Generalized sensitivity analysis capability with the differential operator method in RMC Code

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ABSTRACT

Sensitivity analysis is an important way for us to know how the input parameters will affect the output of a system. Therefore, recently, there is an increased interest in developing sensitivity analysis methods in continuous-energy Monte Carlo Code due to the fact that Monte Carlo method can perform high-fidelity simulations of nuclear reactor. Previous studies mainly focused on developing sensitivity analysis method suitable for analyze eigenvalue. There are relatively few researches for performing sensitivity analysis of generalized response function by using continuous-energy Monte Carlo code. So, in this work, the differential operator method (DOM) has been investigated and implemented in continuous-energy Reactor Monte Carlo code (RMC) to perform sensitivity analysis of generalized response function in the form of ratios of reaction rate. The DOM implemented in RMC is based on the analog Monte Carlo transport mode and non-analog Monte Carlo transport mode. The correctness of the newly implemented method has been verified by comparing the results with those calculated by using the collision history-based method through the Jezeble and Flattop benchmark problems. In general, the results given by the DOM agree well with those obtained by the collision history-based method with an accuracy of 5%. Moreover, it is also shown that the non-analog Monte Carlo transport mode can obtain lower relative standard deviation of the sensitivity coefficients than the analog Monte Carlo transport mode.

KEYWORDS: Monte Carlo, sensitivity analysis, differential operator method, RMC

1. INTRODUCTION

With increased demand for economic and safety in nuclear reactor design, sensitivity and uncertainty analysis draw many research interests. In the past few years, a lot of research about eigenvalue sensitivity analysis have been conducted and several continuous-energy Monte Carlo codes such as SCALE[1], SERPENT2[2], MCNP6[3,4], Open MC[5], MONK[6], MORET5[7], McCARD[8] and the Reactor Monte Carlo (RMC) code[9–12] have been equipped with capability for performing sensitivity analysis of eigenvalue. Besides eigenvalue, some generalized responses such as reaction rate ratios, reaction rates and neutron fluxes are also important in the reactor design. So, methods suitable for performing sensitivity analysis of these different generalized response functions based on the continuous-energy Monte Carlo codes should be investigated. In the previous work, the collision history-based method is implemented in the continuous-energy Reactor Monte Carlo (RMC) code[13,14] to perform sensitivity analysis of reaction rate ratios and bilinear ratios. However, this method is based on analog Monte Carlo transport. Since most

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Monte Carlo codes are based on nonanalog Monte Carlo transport, sensitivity analysis methods suitable for nonanalog Monte Carlo transport should be developed. The GEAR-MC method[15–17] was implemented in RMC[18] based on nonanalog Monte Carlo transport. However, this method is faced with huge memory consumption problem when performing sensitivity analysis. Therefore. the superhistory-based differential operator method (SH-DOM)［19] and the superhistory-based GEAR-MC method[20,21] were developed and implemented in RMC and both of them are based on the nonanalog Monte Carlo transport. In this work, the sensitivity coefficients produced by the differential operator method based on analog Monte Carlo transport and nonanalog Monte Carlo transport are compared.

Section 2 briefly reviews the differential operator method. Section 3 compares the sensitivity coefficients produced by the differential operator method with those produced by the collision history-based method for Jezebel and Flattop benchmark problems. Section 4 gives the conclusion.

2. METHODOLOGY

The DOM is first briefly reviewed here and the thorough description can be found in previous studies[22,23] for more thorough descriptions of DOM

The sensitivity coefficient is defined as

$$S_x^R = \frac{dR/R}{dx/x} \tag{1}$$

where R is the response function, x is the input parameters such as nuclear data.

In this work, the following response function is investigated

$$R = \frac{\langle x_1, \Psi \rangle}{\langle x_2, \Psi \rangle} \tag{2}$$

where $\langle \ \rangle$ means the response function is integrated over some phase space, x_1 and x_2 are the macroscopic cross sections for different reaction types and Ψ is the neutron flux.

Substituting Eq. (2) into Eq. (1) yields

$$\begin{aligned} S_x^R &= \frac{x\frac{\partial}{\partial x}\langle x_1 \Psi \rangle}{\langle x_1 \Psi \rangle} - \frac{x\frac{\partial}{\partial x}\langle x_2 \Psi \rangle}{\langle x_2 \Psi \rangle} = \\ &\frac{1}{Y}\sum_{l=1}^{L_{\text{tot}}} \frac{1}{N_1 w_0^1} \sum_{n=1}^{N_1}\sum_{k=1}^{K_n} q_{n,k}^1 d_{n,k}^1[Y] \left(\frac{q_{0,n,k}^{'(1)}}{q_{0,n,k}^1} + \frac{d_{n,k}^{'(1)}[Y]}{d_{n,k}^1[Y]} + \sum_{j=l-\gamma}^{l-1} \left[\frac{q_{0,n,f}^{'(1)}}{q_{0,n,f}^1} + \frac{d^{'(j)}[k_{\text{eff}}]}{d^{j}[k_{\text{eff}}]} \right] \right) \bullet x \\ &- \frac{1}{Z}\sum_{l=1}^{L_{\text{tot}}} \frac{1}{N_1 w_0^1} \sum_{n=1}^{N_1}\sum_{k=1}^{K_n} q_{n,k}^1 d_{n,k}^1[Z] \left(\frac{q_{0,n,k}^{'(1)}}{q_{0,n,k}^1} + \frac{d_{n,k}^{'(1)}[Z]}{d_{n,k}^1[Z]} + \sum_{j=l-\gamma}^{l-1} \left[\frac{q_{0,n,f}^{'(1)}}{q_{0,n,f}^1} + \frac{d^{'(j)}[k_{\text{eff}}]}{d^{j}[k_{\text{eff}}]} \right] \right) \bullet x \end{aligned} \tag{3}$$

where $Y = \langle x_1 \Psi \rangle$. $Z = \langle x_2 \Psi \rangle$. L_{tot} is the total number of active cycles. N_1 is the total number of neutrons in generation l . w_0^1 is the initial weight of every particle in cycle l . K_n is the total number of collisions in history n during generation l . The quantity $q_{0,n,k}^{'(1)} / q_{0,n,k}^1$ represents the relative change in the probability of the random walk of neutron n entering collision k in the l^{th} generation in which the source perturbation effect is not considered. $q_{0,n,f}^{'(1)} / q_{0,n,f}^1$ represents the relative change in the probability of the random walk of the ancestor neutron in the j^{th} generation with source perturbation effect taken into account. $d_{n,k}^{'(1)}[Y] / d_{n,k}^1[Y]$ is the perturbation in x of the contribution to reaction rate Y in the random walk of neutrons in the l^{th} generation. The subscript f represents the collision in generation j that caused fission and produced the neutron. Furthermore, $d^{'(j)}[k_{\mathrm{eff}}]/d^{j}[k_{\mathrm{eff}}]$ is the perturbation in x of the contribution to the eigenvalue in the random walk of the $(l-\gamma)^{th}$ ancestor. The DOM flow chart is shown in Fig. 1. The sensitivity coefficients will be estimated until all the particles in a block have been simulated. In DOM, w_{SP} is estimated in the latent generation and w_{NSP} is calculated in asymptotic generation as illustrated in Fig. 1.

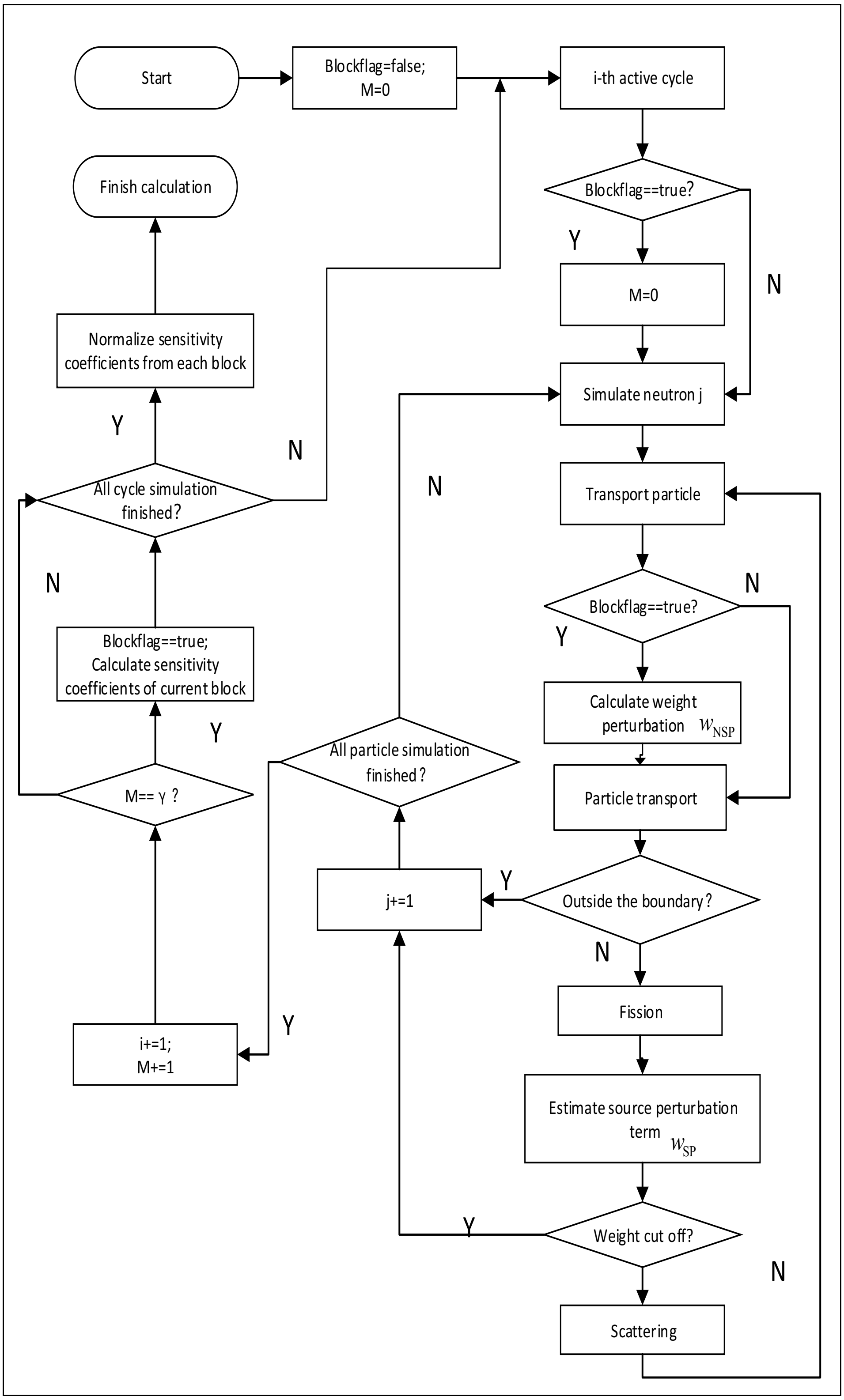

Fig. 1. DOM flow chart.

3. RESULTS

In this work, the differential operator method for the analog Monte Carlo transport and the nonanalog Monte Carlo transport are implemented in continuous-energy Reactor Monte Carlo (RMC) code. The newly developed method was verified by comparing with results calculated by the collision history-based method in Jezebel and Flattop benchmark problems. All the calculations are performed on the Sunway TaihuLight supercomputer with 10 nodes. Each node has 24 CPUs (Intel Xeon E5-2680 v3 at 2.5 GHz) sharing 128 gigabytes of memory. The cross-section used in the calculation are produced from ENDF/B-VII nuclear data library. The following response function is selected

$$R = \frac{\iiint \Sigma_f^{^{238}\mathrm{U}}(r,E)\Psi(r,E,\Omega)dEdrd\Omega}{\iiint \Sigma_f^{^{235}\mathrm{U}}(r,E)\Psi(r,E,\Omega)dEdrd\Omega} \tag{4}$$

where $\Psi(r,E,\Omega)$ is the neutron flux. $\Sigma_f^{^{238}\mathrm{U}}$ and $\Sigma_f^{^{235}\mathrm{U}}$ represents fission cross section for U-238 and U-235 respectively. The asymptotic generation is set to be ten to consider the fission source perturbation effect correctly[2,3].

3.1. Jezebel benchmark

The Jezebel benchmark[24] is a bare metallic sphere of plutonium. Sensitivity coefficients were evaluated for five isotopes. The response function is integrated over a small central sphere of 1-cm radius. The energy–integrated F28/F25 sensitivity coefficients for this benchmark are presented in Table I. As can be seen from the Table I, relative differences between differential operator method and collision history-based method are within 5%. This means differential operator method generally agrees with collision history-based method. Moreover, the sensitivity coefficients produced by the nonanalog Monte Carlo transport based differential operator method have lower relative standard deviation (RSD) than those produced by the analog Monte Carlo transport based differential operator method. This is because the variance reduction techniques used in the nonanalog Monte Carlo transport.

Table I. Sensitivity coefficients for the Jezebel benchmark.

Nuclide	Reaction type	Non-analog DOM	RSD (%)	Analog DOM	RSD (%)	Collision-history	RSD (%)	Relative difference Non-analog DOM/Collision-history-based method (%)	Relative difference Analog DOM/Collision-history-based method (%)
Pu-239	nubar	1.95E-03	4.20	2.02E-03	4.49	1.99E-03	0.85	-2.05	1.53
Pu-239	Inelastic	-1.62E-01	0.14	-1.62E-01	0.16	-1.60E-01	0.04	1.28	1.01
Pu-239	n,2n	-1.99E-03	0.84	-2.00E-03	0.95	-2.00E-03	0.23	-0.35	0.17
Pu-239	Fission	4.97E-02	0.20	5.17E-02	0.28	5.18E-02	0.11	-4.02	-0.18
Pu-239	n,gamma	1.02E-02	0.11	1.02E-02	0.13	1.01E-02	0.09	1.08	0.91
Pu-239	Elastic	-6.38E-02	0.59	-6.34E-02	0.68	-6.40E-02	0.18	-0.27	-0.90
Pu-239	Total	-1.68E-01	0.26	-1.65E-01	0.30	-1.64E-01	0.08	2.34	0.64
Pu-240	nubar	-1.85E-03	4.23	-1.88E-03	4.64	-1.84E-03	0.89	0.54	2.28
Pu-240	Elastic	-4.01E-03	2.21	-4.14E-03	2.41	-4.06E-03	0.64	-1.15	2.08
Pu-240	Inelastic	-8.09E-03	0.62	-8.04E-03	0.70	-8.06E-03	0.18	0.46	-0.19
Pu-241	n,gamma	3.96E-05	0.11	3.96E-05	0.13	4.07E-05	1.88	-2.52	-2.60
Ga-69	Total	-3.93E-03	1.41	-4.09E-03	1.53	-3.95E-03	0.40	-0.41	3.45
Ga-69	Inelastic	-2.24E-03	1.06	-2.26E-03	1.18	-2.24E-03	0.30	-0.05	1.01

Ga-71	Total	-2.95E-03	1.54	-3.09E-03	1.66	-2.98E-03	0.44	-1.28	3.67
Ga-71	Inelastic	-1.79E-03	1.14	-1.82E-03	1.28	-1.81E-03	0.33	-1.17	0.65
Ga-71	n,gamma	3.74E-05	0.27	3.70E-05	0.30	3.87E-05	1.49	-3.38	-4.40

3.2. Flattop benchmark

The Flattop benchmark[24] is a sphere of plutonium-gadolinium alloy that is reflected by a region of depleted uranium. The response function is also calculated in a small central sphere of 1-cm radius. Table II presents the energy –integrated sensitivity coefficients for seven different isotopes. Fairly good agreement between the differential operator method and the collision history-based method can be appreciated. The biggest difference between the differential operator method and the collision history predictions occurs in the Pu-239 elastic where the difference is 7.17%. This difference is caused by the large relative standard deviations of the sensitivity results produced by the analog based differential operator method and the collision history-based method. It can also be noted that the fission cross section sensitivity coefficients for U-235 and U-238 are larger than the other nuclear data. This is due to the fact that fission event of U-235 and U-238 contribute directly and dominantly to F28/F25.

Table II. Sensitivity coefficients for the Flattop benchmark.

Nuclide	Reaction type	Non-analog	RSD (%)	Analog	RSD (%)	Collision-history	RSD (%)	Relative difference Non-analog DOM/Collision-history-based method (%)	Relative difference Analog DOM/Collision-history-based method (%)
Pu-241	Inelastic	-4.76E-04	5.65	-4.53E-04	6.92	-4.53E-04	1.81	5.22	0.00
Pu-240	Inelastic	-6.14E-03	1.67	-6.33E-03	1.85	-6.17E-03	0.49	-0.43	2.59
Pu-240	n,gamma	1.25E-03	0.19	1.26E-03	0.88	1.26E-03	0.54	-1.04	-0.32
Pu-239	Total	5.88E-02	1.78	5.94E-02	2.05	6.22E-02	0.53	-5.51	-4.50
Pu-239	Elastic	-1.42E-02	6.20	-1.54E-02	6.46	-1.44E-02	1.86	-1.69	7.17
Pu-239	Inelastic	-1.15E-01	0.39	-1.15E-01	0.46	-1.14E-01	0.12	1.33	0.78
Pu-239	n,2n	-1.42E-03	2.04	-1.47E-03	2.33	-1.45E-03	0.58	-2.20	1.26
Pu-239	Fission	1.71E-01	0.22	1.72E-01	0.26	1.73E-01	0.08	-1.34	-0.57
Pu-239	n,gamma	1.88E-02	0.19	1.87E-02	0.22	1.86E-02	0.14	0.67	0.62
Pu-239	nubar	5.70E-02	0.58	5.69E-02	0.66	5.68E-02	0.12	0.29	0.22
U-238	Total	8.04E-01	0.20	8.04E-01	0.23	8.03E-01	0.06	0.18	0.15
U-238	Elastic	-1.23E-01	1.19	-1.23E-01	1.40	-1.25E-01	0.36	-1.41	-1.61
U-238	Inelastic	-8.14E-02	0.80	-8.19E-02	0.91	-8.04E-02	0.25	1.22	1.88
U-238	n,2n	-1.31E-03	3.75	-1.24E-03	4.46	-1.25E-03	1.09	4.49	-0.69
U-238	Fission	9.65E-01	0.02	9.65E-01	0.03	9.65E-01	0.01	-0.01	-0.01
U-238	n,gamma	4.52E-02	0.35	4.51E-02	0.39	4.44E-02	0.12	1.90	1.73
U-238	nubar	-4.98E-02	0.55	-4.97E-02	0.64	-4.97E-02	0.12	0.14	0.01
U-235	Total	-1.01E+00	0.02	-1.01E+00	0.02	-1.01E+00	0.00	-0.01	0.03
U-235	Fission	-1.00E+00	0.01	-1.00E+00	0.01	-1.00E+00	0.00	0.00	0.01
U-235	n,gamma	6.15E-04	0.36	6.14E-04	0.40	6.13E-04	0.81	0.23	0.16
U-235	nubar	-7.61E-03	1.22	-7.65E-03	1.37	-7.52E-03	0.27	1.20	1.69
Ga-71	Total	-1.79E-03	5.86	-1.81E-03	6.21	-1.70E-03	1.82	4.94	6.49
Ga-71	Inelastic	-1.44E-03	2.75	-1.48E-03	2.97	-1.44E-03	0.82	0.31	3.19
Ga-71	n,gamma	7.28E-05	0.28	7.31E-05	0.37	7.55E-05	2.12	-3.62	-3.22

Ga-69	Total	-2.25E-03	5.69	-2.35E-03	5.99	-2.30E-03	1.64	-2.01	2.37
Ga-69	Inelastic	-1.82E-03	2.43	-1.78E-03	2.92	-1.78E-03	0.75	2.57	0.33

4. CONCLUSIONS

In this work, the differential operator method is implemented in the continuous-energy Monte Carlo code RMC to perform generalized sensitivity analysis for response function in the form of reaction rate ratios. The newly developed method is verified by comparing with the results obtained by using collision history-based method. The differential operator method produce response sensitivity coefficients that agreed well with those produced by the collision history-based method. Results also shown that the non-analog Monte Carlo transport mode can obtain lower relative standard deviation of the sensitivity coefficients than the analog Monte Carlo transport mode.

Future work will focus on extending the differential operator method to perform sensitivity analysis of reaction rates and applying the superhistory algorithm to solve the huge memory consumption problem encountered when using this method to perform sensitivity analysis of reaction rates.

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