric efficiency does not change between different configurations, then the detection efficiency also remains static. The intrinsic efficiency of the detectors is not considered because the incident neutron spectrum should not significantly change between different configurations of an experiment. If all of these parameters can be considered static between configurations, then only two parameters are unique to each configuration: k_p and l .

This methodology should work for systems containing between two and four unique measurements. Using less than four unique measurements, requires the assumption of at least one neutronic parameter. The number of assumptions as a function of number of unique measurements is shown in Table 1. If using more than four unique measurements, the system is over-determined.

Equations 11, 12, and 13 comprise the system of equations used to numerically solve for the kinetics parameters. Note the use of the subscript i which can stand for any unique measurement.

$$A_i = \frac{\Psi \epsilon (\beta_{eff} - 1)}{\beta_{eff} + k_{p,i} - 1} \quad (11)$$

Table 1: Number of Equations, Variables, and Assumptions Required to Create Well-Posed Problem for Different Numbers of Measurements

Number of Measurements	Number of Equations	Number of Variables	Number of Assumptions
1	3	6	3
2	6	8	2
3	9	10	1
4	12	12	0

$$B_i = \frac{\epsilon D_v k_{p,i}^2}{2l_i (1 - k_{p,i})} \quad (12)$$

$$\alpha_i = \frac{k_{p,i} - 1}{l_i} \quad (13)$$

For simplicity, a system using two unique measurements is examined in this work. Using a system of two unique measurements requires assumption of two parameters. For a system with two measurements, two assumptions are required. For the following system of equations, Diven's parameter D_v and the effective delayed neutron fraction β_{eff} are assumed to be independently quantified known parameters. So this system is designed to solve for Ψ , ϵ , $k_{p,1}$, $k_{p,2}$, l_1 , and l_2 using Eqns. 14 through 19.

$$A_1 = \frac{\Psi \epsilon (\beta_{eff} - 1)}{\beta_{eff} + k_{p,1} - 1} \quad (14)$$

$$B_1 = \frac{\epsilon D_v k_{p,1}^2}{2l_1 (1 - k_{p,1})} \quad (15)$$

$$\alpha_1 = \frac{k_{p,1} - 1}{l_1} \quad (16)$$

$$A_2 = \frac{\Psi \epsilon (\beta_{eff} - 1)}{\beta_{eff} + k_{p,2} - 1} \quad (17)$$

$$B_2 = \frac{\epsilon D_v k_{p,2}^2}{2l_2 (1 - k_{p,2})} \quad (18)$$

$$\alpha_2 = \frac{k_{p,2} - 1}{l_2} \quad (19)$$

3. RESULTS

The methodology developed in Section 2.3 was tested using some data obtained while measuring a thermal highly enriched uranium (HEU) system [11]. The system contained HEU foils, 9 x 9 x

0.003 in., laminated in plastic. These foils were interleaved between 14 x 14 x 0.5 in. polyethylene plates. The entire system was reflected both on the top and bottom by 3 in. of polyethylene. For this series of experiments, measurements were taken on a critical configuration of 16 foils as well as several subcritical configurations [11]. The configurations examined for this work contain 13, 14, and 15 foils and are all subcritical. The detectors were placed in a specially designed polyethylene plate such that the detectors could be inside the system as shown in Fig. 1. Mathematica was



Figure 1: The Rossi- α measurement system in the thermal HEU system.

used to perform the numerical analysis [12]. Using values of D_ν [4] from literature and assuming a value for β_{eff} , the results for the other kinetics parameters are determined. The values assumed for D_ν and β_{eff} are 0.795 and 0.008, respectively. The initial parameters for each of the three examined configurations are given in Table 2. The values obtained from the numerical solution in

Table 2: Data from Rossi- α Analysis

Number of Foils	A ($\frac{detections}{s}$)	B ($\frac{detections}{s}$)	α (s^{-1})
15	3.202E6	1.423E5	-344.8
14	4.133E5	4.282E4	-756.5
13	2.374E5	3.246E4	-1273.8

Mathematica are given in Table 3. The average of the two calculated values of k_p reported in Table 2 are compared to values calculated from the measured value of α while assuming a prompt neutron lifetime l and from simulation using MCNP[®] 6.1 * [13]. Only the prompt multiplication factor k_p is

*MCNP[®] and Monte Carlo N-Particle[®] are registered trademarks owned by Triad National Security, LLC, manager and operator of Los Alamos National Laboratory. Any third party use of

examined in Table 4 because the work referenced assumed the neutron lifetime remained constant and had a value of $42.63 \mu\text{s}$ [11]. This result for the neutron lifetime is near the values obtained by the method developed in this work. The values obtained for the neutron lifetime in this method seem to range between 30 and $40 \mu\text{s}$, in theory these neutron lifetimes should be nearly constant or get slightly shorter as deeper sub-criticality is achieved. In this work there appears to be no trend, this is likely due to the uncertainty in the parameter from this method. This will be an effort of future works.

Table 3: Results for Kinetics Parameters

Number of Foils in 1	Number of Foils in 2	$k_{p,1}$	$k_{p,2}$	l_1 (μs)	l_2 (μs)	Ψ ($\frac{\text{fissions}}{\text{s}}$)	ϵ ($\frac{\text{detections}}{\text{fission}}$)
15	14	0.9897	0.9742	29.87	34.10	6.6E4	0.1124
15	13	0.9896	0.9593	30.23	31.92	6.8E4	0.1152
14	13	0.9684	0.9510	41.71	38.47	5.8E4	0.1702

Table 4: Comparison of Results to Previously Published Values

Number of Foils	k_p		
	This Paper	From α	From Simulation
15	0.9896	0.9856	0.9849
14	0.9713	0.9689	0.9679
13	0.9552	0.9489	0.9453

4. CONCLUSIONS

This work has examined a new method to obtain additional kinetics information from Rossi- α measurements. This methodology requires multiple configurations of a single experiment to be measured, as well as adds some additional assumptions. Based on the proof of concept comparisons with measured data, this new methodology has been shown to be promising at obtaining additional kinetics information. The work is far from over and still needs more validation. The

such registered marks should be properly attributed to Triad National Security, LLC, including the use of the designation as appropriate.

promising results obtained include values of the prompt multiplication factor k_p agree well with alternative experiments and simulations.

Looking forward, validation using more data of both thermal and fast neutron measurements needs to be completed, and investigation of how well Ψ and ϵ agree with the expected values needs to be completed. While these results are promising, there is still much work to complete on the development of this method including the propagation of uncertainty for all parameters which will hopefully shed some light onto the neutron lifetime results. Once the method has been validated, investigation of systems with three or four unique measurements solved simultaneously is of interest.

ACKNOWLEDGEMENTS

This work was supported by the DOE Nuclear Criticality Safety Program, funded and managed by the National Nuclear Security Administration for the Department of Energy.

REFERENCES

- [1] R. E. Uhrig. *Random Noise Techniques in Nuclear Reactor Systems*. Ronald, New York, NY (1970).
- [2] M. M. R. Williams. *Random Processes in Nuclear Reactors*. Maxwell House, Elmsford, NY (1974).
- [3] H. C. Paxon and G. E. Hansen. “Critical Assemblies and Associated Measurements.” Unpublished.
- [4] G. R. Keepin. *Physics of Nuclear Kinetics*. Addison-Wesley, Reading, MA (1965).
- [5] M. Hayashi. “Comments on” Neutron Lifetime, Fission Time, and Generation Time”.” *Nuclear Science and Engineering*, **volume 121**, pp. 492–492 (1995).
- [6] F. de Hoffman. “Statistical Fluctuations in the Water Boiler and Time Dispersion of Neutrons Emitted per Fission.” Technical Report LA-101, Los Alamos National Laboratory, Los Alamos, NM (1944).
- [7] G. Hansen, H. H. Helmick, and J. D. Orndoff. “Neutron Counting Statistics in Basic Fast Critical Assemblies.” Technical Report LA-UR-85-3859, Los Alamos National Laboratory, Los Alamos, NM (1985).
- [8] G. Spriggs, G. Hansen, E. Martin, E. Plassman, R. Pederson, J. Schlessler, T. Krawczyk, G. Smolen, and J. Tanner. “Subcritical Measurements of the WINCO Slab Tank Experiment using the Source-jerk Technique.” *ANS Conference Proceedings* (1989).
- [9] R. Feynman. “Statistical Behavior of Neutron Chains.” Technical Report LA-591(DEL), Los Alamos National Laboratory, Los Alamos, NM (1946).
- [10] C. P. Baker. “Time Scale Measurements by the Rossi Method.” Technical Report LA-617, Los Alamos National Laboratory, Los Alamos, NM (1947).
- [11] G. McKenzie. “Limits on Subcritical Reactivity Determination using Rossi- α and Related Methods.” *University of Illinois* (2018).
- [12] W. R. Inc. “Mathematica, Version 12.0.” URL <https://www.wolfram.com/mathematica>.
- [13] T. Goorley. “Initial MCNP6 Release Overview.” *Nuclear Technology* (2012).