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EFFECT OF β ON EFFECTIVE MULTIPLICATION FACTOR IN $1/f^\beta$ SPECTRUM RANDOM SYSTEM

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ABSTRACT

We investigated the effect of β on effective multiplication factor(k_{eff}) in the $1/f^\beta$ spectrum random system. The random system was generated by the $1/f^\beta$ noise model. The model is a continuous space model based on the Randomized Weierstrass function and describes the component spatial distribution with a power spectrum of $1/f^\beta$, where f and β are the frequency domain variable and the characteristic parameter related to randomness, respectively. In this work, the two-group Monte Carlo calculations were performed to obtain the k_{eff} for a simple cubic geometry that consisted of two materials (fuel burned to 12 GWd/t and concrete). A large number of replicas having different spatial distribution and characterized by the representative β values were generated using the model, and the distribution on k_{eff} was analyzed. We found the dependency on β of standard deviation, skewness, and kurtosis of k_{eff} distribution. This result is expected to help to predict the k_{eff} distribution due to the randomizing model.

KEYWORDS: criticality safety, fuel debris, random media.

1. INTRODUCTION

The criticality control of the fuel debris is significant for decommissioning of the Fukushima Daiichi Nuclear Power Plant. For the criticality control, it is important to obtain information about an inside of the fuel debris such as the composition that is a set of average values of material concentration including fuel, and the component spatial distribution that is the local variation of the concentration. However, the fuel debris might have been formed by the mixing of fuel and structure materials through some interactions such as fuel coolant interaction and melted corium concrete interactions without control. Therefore, the inside of the fuel debris is not known and expected to be non-uniform.

Under this situation, the systematic computational investigations of the effect of the composition on the criticality have been conducted by using the simplified model consisting of a fuel sphere and a moderator shell as a conservative model of the component spatial distribution [1-3]. On the other hand, Statistical Geometry Model (STGM) [4] is used to check the difference between homogeneous and heterogeneous models [5,6]. However, the investigation of the effect of the component spatial distribution has not been conducted in a systematic manner.

The $1/f^\beta$ noise model is one of the interesting candidates of the component spatial distribution model [7,8]. This model is a continuously-varying spatial model and approximately describes the distribution with a power spectrum of $1/f^\beta$, where f and β are the frequency domain variable and the characteristic parameter related to randomness, respectively [7,8]. Depending on the β value, the model generates the component spatial distribution which has different characteristics. Moreover, it is well known that the $1/f^{\beta}$ noise describes a diverse range of natural phenomena by changing the β value like lattice vibration (β=1.0), Kolmogorov law of turbulence (β=5/3), Brownian motion (β=2.0), etc. Namely, the fuel debris may be modeled by the $1/f^{\beta}$ noise model [9] as it was formed by way of extreme disorder. Therefore, the investigation of the effect of β on the k_{eff} is helpful to understand the effect of the component spatial distribution on the k_{eff}. This knowledge is expected to be useful for the criticality control.

In this work, we investigated the β dependence of the k_{eff} in the $1/f^{\beta}$ noise model. A large number of random media was generated according to the $1/f^{\beta}$ noise model. The distributions of the k_{eff} values were analyzed from the view point of standard deviation, skewness, and kurtosis to capture the features of the k_{eff} distributions.

2. CALCULATION METHOD

In this work, to calculate k_{eff} values, two-energy-group Monte Carlo calculations were conducted with a simple geometry including the random media based on the $1/f^{\beta}$ noise model.

A simple cubic geometry shown in Figure 1 was used for this calculation. It consisted of mixture region and reflector region. The mixture region was a cube of 100 cm per side. The moderator region covered the fuel cube. The thickness of the moderator region was 20 cm. The mixture region consisted of fuel burned to 12 GWd/t and normal concrete and the component spatial distribution in it was modeled by the $1/f^{\beta}$ noise model. We changed a mean volume fraction of fuel in the mixture region from 2.3% to 94.2%. The moderator region consisted of only normal concrete.

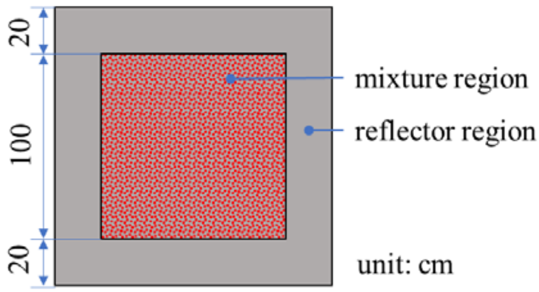

Figure 1. Geometry for calculation of k_{eff} with the $1/f^{\beta}$ noise model.

The $1/f^{\beta}$ noise model generating random media is based on Randomized Weierstrass function (RWF) [7,8]. The volume fraction of material in the random mixture region is changed by a fluctuation of the RWF. The RWF is computed as

$$W(\mathbf{r}) = \frac{\left(\lambda^{\frac{\beta-1}{2}} - 1\right)}{1-\lambda^{-\left(\frac{\beta-1}{2}\right)M}} \sum_{j=1}^{M} B_j \lambda^{-\left(\frac{\beta-1}{2}\right)j} \sin\left(\frac{\lambda^j \mathbf{r}\bullet\mathbf{\Omega}_j}{S} + A_j\right), \lambda > 1, 1 < \beta < 3, \quad (1)$$

where, A_j are independent random variables uniformly distributed on $[0,2\pi)$, B_j are independent Bernoulli random variables with zero mean and unit variance, $\mathbf{r}$ is the space coordinate vector, $\mathbf{\Omega}_j$ are vectors sampled uniformly and independently on the unit sphere, and S is the scaling factor. In this work, we set the values of S and λ at 100-cm and 1.5, respectively. $W(\mathbf{r})$ is less than or equal to 1 and characterized by the value of β. Different $W(\mathbf{r})$ is produced by changing the random variables A_j and B_j. Namely, we can generate a number of independent random media characterized by β value, which was called "replica".

The macroscopic cross section in the mixture region for each reaction $\Sigma_i(\mathbf{r})$ was defined using $W(\mathbf{r})$, Σ_i^F that is the macroscopic cross section of the fuel, and Σ_i^M that is the macroscopic cross section of the moderator as

$$\Sigma_i(\mathbf{r}) = \begin{cases} \left(1 - V^F\left(1 + W(\mathbf{r})\right)\right)\Sigma_i^M + V^F\left(1 + W(\mathbf{r})\right)\Sigma_i^F, & V^F \le 0.5 \\ \left(1 - (1 - V^F)\left(1 - W(\mathbf{r})\right)\right)\Sigma_i^F + (1 - V^F)\left(1 - W(\mathbf{r})\right)\Sigma_i^M, & V^F > 0.5 \end{cases}, \tag{2}$$

where, V^F is the mean volume fraction of fuel. Additionally, the macroscopic cross section in the reflector region Σ_i^R was used. Σ_i^M, Σ_i^F, and Σ_i^R were calculated by MVP code [10] with JENDL-4.0 [11]. The geometries for calculation of the macroscopic cross sections were shown in Figure 2. The geometry for the calculation of Σ_i^M and Σ_i^F was a cube and composed of concrete and fuel regions. The volume of the fuel region was fixed. The volume of the concrete region was adjusted to match a ratio of volume between fuel and concrete regions with a mean volume fraction of concrete in the mixture region. Σ_i^M and Σ_i^F were calculated by tallying in concrete and fuel region, respectively. The geometry for calculation of Σ_i^R was also cube and consist of three regions (fuel, concrete, and outer concrete regions). The outer concrete region was also made of concrete as same as concrete region. Σ_i^R was calculated by tallying in the outer concrete region. The atomic densities of fuel and concrete was based on previous work [1] and shown in Table I.

Table I. Atomic densities (10^{24} atoms/cm^3) of fuel and concrete [1].

Fuel (burn-up of 12GWd/t)				Normal concrete (2.3 g/cm^3)			
Nuclide	Atomic density	Nuclide	Atomic density	Nuclide	Atomic density	Nuclide	Atomic density
U-234	6.9825E-06	Rh-103	1.0233E-05	H-1	1.3742E-02	K-40	5.3883E-08
U-235	6.2168E-04	Cs-133	1.9503E-05	O-16	4.5921E-02	K-41	3.0995E-05
U-238	2.2393E-02	Nd-143	1.5032E-05	C-nat	1.1532E-04	Ca-40	1.4566E-03
Pu-238	2.8950E-07	Nd-145	1.0676E-05	Na-23	9.6397E-04	Ca-42	9.7218E-06
Pu-239	9.1638E-05	Sm-147	4.1588E-06	Mg-24	9.7861E-05	Ca-43	2.0285E-06
Pu-240	1.5755E-05	Sm-149	1.1729E-07	Mg-25	1.2389E-05	Ca-44	3.1344E-05
Pu-241	5.2453E-06	Sm-150	3.4526E-06	Mg-26	1.3640E-05	Ca-46	6.0104E-08
Pu-242	6.7431E-07	Sm-152	1.6767E-06	Al-27	1.7409E-03	Ca-48	2.8099E-06
Am-241	1.5322E-06	Eu-153	9.9686E-07	Si-28	1.5326E-02	Fe-54	2.0014E-05
Mo-95	1.7814E-05	Gd-155	9.8586E-07	Si-29	7.7601E-04	Fe-56	3.1650E-04
Tc-99	1.7775E-05	O-16	4.6983E-02	Si-30	5.1513E-04	Fe-57	7.5915E-06
				K-39	4.2949E-04	Fe-58	9.6620E-07

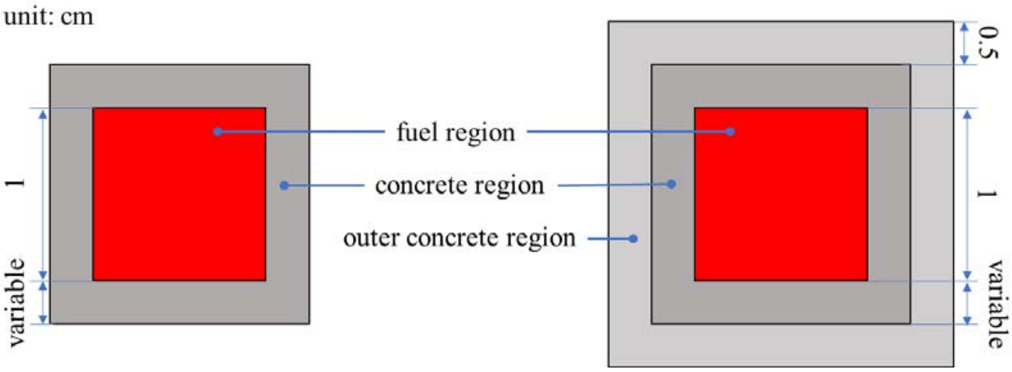

Figure 2. Geometries to calculate the macroscopic cross sections. The left and right geometries were used to calculate (Σ_i^M and Σ_i^F) and Σ_i^R, respectively.

3. RESULTS AND DISCUSSION

The k_{eff} values of independent random media that was characterized by different β values in different mean volume fractions of fuel (2.3%, 12.5%, and 94.2%) are shown as histograms in Figure 3. It is found that the shapes of these histograms were different from normal and changed with the β value and the mean volume fraction. To capture the feature of the distributions, standard deviation, skewness, and kurtosis of them were calculated and compared. The feature was considered from the component spatial distribution generated by the $1/f^{\beta}$ noise model.

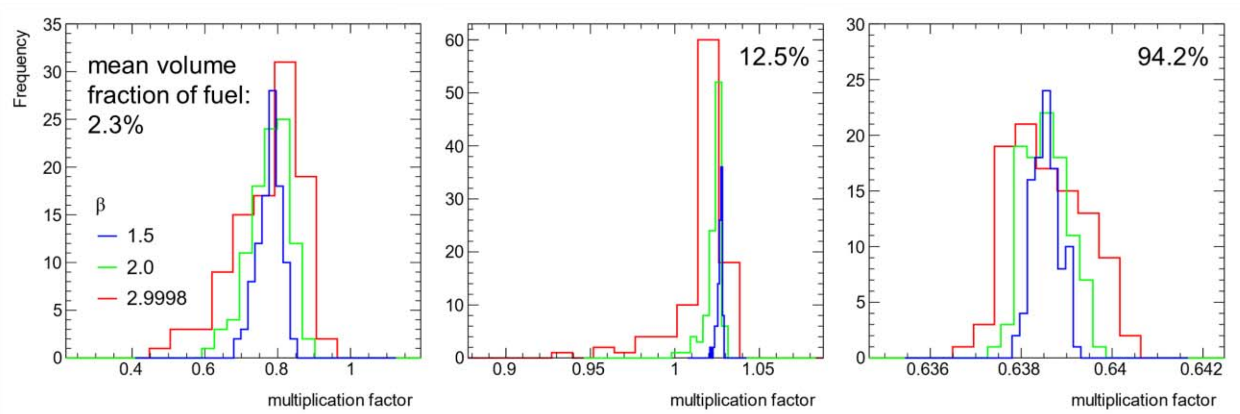

Figure 3. Histograms of the k_{eff} values of random media which had different β values (1.5, 2.0, and 2.9998) in different mean volume fraction of fuel (2.3%, 12.5%, and 94.2%). The blue, green, and red line denote the results in the case that the β values are 1.5, 2.0, and 2.9998, respectively.

First, the standard deviations of the k_{eff} distributions are shown as a function of β in Figure 4, which show a strong positive correlation between them. That implies a larger value of β gives a larger standard deviation to the k_{eff} distribution and a k_{eff} value can have a large uncertainty. Therefore, the random media with the large β values should be paid attention to for criticality safety. Figure 4 also shows a negative dependence between the magnitude of standard deviation and the mean volume fraction of fuel, which may indicate that a system with a small mean volume fraction can have a high value of standard deviation.

The fitting results are also shown in figure 4. The data of standard deviation were fitted by a linear function using the least square method. The fitting results show good agreement with the data. It is expected the fitting results are useful for the estimation of variation of the k_{eff} value of the random media.

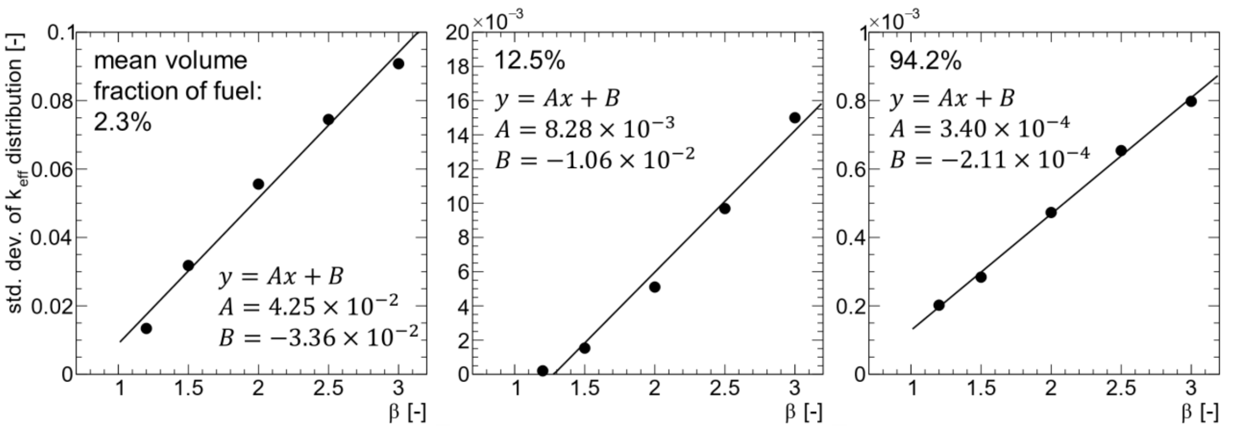

Figure 4. Standard deviation of the k_{eff} distribution for different mean volume fraction of fuel. The fitting results by linear functions are also shown. Left, center, and right figures correspond to the cases that the mean volume fraction of fuel are 2.3, 12.5, and 94.2%, respectively.

Next, skewness (Sk) and kurtosis (Ku) of the k_{eff} distributions were calculated to measure the asymmetry and the sharpness of the distributions. They are defined as

$$Sk = \frac{1}{n}\sum_{i=1}^{n}\frac{\left(k_{eff_i} - \mu\right)^3}{\sigma^3}, \tag{3}$$

$$Ku = \frac{1}{n}\sum_{i=1}^{n}\frac{\left(k_{eff_i} - \mu\right)^4}{\sigma^4} - 3, \tag{4}$$

where, k_{eff_i} is the k_{eff} value of replica i, n is the number of replicas, μ is the mean value of k_{eff_i}, and σ is the standard deviation value of k_{eff_i}. Figures 5, 6 show the Sk and Ku as a function of β, respectively. The variation of Sk and Ku values related to β value showed different tendencies with the mean volume fractions.

In the case of the mean volume fraction of 94.2%, the value of Sk and Ku increased and decreased with the β value, respectively. The increase of Sk value means the right tail of the distribution was getting longer and thicker. It indicated the random media which had the larger k_{eff} was likely to be generated as β value increase. The decrease of Ku value means there were a few random media that had the k_{eff} value differing significantly from other k_{eff} values.

On the other hand, in the case of the mean volume fraction of 2.3% and 12.5%, the value of Sk and Ku decreased and increased with the β value, respectively. Thus, the tail of the k_{eff} distribution extended to the left as β increases and there were random media that had significantly small k_{eff} value.

In this way, those tendencies of Sk and Ku are useful to estimate the asymmetry of the k_{eff} distribution and outlier of the k_{eff} values. The results of the β dependence may be used to predict the values of Sk and Ku. Moreover, Sk and Ku showed a dependency on the mean volume fraction. e.g. A system with small mean volume fraction is likely to have negative Sk and form a skewed distribution to the left side. A system with large mean volume fraction is likely to have small Ku and have a relatively small probability to have outlier k_{eff} values. Therefore, Sk and Ku may be useful to understand the characteristics of the mean volume fraction in the system although further investigations are required.

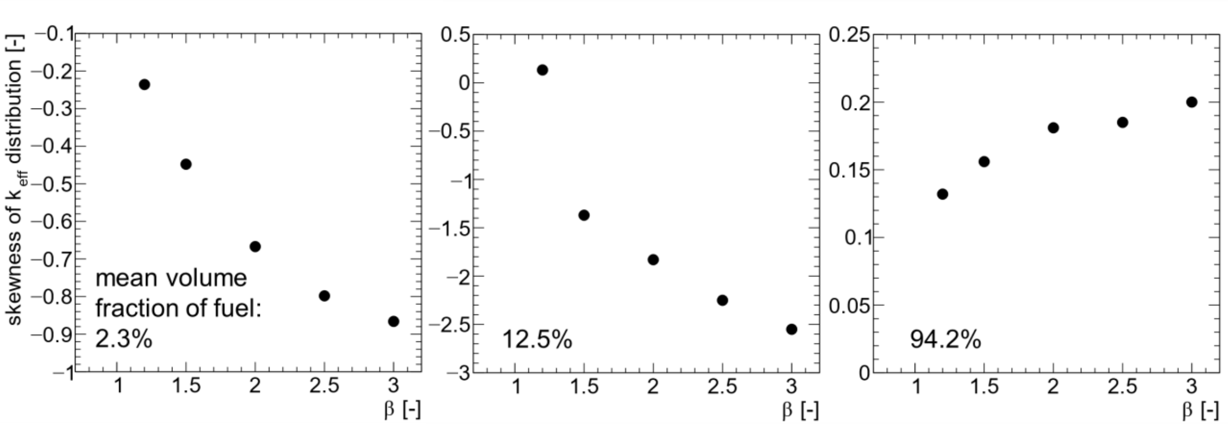

Figure 5. Skewness of the k_{eff} distribution for different mean volume fraction of fuel. The other meanings of figures are the same as Figure 4.

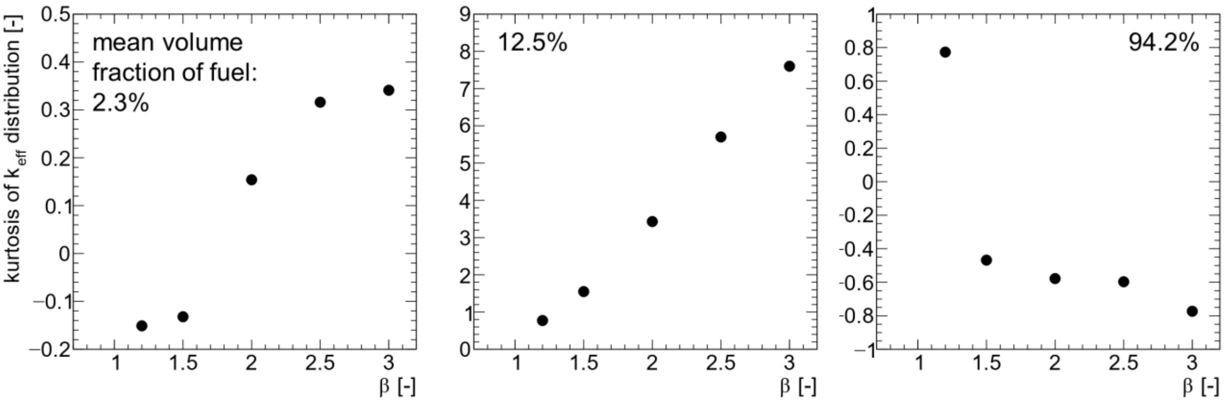

Figure 6. Kurtosis of the k_{eff} distribution for different mean volume fraction of fuel. The other meanings of figures are the same as Figure 4.

Finally, these tendencies were partly explained by the uniformity of the random media generated by the $1/f^{\beta}$ noise model. As a sample of random media generated by the $1/f^{\beta}$ noise model, Figure 7 shows 2-dimensional random media which were characterized by different β values (1.2, 1.5, 2.0, and 2.9998) and generated with the same random variable series. In this figure, a mean volume fraction of fuel was 12.5% and the contour color indicated the fuel volume fraction in the region.

Figure 7 shows that the state of the variation of the fuel fraction was changing with the β value. In the case of small β value, the spatial distribution of the fuel fraction changed little by little, and the variation of it was close to uniform. This tendency caused the standard deviation of the k_{eff} distribution to be small. On the other hand, with the increase of β value, the fuel fraction was varied smoothly, and the fuel-rich or -poor regions were locally generated. The fuel-rich or poor region changed the neutron spectra in the region locally. The change of the neutron spectra caused the change of the k_{eff} value. Namely, the variation range of k_{eff} values was getting larger with β value since a random media which have a various moderation condition was generated.

Moreover, the mean volume fraction might influence on the asymmetry of the k_{eff} distribution. In the case that the mean volume fraction is close to the optimal moderation condition like the result of 12.5%, the change of the local moderation condition had a negative effect on the k_{eff}. Therefore, the k_{eff} distribution of the random media being close to the optimal moderation condition might have a negative skew.

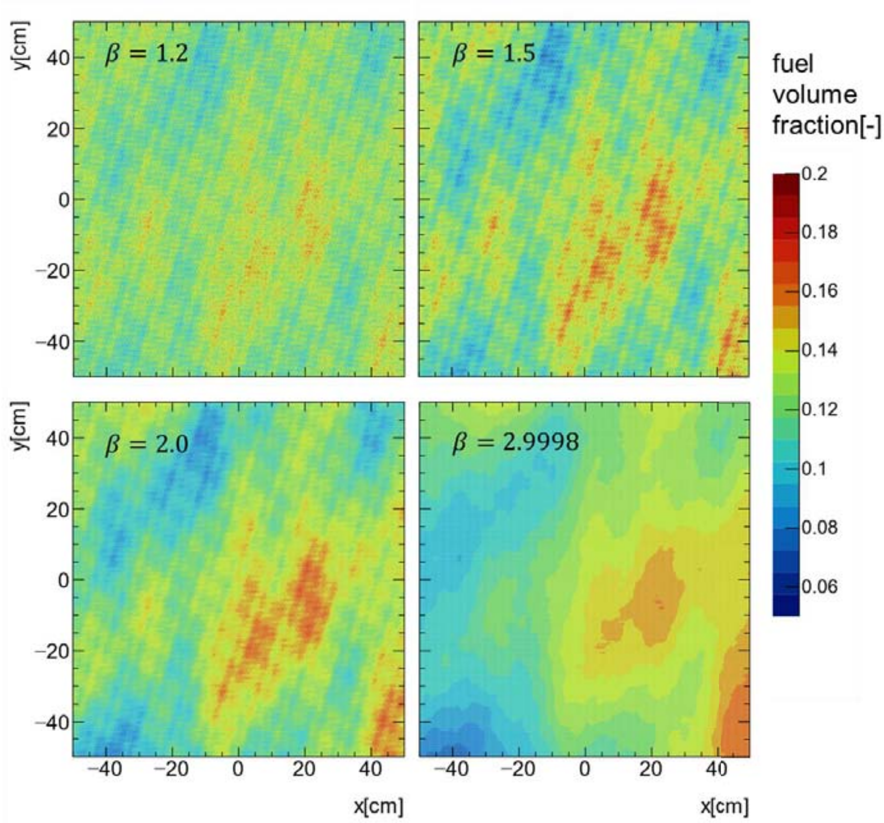

Figure 7. Sample of 2-dimensional random media which were generated by the $1/f^{\beta}$ noise model and characterized by different the β value with the same random variable series. The mean volume fraction of fuel was 12.5%. The upper left, upper right, lower left, and lower right figures show the case of β=1.2, 1.5, 2.0, and 2.9998, respectively. The contour color indicated the volume fraction of fuel in the region.

4. CONCLUSIONS

The effect of β on the k_{eff} of the random media generated with the $1/f^{\beta}$ noise model was investigated. We calculated the k_{eff} values of a large number of random media using two-group Monte Carlo calculations. The distributions of the k_{eff} values were different from normal. It is found that standard deviation, skewness, and kurtosis of the k_{eff} distribution varied with the β value. The value of standard deviation tended to rise with the β value. Skewness and kurtosis show the different tendencies for the different mean volume fractions of fuel due to the transition of non-uniformity nature of the component spatial distribution characterized by β. Those results are expected to be useful for predicting the feature of the k_{eff} distribution such as the variation, the asymmetry, and outlier of the k_{eff} values.

This report also indicates that the standard deviation, skewness, and kurtosis depended on not only the β value but also the mean volume fraction of fuel due to the change in the moderation conditions. The mean volume fraction has a strong influence on the magnitude and tendency of them. Therefore, we plan to investigate the dependence of the mean volume fraction in future work. The dependences of β value and mean volume fraction on k_{eff} values are expected to be useful for predicting the effect of the component spatial distribution generated by the $1/f^{\beta}$ noise model on the k_{eff} for the criticality control.

The $1/f^{\beta}$ noise model has been improved to deal with the range of $-1 < \beta < 3$ [8]. This model is implemented in a new continuous energy Monte Carlo solver, Solomon[12]. By using Solomon, it is expected to be able to investigate the dependence of the β and other parameters in more detail.

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