Excitation of Some Nitrogen-Like Ions: Calculations and Comparison with Measurements

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Abstract. Electronic energies of one-electron excited states $1s^22s^22p^2ns\ ^4P$, $1s^22s^22p^2ns\ ^2P$, $1s^22s^22p^2np\ ^4S$, $1s^22s^22p^2np\ ^2S$, $1s^22s^22p^2np\ ^4P$, $1s^22s^22p^2np\ ^2P$, $1s^22s^22p^2np\ ^4D$, $1s^22s^22p^2np\ ^2D$, $1s^22s^22p^2nd\ ^4P$, $1s^22s^22p^2nd\ ^2P$, $1s^22s^22p^2nd\ ^4D$, $1s^22s^22p^2nd\ ^2D$, $1s^22s^22p^2nd\ ^4F$, $1s^22s^22p^2nd\ ^2F$ of nitrogen-like ions with *n*=3-12 and Z=7-50 have been calculated. For all the aforementioned electronic configurations, fine-structure levels have been determined for states with different total momentum J. The comparison of theoretical results with the available experimental data shows that theory and measurements agree well. In particular, relative accuracy of 10^{-4} has been achieved in the electronic energy of N-like at Z > 10, while the typical deviation in the calculated fine-structure levels from experimental data is in the order of 10^{-2}

1 Introduction

Knowledge of the energy levels of multiply charged ions with large nuclear charge Z is critically important for solving multiple emerging problems of plasma physics and astrophysics. Obtaining accurate energy levels is not an easy task because it requires accurate simultaneous accounting for relativistic and radiation effects. In the literature, there exist relevant quantum-mechanical studies concerning atomic energy levels and other related properties, for example [1-11]. The studies are dedicated to both the general theoretical approach [1-5] and specific methods for calculating energy levels of multielectron ions with varying Z [6-11]. A large fraction of complexity in getting accurate energy level is related to the fact that the Schrödinger equation is nonrelativistic, and, thus, the theoretical description of ions with $Z>>1$ requires obtaining relativistic corrections. The Dirac's approach [3] is considered by many to be more suitable for spectroscopic calculations. In both scenarios, taking into account the radiation effects is needed.

The commonly used handbooks [12]- [13] contain a compilation of experimental data on the energy levels of multiply charged ions with variable Z. Due to the complexity of the spectroscopic measurements of such ions, the amount of existing experimental data is quite small. On the other hand, direct quantum-mechanical simulations have very large, sometimes prohibitively high, computational costs and encounter significant difficulties related with the use of a number of approximations. The modified Bohr model [14] allows obtaining a large amount of data on the energies of various configurations of multiply charged ions with varying Z at nearly zeros computational costs. In this work, the modified Bohr model [14] is used to obtain energy levels of multiply charged N-like ions. In particular, energies of one-electron excited states $1s^22s^22p^2ns\ ^4P$, $1s^22s^22p^2ns\ ^2P$, $1s^22s^22p^2np\ ^4S$, $1s^22s^22p^2np\ ^2S$, $1s^22s^22p^2np\ ^4P$, $1s^22s^22p^2np\ ^2P$, $1s^22s^22p^2np\ ^4D$, $1s^22s^22p^2np\ ^2D$, $1s^22s^22p^2nd\ ^4P$, $1s^22s^22p^2nd\ ^2P$, $1s^22s^22p^2nd\ ^4D$, $1s^22s^22p^2nd\ ^2D$, $1s^22s^22p^2nd\ ^4F$, $1s^22s^22p^2nd\ ^2F$ of N-like ions with *n*=3-12 and Z=7-50 have been calculated and compared with available experimental data.

2 Model and Method

As for the ground state of boron-like and carbon-like ions [15], [16], [17], the energy of the ground state of nitrogen-like ions, 1s22s22p3, is described assuming the formation of a hybrid state of all (five) 2s22p3 outer electrons with the identical wave function parameter. The interaction potential of 2s electrons and 2p electrons with internal 1s electrons is described in the same way as in lithium-like ions [18-22]. The parameter of the hybridized wave function for ground state is determined by minimizing the sum of electronic energies of outer electrons. In this way, we get accurate energies of the ground state of ions with varying Z. An empirical parameter in this model is the screening coefficient of the nuclear charge due to the interaction of electrons in the 2s22p3 configuration. Excited states 1s22s2p4 are also considered as hybridized states with the identical wave function parameter of outer electrons.

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3 Results and discussion

After the accurate theoretical description of hybridized energy states of carbon-like ions is achieved, we begin modeling of multiple one-electron excitations of N-like ions, where the outer electron is in the state with *n* > 2. The ionic core $1s^22s^22p^2$ yields multiple series of one-electron excited states of N-like ions $1s^22s^22p^2nl$ with n>2 and varying *l*=0,1,2,...,n-1. Different sets of similar series $1s^22s2p^3nl$, $1s^22p^4nl$ are realized in the case, when the carbon-like core is in excited $1s^22s2p^3$ or $1s^22p^4$ states. The parameters for calculating the energy of one-electron excited states of nitrogen-like ions are given in Table 1.

Table 1. Parameters of one-electron excited states of N-like ions

State	SIGM3	A_{13}	B_{13}	A_{23}	B_{23}
Third shell					
$1s^22s^22p^2(^3P)ns^4P$	0.0	0.8045	0.15	0.359	0.135
$1s^22s^22p^2(^3P)ns^2P$	0.0	0.8045	0.15	0.333	0.118
$1s^22s^22p^2(^3P)np^4P$	0.0	0.2455	-0.084	0.288	0.07
$1s^22s^22p^2(^3P)np^2P$	0.0	0.2455	-0.084	0.248	0.05
$1s^22s^22p^2(^3P)np^4D$	0.0	0.2455	-0.084	0.300	0.075
$1s^22s^22p^2(^3P)np^2D$	0.0	0.2455	-0.084	0.27	0.055
$1s^22s^22p^2(^3P)np^4S$	0.0	0.2455	-0.084	0.252	0.075
$1s^22s^22p^2(^3P)np^2S$	0.0	0.2455	-0.084	0.322	0.081
$1s^22s^22p^2(^3P)nd^4P$	0.0	0.0105	-0.003	0.087	-0.055
$1s^22s^22p^2(^3P)nd^2P$	0.0	0.0105	-0.003	0.130	-0.105
$1s^22s^22p^2(^3P)nd^4F$	0.0	0.0105	-0.003	0.130	-0.075
$1s^22s^22p^2(^3P)nd^2F$	0.0	0.0105	-0.003	0.093	-0.065
$1s^22s^22p^2(^3P)nd^4D$	0.0	0.0105	-0.003	0.088	-0.045
$1s^22s^22p^2(^3P)nd^2D$	0.0	0.0105	-0.003	0.033	-0.045
$1s^22s2p^3(^5S)3s^4S$	0.0	0.8045	0.150	0.265	0.218
$1s^22s2p^3(^5S)3s^6S$	0.0	0.8045	0.150	0.35	0.130
$1s^22s2p^3(^5S)3p^4P$	0.0	0.2455	-0.084	0.22	0.103
$1s^22s2p^3(^5S)3p^6P$	0.0	0.2455	-0.084	0.272	0.103
$1s^22s2p^3(^5S)3d^4D$	0.0	0.0105	-0.003	0.004	-0.022
$1s^22s2p^3(^5S)3d^6D$	0.0	0.0105	-0.003	0.096	-0.035

4 Energy levels of $1s^22s^22p^2(^3P)ns\ ^4P$ states of N-like ions.

The constants of interaction of 2s and 2p electrons with 1s electrons are same as in the states of lithium-like ions $1s^22s\ ^2S$ and $1s^22p\ ^2P$, respectively, or in $1s^22s^22p^2\ ^3P$ states of carbon-like ions. Electrons in ns states interacts with the $1s^2$ shell in the same way as in lithium-like ions. For ns states, the parameters of interaction of electrons of the second and third shells were chosen. The calculated values of the energy $1s^22s^22p^2(^3P)3s\ ^4P_{5/2}$ – $1s^22s^22p^3\ ^4S_{3/2}$ and fine splitting $1s^22s^22p^23s\ ^4P_{5/2}$ – $1s^22s^22p^23s\ ^4P_{1/2}$ for ions with Z = 7-20 are shown in Table. 2. The same table shows a comparison of the calculation with the experimental data the handbooks [12]- [13]. As seen from the comparison, the theoretical and measured values of electronic energies and fine splitting are in good agreement. The fine splitting 1s22s22p23s 4P5/2 –1s22s22p23s 4P1/2 is equal to the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$ calculated taking into account the effect of the third shell electron on the energy levels of the second shell. As the principal quantum number *n* of the third shell electron increases, $1s^22s^22p^2ns\ ^4P_{5/2}$ –$1s^22s^22p^2ns\ ^4P_{1/2}$ increases, approaching the value $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$ for carbon-like ions. The comparison of data shows that $1s^22s^22p^2ns\ ^4P_{5/2}$ –$1s^22s^22p^2ns\ ^4P_{1/2}$ at Z = 18 was probably measured with a large error.

Table 2. Comparison with experiment of the calculated energies (in eV) of $1s^22s^22p^2(^3P)3s\ ^4P_{5/2}$ states of N-like ions and fine splitting $1s^22s^22p^23s\ ^4P_{5/2}$ – $1s^22s^22p^23s\ ^4P_{1/2}$

Z	$1s^22s^22p^2(^3P)3s\ ^4P_{5/2}$– $1s^22s^22p^3\ ^4S_{3/2}$			$1s^22s^22p^23s\ ^4P_{5/2}$–$1s^22s^22p^23s\ ^4P_{1/2}$		
	E_{th}	E_{exp}	$E_{th} – E_{exp}$	E_{th}	E_{exp}	$E_{th} – E_{exp}$
7	10.2754	10.3362	-0.06073	0.01335	0.00998	0.00337
8	22.9071	22.9992	-0.09210	0.03171	0.03271	-0.00101
9	39.3130	39.3328	-0.01984	0.06472	0.06549	-0.00077
10	59.4422	59.4696	-0.02738	0.11870	0.11779	0.00091
11	83.4252	83.4119	0.01329	0.20102	0.20098	0.00004
12	111.176	111.146	0.03021	0.32021	0.30996	0.01025
13	142.709	142.697	0.01227	0.48589	0.47362	0.01226
14	178.048	178.058	-0.01004	0.70881	0.69556	0.01326
15	217.199	217.203	-0.00494	1.00091	0.97328	0.02762
16	260.143	260.178	-0.03502	1.37523	1.36384	0.01139
17	306.913	306.955	-0.04234	1.84603		
18	357.497	357.611	-0.11390	2.42873	2.14494	0.28379
19	411.889			3.13996		
20	470.116			3.99758		

Comparison with experiment of the calculated energies of $1s^22s^22p^2(^3P)3p\ ^4D_{7/2}$, $1s^22s^22p^2(^3P)3p\ ^4P_{5/2}$ states of N-like ions and fine splitting $1s^22s^22p^23p\ ^4D_{7/2}$ – $1s^22s^22p^23p\ ^4D_{1/2}$ and $1s^22s^22p^23p\ ^4P_{5/2}$ – $1s^22s^22p^23p\ ^4P_{1/2}$ values is given in Tables 3 and 4. The fine splitting $1s^22s^22p^23p\ ^4D_{7/2}$ – $1s^22s^22p^23p\ ^4D_{1/2}$ is equal to the $1s^22s^22p^23p\ ^4P_{5/2}$ – $1s^22s^22p^23p\ ^4P_{1/2}$ calculated taking into account the effect of the third shell electron on the state on the second electronic shell. Its slight difference from $1s^22s^22p^23s\ ^4P_{5/2}$ – $1s^22s^22p^23s\ ^4P_{1/2}$ is explained by the different influence of 3p and 3s electrons on the states of $22s^22p^2$ electrons. The fine splitting $1s^22s^22p^23p\ ^4P_{5/2}$ – $1s^22s^22p^23p\ ^4P_{1/2}$ value is equal to half of $1s^22s^22p^2\ ^3P_2$ – $1s^22s^22p^2\ ^3P_0$

calculated with accounting for the effect of the third shell electron on the second shell.

Table 3. Comparison with experiments of the calculated energies (in eV) of $1s^22s^22p^2(^3P)3p\ ^4D_{7/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23p\ ^4D_{7/2} - 1s^22s^22p^23p\ ^4D_{1/2}$

Z	$1s^22s^22p^2(^3P)3p\ ^4D_{7/2}- 1s^22s^22p^3\ ^4S_{3/2}$			$1s^22s^22p^23p\ ^4D_{7/2}-1s^22s^22p^23p\ ^4D_{1/2}$		
	E_{th}	E_{exp}	$E_{th} - E_{exp}$	E_{th}	E_{exp}	$E_{th} - E_{exp}$
7	11.7395	11.7639	-0.02447	0.01374	0.01375	-0.00002
8	25.6715	25.6652	0.00628	0.03270	0.03369	-0.00099
9	43.3078	43.3036	0.004197	0.06657	0.06987	-0.00329
10	64.6054			0.12165		
11	89.7067			0.20534		
12	118.533			0.32618		
13	151.100			0.49382		
14	187.432			0.71906		
15	227.531			1.01383		
16	271.376			1.39123		
17	318.993			1.86552		
18	370.363			2.45217		
19	425.474			3.16782		
20	484.344			4.03036		

Table 4. Comparison with experiments of the calculated energies (in eV) of $1s^22s^22p^2(^3P)3p\ ^4P_{5/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23p\ ^4P_{5/2} - 1s^22s^22p^23p\ ^4P_{1/2}$

Z	$1s^22s^22p^2(^3P)3p\ ^4P_{5/2}- 1s^22s^22p^3\ ^4S_{3/2}$			$1s^22s^22p^23p\ ^4P_{5/2}-1s^22s^22p^23p\ ^4P_{1/2}$		
	E_{th}	E_{exp}	$E_{th} - E_{exp}$	E_{th}	E_{exp}	$E_{th} - E_{exp}$
7	11.8377	11.8447	-0.00697	0.00689	0.00703	-0.00014
8	25.8834	25.8490	0.03446	0.01641	0.01712	-0.00071
9	43.6356	43.5829	0.05272	0.03340	0.03509	-0.00168
10	65.0483	65.0943	-0.04608	0.06102	0.07761	-0.01659
11	90.2631			0.10298		
12	119.201			0.16353		
13	151.880			0.24753		
14	188.321			0.36035		
15	228.531			0.50798		
16	272.485			0.69696		
17	320.210			0.93444		
18	371.688			1.22814		
19	426.908			1.58640		
20	485.886			2.01816		

Comparison of the calculated energies of $1s^22s^22p^2(^3P)3p\ ^2S_{1/2}$ and $1s^22s^22p^2(^3P)3p\ ^4S_{3/2}$ states of nitrogen-like ions with experiment data is shown in Table 5. Experimental data [12]-[13] are available for ions with Z = 7-9 only. As seen from the comparison, the relative deviation of the theory from measurements is in the order of 10^{-4} - 10^{-3}.

Table 5. Comparison with experiments of the calculated energies (in eV) of $1s^22s^22p^2(^3P)3p\ ^4P_{5/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23p\ ^4P_{5/2} - 1s^22s^22p^23p\ ^4P_{1/2}$

Z	$1s^22s^22p^23p\ ^2S_{1/2}- 1s^22s^22p^3\ ^4S_{3/2}$			$1s^22s^22p^23p\ ^4S_{3/2}- 1s^22s^22p^3\ ^4S_{3/2}$		
	E_{th}	E_{exp}	$E_{th} - E_{exp}$	E_{th}	E_{exp}	$E_{th} - E_{exp}$
7	11.5665	11.6027	-0.03619	11.9957	11.9958	-0.00011
8	25.2895	25.2858	0.00371	26.3050	26.3049	0.00001
9	42.7118	42.7053	0.00658	44.3600	44.3219	0.03817
10	63.7976			66.0871	66.0814	0.00572
11	88.6902			91.6207		
12	117.310			120.879		
13	149.674			153.879		
14	185.803			190.641		
15	225.701			231.171		
16	269.346			275.446		
17	316.763			323.491		
18	367.934			375.290		
19	422.847			430.830		
20	481.519			490.129		

Comparison of the calculated energies of $1s^22s^22p^2(^3P)3p\ ^2D_{5/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23p\ ^2D_{5/2} - 1s^22s^22p^23p\ ^2D_{3/2}$ with experimental data is given in Table 6. Fine splitting $1s^22s^22p^23p\ ^2D_{5/2} - 1s^22s^22p^23p\ ^2D_{3/2}$ is equal to 2/3 of $1s^22s^22p^2\ ^3P_2 - 1s^22s^22p^2\ ^3P_0$ calculated taking into account the effect of the third shell electron on the electrons of the second shell.

Table 6. Comparison with experiments of the calculated energies (in eV) of $1s^22s^22p^2(^3P)3p\ ^2D_{5/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23p\ ^2D_{5/2} - 1s^22s^22p^23p\ ^2D_{3/2}$

Z	$1s^22s^22p^2(^3P)3p\ ^2D_{5/2} - 1s^22s^22p^3\ ^4S_{3/2}$			$1s^22s^22p^23p\ ^2D_{5/2} - 1s^22s^22p^23p\ ^2D_{3/2}$		
	E_{th}	E_{exp}	$E_{th} - E_{exp}$	E_{th}	E_{exp}	$E_{th} - E_{exp}$
7	12.0203	12.0097	0.01063	0.00920	0.00947	-0.00026
8	26.2501	26.2492	0.00088	0.02194	0.01124	0.01070
9	44.1840	44.1846	-0.00063	0.04467	0.04840	-0.00373
10	65.7740			0.08159		
11	91.1626			0.13767		
12	120.271			0.21858		
13	153.119			0.33078		
14	189.727			0.48147		
15	230.102			0.67862		
16	274.221			0.93096		
17	322.110			1.24802		
18	373.752			1.64011		
19	429.134			2.11834		
20	488.275			2.69464		

A comparison of the calculated energies of $1s^22s^22p^2(^3P)3p\ ^2P_{3/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23p\ ^2P_{3/2} - 1s^22s^22p^23p\ ^2P_{1/2}$ with measurements is given in Table 7. The fine splitting $1s^22s^22p^23p\ ^2P_{3/2} - 1s^22s^22p^23p\ ^2P_{1/2}$ is 1/3 of $1s^22s^22p^2\ ^3P_2 - 1s^22s^22p^2\ ^3P_0$ calculated taking into account the effect of the third shell electron on the electrons of the second shell.

Table 7. Comparison of the calculated energies (in eV) of $1s^22s^22p^2(^3P)3p\ ^2P_{3/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23p\ ^2P_{3/2} - 1s^22s^22p^23p\ ^2P_{1/2}$ with experiments.

Z	$1s^22s^22p^2(^3P)3p\ ^2P_{3/2} - 1s^22s^22p^3\ ^4S_{3/2}$			$1s^22s^22p^23p\ ^2P_{3/2} - 1s^22s^22p^23p\ ^2P_{1/2}$		
	E_{th}	E_{exp}	$E_{th} - E_{exp}$	E_{th}	E_{exp}	$E_{th} - E_{exp}$
7	12.1506	12.1265	0.02418	0.00461	0.00443	0.00019
8	26.5607	26.5613	-0.00057	0.01101	0.00741	0.00360
9	44.6888	44.6884	0.00040	0.02241	0.01077	0.01164
10	66.4770			0.04094		
11	92.0647			0.06906		
12	121.372			0.10964		
13	154.419			0.16588		
14	191.225			0.24140		
15	231.798			0.34019		
16	276.114			0.46661		
17	324.200			0.62543		
18	376.038			0.82181		
19	431.618			1.06132		
20	490.955			1.34991		

The energies of $1s^22s^22p^2(^3P)3d\ ^{4,2}F$, $1s^22s^22p^2(^3P)3d\ ^{4,2}D$, $1s^22s^22p^2(^3P)3d\ ^4P$ states of nitrogen-like ions were calculated. The constants of interaction of 2s and 2p electrons with 1s electrons were the same as in the case of the states of lithium-like ions $1s^22s\ ^2S$ and $1s^22p\ ^2P$, respectively or $1s^22s^22p^2\ ^3P$ states of carbon-like ions. The electron in nd states interacts with the $1s^2$ shell in the same way as in lithium-like ions. For nd states, the parameters of interaction of electrons of the second and third shells were chosen. The calculated values of the energy 1s22s22p2 (3P) 3d 4F9 / 2– 1$1s^22s^22p^2\ ^3P$ s22s22p3 4S3 / 2 and fine splitting 1s22s22p23d 4F9 / 2–1s22s22p23d 4F3 / 2 for ions with Z = 7-20 are given in Table. 8. The same table shows a comparison of the calculated values with experimental data taken from the handbooks [12], [13]. The calculated values are in good agreement with the experiment. The fine splitting $1s^22s^22p^23d\ ^4F_{9/2} - 1s^22s^22p^23d\ ^4F_{3/2}$ is equal to the fine splitting $1s^22s^22p^2\ ^3P_2 - 1s^22s^22p^2\ ^3P_0$ calculated with accounting for the impact of the third shell electron on the second shell electrons. Its slight difference from $1s^22s^22p^23s\ ^4P_{5/2} - 1s^22s^22p^23s\ ^4P_{1/2}$ is explained by the different influence of 3d and 3s electrons on the state of $2s^22p^2$ electrons.

Table 8. Comparison of the calculated energies (in eV) of $1s^22s^22p^2(^3P)3d\ ^4F_{9/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23d\ ^4F_{9/2}$–$1s^22s^22p^23d\ F_{3/2}$ with experiments.

Z	$1s^22s^22p^2(^3P)3d\ ^4F_{9/2}$– $1s^22s^22p^3\ ^4S_{3/2}$			$1s^22s^22p^23d\ ^4F_{9/2}$–$1s^22s^22p^23d\ ^4F_{3/2}$		
	E_{th}	E_{exp}	$E_{th} - E_{exp}$	E_{th}	E_{exp}	$E_{th} - E_{exp}$
7	13.1289	12.9896	0.13938	0.01401	0.01265	0.00136
8	28.7120	28.6939	0.01808	0.03357	0.01664	0.01693
9	48.0746	48.0722	0.00240	0.06844	0.05805	0.01039
10	71.1253			0.12498		
11	97.9919			0.21067		
12	128.591			0.33412		
13	162.941			0.50505		
14	201.064			0.73432		
15	242.968			1.03395		
16	288.635			1.41711		
17	338.093			1.89813		
18	391.329			2.49255		
19	448.335			3.21711		
20	509.133			4.08975		

Comparison with experiments of the calculated energies of $1s^22s^22p^2(^3P)3d\ ^4D_{7/2}$– $1s^22s^22p^3\ ^4S_{3/2}$, $1s^22s^22p^2(^3P)3d\ ^4P_{5/2}$– $1s^22s^22p^3\ ^4S_{3/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23d\ ^4D_{7/2}$–$1s^22s^22p^23d\ ^4D_{1/2}$ and $1s^22s^22p^23d\ ^4P_{5/2}$ – $1s^22s^22p^23d\ ^4P_{1/2}$ is given in Tables 9 and 10. The fine splitting interval $1s^22s^22p^23d\ ^4D_{7/2}$–$1s^22s^22p^23d\ ^4D_{1/2}$ is 0.33 of the fine splitting calculated with accounting for the impact of the third shell electron on the second shell electrons. The fine splitting $1s^22s^22p^23d\ ^4P_{5/2}$ – $1s^22s^22p^23d\ ^4P_{1/2}$ is equal to half of the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$.

Table 9. Comparison of the calculated energies (in eV) of $1s^22s^22p^2(^3P)3d\ ^4D_{7/2}$– $1s^22s^22p^3\ ^4S_{3/2}$ and $1s^22s^22p^3\ ^4S_{3/2}$ states of nitrogen-like ions and fine splitting $1s^22s^22p^23d\ ^4D_{7/2}$–$1s^22s^22p^23d\ ^4D_{1/2}$ with experiments.

Z	$1s^22s^22p^2(^3P)3d\ ^4D_{7/2}$– $1s^22s^22p^3\ ^4S_{3/2}$			$1s^22s^22p^23d\ ^4D_{7/2}$–$1s^22s^22p^23d\ ^4D_{1/2}$		
	E_{th}	E_{exp}	$E_{th} - E_{exp}$	E_{th}	E_{exp}	$E_{th} - E_{exp}$
7	13.1004	13.0209	0.07945	0.00467	0.00409	0.00058
8	28.8158	28.8580	-0.04222	0.01120	0.00523	0.00597
9	48.3931	48.3801	0.01301	0.02285	0.01116	0.01169
10	71.7037			0.04175		
11	98.8576			0.07040		
12	129.762			0.11167		
13	164.429			0.16880		
14	202.878			0.24544		
15	245.115			0.34558		
16	291.119			0.47362		
17	340.917			0.63436		
18	394.496			0.83296		
19	451.847			1.07502		
20	512.992			1.36653		

5 Conclusion

Electronic energies of one-electron excited states $1s^22s^22p^2ns\ ^4P$, $1s^22s^22p^2ns\ ^2P$, $1s^22s^22p^2np\ ^4S$, $1s^22s^22p^2np\ ^2S$, $1s^22s^22p^2np\ ^4P$, $1s^22s^22p^2np\ ^2P$, $1s^22s^22p^2np\ ^4D$, $1s^22s^22p^2np\ ^2D$, $1s^22s^22p^2nd\ ^4P$, $1s^22s^22p^2nd\ ^2P$, $1s^22s^22p^2nd\ ^4D$, $1s^22s^22p^2nd\ ^2D$, $1s^22s^22p^2nd\ ^4F$, $1s^22s^22p^2nd\ ^2F$ of nitrogen-like ions with *n*=3-12 and Z=7-50 have been calculated. For all the aforementioned electronic configurations, fine-structure levels have been determined for states with different total momentum J. The comparison of theoretical results with the available experimental data shows that theory and measurements agree well. In particular, relative accuracy of 10^{-4} has been achieved in the electronic energy of N-like at Z > 10, while the typical deviation in the calculated fine-structure levels from experimental data is in the order of 10^{-2}.

The dependences of the fine structure on the nuclear charge Z and the principal quantum number *n* for a number of configurations of nitrogen-like ions have been determined. The fine splitting $1s^22s^22p^23s\ ^4P_{5/2}$ – $1s^22s^22p^23s\ ^4P_{1/2}$ of the energy levels of N-like ions is equal to the fine splitting of $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$ of the energy levels of carbon- like ions calculated taking into account the influence of the third shell electron on the state of the second shell electrons. With the increase in the principal quantum number *n* of the third shell electron, the fine splitting $1s^22s^22p^2ns\ ^4P_{5/2}$ – $1s^22s^22p^2ns\ ^4P_{1/2}$ slightly increases, reaching the value of the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$ for carbon-like ions. The fine splitting $1s^22s^22p^23p\ ^4P_{5/2}$ –

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$1s^22s^22p^23p\ ^4P_{1/2}$ is equal to 0.5 of the fine splitting $1s^22s^22p^2\ ^3P_2$ – $1s^22s^22p^2\ ^3P_0$. The fine splitting $1s^22s^22p^23p\ ^2P_{3/2}$ – $1s^22s^22p^23p\ ^2P_{1/2}$ is equal to the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$. The fine splitting $1s^22s^22p^23p\ ^2D_{5/2}$ – $1s^22s^22p^23p\ ^2D_{3/2}$ is equal to 2/3 of $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$. The fine splitting $1s^22s^22p^23p\ ^2P_{3/2}$ – $1s^22s^22p^23p\ ^2P_{1/2}$ is equal to 0.33 of the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$. The fine splitting $1s^22s^22p^23d\ ^4F_{9/2}$–$1s^22s^22p^23d\ ^4F_{3/2}$ is equal to the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$.Its slight difference from the $1s^22s^22p^23s\ ^4P_{5/2}$ —$1s^22s^22p^23s\ ^4P_{1/2}$ is explained by the different influence of 3d and 3s electrons on the state of $2s^22p^2$ electrons. The fine splitting $1s^22s^22p^23d\ ^4D_{7/2}$–$1s^22s^22p^23d\ ^4D_{1/2}$ is equal to 0.33 of the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$. The fine splitting $1s^22s^22p^23d\ ^4P_{5/2}$ – $1s^22s^22p^23d\ ^4P_{1/2}$ is equal to 0.5 of the fine splitting $1s^22s^22p^2\ ^3P_2$ – $1s^22s^22p^2\ ^3P_0$. The fine splitting $1s^22s^22p^23d\ ^2D_{5/2}$–$1s^22s^22p^23d\ ^2D_{3/2}$ is equal to 0.2 of the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$. In the case of the fine splitting of $1s^22s^22p^23d\ ^2P$, the energy $1s^22s^22p^23d\ ^2P_{3/2}$ is lower than that of $1s^22s^22p^23d\ ^2P_{1/2}$ and fine splitting $1s^22s^22p^23d\ ^2P_{1/2}$ –$1s^22s^22p^23d\ ^2P_{3/2}$ is equal to 0.33 of $1s^22s^22p\ ^2P_{3/2}$– $1s^22s^22p\ ^2P_{1/2}$.

Experimental data for the energy levels of outer electrons in N-like ions with the principal quantum number $n > 3$ are very limited. However, taken together, they confirm the correctness of the proposed theoretical approach, which was validated in more detail at $n = 3$. In many cases, the fine splitting of N-like ions is associated with the splitting of the energy of the $1s^22s^22p^2$ state of carbon-like ions and does not decrease with increasing n. Instead, it slightly increases, approaching the value of the fine splitting $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$ for carbon-like ions. The fine splitting of the energy levels of *nl* electrons is small compared to the interval $1s^22s^22p^2\ ^3P_2$ –$1s^22s^22p^2\ ^3P_0$ and decreases sharply with increasing n.

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