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Modeling Cognitive Activity of the Human Brain by the Mathematical Apparatus of Quantum Mechanics

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Abstract. This paper discusses the possible approaches to modeling the cognitive activity of the human brain using the mathematical apparatus of quantum mechanics (primarily – potential wells and virtual particles) in terms of the theory of information representations. The article briefly describes the proposed theory of information representations, draws analogies, and identifies common features of information representations of the human mind and Feynman's virtual particles. The human mind is represented as a one-dimensional potential well with finite walls of different sizes and internal potential barrier simulating the boundary between consciousness and subconsciousness. This creates a foundation for a mathematical apparatus that can make it possible to forecast particular cognitive functions of the human brain. The results of these studies can be used to create predictive models of various cognitive disorders (diseases) and to be used in diagnostics.

1 Introduction

Natural science aspects of how the environmental information factors impact living systems are still relatively poorly studied [1]. There has been quite a number of new electrophysiological methods created, primarily, the registration of evoked and event-related potentials recorded in experimental behavioral situations, that made it possible to approach the study of physiological mechanisms of individual stages of information processing: sensory analysis, attention mobilization, image formation, extraction of memory standards, decision making, etc. The study of time parameters of electrophysiological reactions to stimuli of different types and under different conditions made it possible for the first time to register time parameters, i.e. to estimate the duration of individual stages of information processing directly at the level of brain substrate [1-2]. Many fundamental facts were also obtained using classical methods of psychology, which were used to study and analyze the behavior of subjects in various social situations.

The problem of describing the processes of information transfer and processing by an individual is fundamental for modern cognitive science. Relatively recently, unique natural science models of information transfer from individual to individual [2-5], cognitive activity modeling [6-13], and others have appeared. However, many of the presented models are either poorly scalable or insufficiently formalized and therefore do not allow to fundamentally explain the processes of information transfer and its distortion resulting from interaction with the external communicative environment.

Moreover, one aspect of human cognitive activity is the fact that a person's thinking is not code-based (as compared to a computer), but is a result of the interaction of multiple images. Although these images have quite specific material structure at the heart of their functioning (namely, electrical and chemical activity in the human brain), from the point of view of conventional mathematical models their description is difficult for several reasons [11].

Therefore, this paper proposes new methods for describing the activity of information representations (which, in turn, simulate the cognitive activity of an individual) based on the mathematical apparatus of quantum physics (potential wells and virtual particles used in physics to describe fundamental interactions [14]).

2 A brief overview of the theory of information representations

The proposed theory is based on the idea of a universal cognitive unit [11] of information in the human mind - an information representation (or image), the space in which it exists, its topology and properties. Information representations (hereinafter referred to as IRs) can be defined as displays of objects and events in any feature space.

Correspondingly, the theory of information representations (hereinafter referred to as TIR) is a way to describe information interactions of individuals, as well as several cognitive functions of a person.

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TIR views the human mind as a huge structured number of interacting images, which are constantly affected by external factors.

Images with higher energy (we introduce the notion of energy E to describe the communicative activity of images) are located "above" and closer to the edge of the individual's information image space, which is why they interact with external elements much more often; meanwhile, images with lower energy and longer response rate are located closer to the center of the space and relatively rarely interact actively with external stimuli.

TIR gives an alternative perspective on some characteristic regularities in the human mind, allowing for a correct interpretation and explanation of some of them.

The information image can not be communicated between individuals without changes. Each IR is unique because each individual has specific individual experiences.

Besides, an individual is unable to convey the image that exists in their mind, in their IR space to another individual directly. For this purpose, they use various communication apparatuses formed within the social superstructure of communication or the communication field (CF). A communication field (CF) is an information community of individual experiences and collective unconsciousness formed as a result of an individual's presence in society. The communicative apparatus includes speech, visual, tactile, symbolic, and other ways of information transmission.

In previous works, the mathematical model of interaction of information representations was based on diffusion equations (first of all, on the Langevin equation). This approach allowed to describe several special cases of cognitive processes but also had some limitations related to the specificity of IRs. More details about TIR and its application to real cognitive processes and experimental testing are available in [15].

3 A brief overview of the theory of information images

What is the peculiarity of the theory of virtual particles and why should it be used to describe cognitive processes in the human brain? Virtual particle (VP) is quanta of relativistic wave fields participating in vacuum fluctuations. From the general quantum-mechanical point of view, VP can be considered as particles arising in intermediate states of transition processes and interaction of particles. VPs have the same quantum numbers as ordinary real particles, the quantum numbers of which, in turn, correspond to one of the real elementary particles (with mass m), for which, however, the usual connection between energy and momentum is not satisfied.

Which means that for them:

$$E^2 \neq m^2c^4 + p^2c^2$$

Where c is the speed of light in vacuum.

Therefore, a measure of "virtuality" of the particle was introduced, which is expressed as:

$$Q^2 = E^2 - m^2c^4 - p^2c^2 \quad (1)$$

Virtual particles cannot "travel infinitely"; they are generated and must either be absorbed by some particle or disintegrate [14]. Basically, virtual particles are the way the interaction takes place.

This is very close to the concept of IRs for human interaction, because they are also present only in the virtual world, but also have material properties, can have a concrete impact on the matter (on the human body), and are designed to describe the processes of exchange, generation, and processing of information by an individual. IRs also cannot "exist infinitely"; their existence is conditioned exclusively by the presence of a perceptive individual (imagine a picture that no one is looking at). Therefore, each information image must also be absorbed or embodied in a material form [15].

Additionally, VPs are responsible for the quantum mechanism of interaction of particles - they are the carriers of interactions. For example, the scattering of charged particles due to electromagnetic interactions between them on quantum field representations is carried out through the exchange of virtual photons.

Virtual particles exist in intermediate short duration states, for which the usual ratio between energy, pulse and mass is not met. Other characteristics of virtual particles are electric charge, spin, baryon charge, etc. - are the same as for corresponding real particles [14].

The concept of "Virtual Particles" and virtual processes is central to modern quantum field theory. In this theory, the interaction of particles and their mutual transformations are considered as the generation or absorption by one free particle of other (virtual) particles. Any particle continuously emits and absorbs VPs of different types. For example, a proton emits and absorbs virtual pi-mesons (and other VPs) and thus appears surrounded by a cloud. In this case, the number of VPs is uncertain.

Here we can also draw an analogy with an individual who generates a "cloud" of their information images, which essentially represent their mind and accumulated individual and social experience [15].

In addition, from the point of view of classical physics, a free particle (a particle which is not affected by external forces, i.e. resting or moving evenly and linearly) can neither generate nor absorb another particle (for example, a free electron can neither emit nor absorb a photon), since such an occurrence would violate either the law of conservation of energy or the law of conservation of momentum. Indeed, the resting electron has the minimum possible energy. That is why such an electron can not emit a photon that always possesses energy, as this would violate the law of energy conservation. If the electron moves at a constant speed, it also cannot (due to its kinetic energy) generate a photon, because this would violate the law of conservation of momentum: the electron's momentum loss caused by the energy loss from photon generation would be larger than the impulse of the photon corresponding to its energy (due to the difference in masses of these particles). The same applies to the process of absorption of the photon by a free electron.

However, the situation in quantum mechanics is quite different. According to the fundamental principle of quantum mechanics - the uncertainty principle (see Indeterminacy Relation), the energy of any particle "existing" in a small interval of time Δt is not fixed. The energy and momentum are continuously fluctuating, and during small intervals of time, the law of energy conservation may be "temporarily violated" (in the classical sense), while processes occurring at small scales may be accompanied by "local violations" of the law of conservation of momentum [14].

Moreover, it would not be accurate to refer to conservation laws in the usual sense in the case of IR, which, of course, are limited to certain energy indicators based on their material carriers, i.e. – human beings. However, if we are not talking about the entire IR system, but about its individual information images, it is obvious that the classical connection (1) will also be broken (in previous models IRs were presented as a higher number of interacting particles [15]).

Sometimes, for the sake of clarity, the concept of "virtual particles" is explained somewhat differently. Namely, it is mentioned that in the process of interaction, the law of energy conservation is satisfied with some permissions. This does not contradict quantum mechanics: according to the indeterminacy principle, an event lasting for a finite period of time does not allow to determine the energy with an accuracy exceeding a certain limit. Roughly speaking, intermediate particles "borrow energy" for a short time. In this case, ordinary particles can be generated and disappear in the process of interaction, only with a small violation of the law of energy conservation.

The interaction of ordinary, real particles in the vast majority of cases occurs via emission and absorption (exchange) of VPs. The energy and momentum of real particles before and after the reaction remain unchanged, and during the reaction, the laws of preservation of these values are not satisfied. The whole theory is constructed in such a way that any reaction can be represented as a result of various virtual processes occurring during a short reaction time [16].

Additionally, the interaction of individuals is a process of emission and absorption (processing and perception) of IRs. Thus, we can describe any information interaction as a result of virtual processes related to IR.

Thus, there are quite a few analogies between the processes of interaction of particles in quantum field theory and individuals in the theory of information images. Of course, such analogies cannot be interpreted as a sign of a complete concordance of the two theories; however, it should be taken into account that an individual is also a part of nature, and, therefore, those patterns, laws, phenomena that surround us (not only phenomena related to VPs but also diffusion, laws of conservation, etc.) must somehow manifest in individuals and their cognitive processes.

However, this can only be reliably verified by using the appropriate mathematical apparatus to predict specific cognitive processes and verify results in an experimental study.

4 Mathematical apparatus

The classical method for describing a virtual process (a process involving virtual particles) is the Feynman diagram method [17]. However, the direct application of such a device for IRs would hardly be possible. First, it is necessary to perform parameterization of IRs and compare these values with VP parameters.

Interactions of virtual particles are described by the propagator (distribution function) - a function that determines the amplitude of probability of distribution of a relativistic field (particle) between two stages of interaction. The propagator is determined through the evolutionary operator:

$$D(x,t;x',t') = \langle x|\hat{U}(t,t')|x'\rangle$$

It allows considering the influence of virtual particles. In fact, the propagator is a function of the Green's wave equation. In general, in elementary particle physics and statistical physics, Green's functions are often used as propagators in Feynman diagrams (and the expression "Green's function" is often applied in general to the correlation function in quantum field theory).

Within the model, the interaction of two individuals is represented as the interaction of two systems with the help of communication fields (which are quantized through IR - as an analog of VP) in the information environment. Also, of interest is the impact on one individual of a given information environment (e.g., media, Internet resources, social environment, etc.).

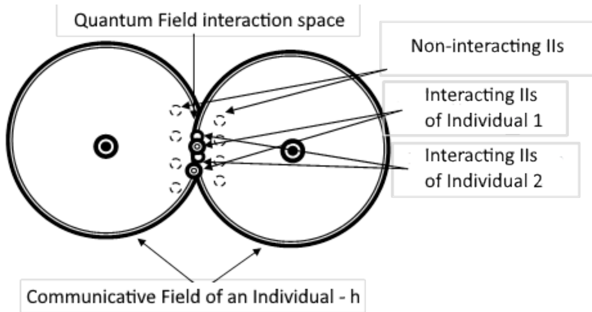

Fig. 1. Interaction of communicative fields of two individuals

The interaction function for these two individuals or an individual and the external action should be written accordingly.

Since we are talking about the IR space, it is obvious that this space will have certain coordinates; the particles simulating the IR movement in this space will also have a certain momentum, energy. At the same time, we should keep in mind the presence of local minimum potential particle energy in the space. Thus, an individual's mind can be represented as a potential well, inside of which information representations make vibrational movements. It is clear, that it is common for physics to solve the problem of quantization of the field inside the potential well to determine energy levels (by solving the stationary Schrödinger equation [18]). However, in this case, we know about the presence of particles in advance.

The potential well can be represented in the simplest form as - **Fig.2.**

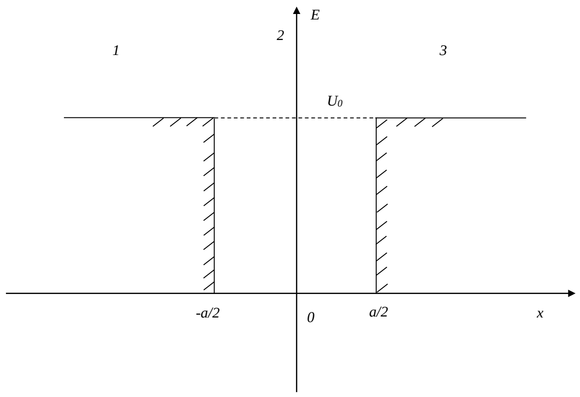

Fig. 2. The most basic one-dimensional potential well with finite walls.

$$U(x) = \begin{cases} 0, -\frac{a}{2} < x < \frac{a}{2}, \text{region } 2 \\ U_0, \quad x \geq \frac{a}{2}, \text{region } 3 \\ \quad\;\; x \leq -\frac{a}{2}, \text{region } 1 \end{cases} \quad (2)$$

When a particle crosses the potential barrier in the form of a wall, it simulates the information interaction of an individual (i.e., particle disturbance, the transition from one energy level to a higher one).

A more interesting case of a potential well is the well uneven walls and an additional internal barrier. Such barrier and heterogeneity of walls simulate certain specific properties of the human mind, in particular, the conditional division into consciousness-subconsciousness, complexity of interaction with the external environment, etc. Theoretically, depending on the mental state of the modeled individual (or their individual cognitive function), such barriers may be more complex and repeated multiple times - Fig.3.

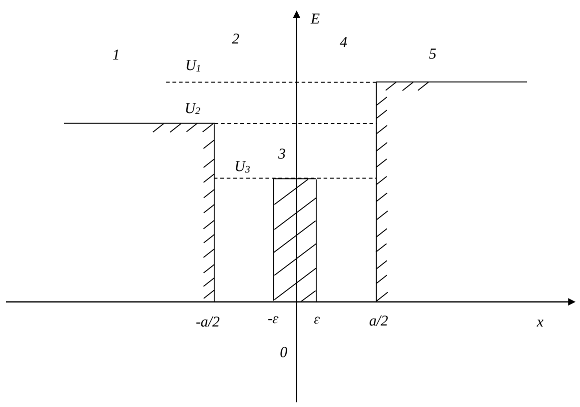

Fig .3. One-dimensional potential well with finite walls. A version with uneven walls and internal potential barrier.

$$U(x) = \begin{cases} U_1, x \geq \frac{a}{2} \\ 0, \varepsilon < x < \frac{a}{2} \\ U_2, -\varepsilon \leq x <\leq \varepsilon \\ 0, -\frac{a}{2} < x < -\varepsilon \\ U_0, x \leq -\frac{a}{2} \end{cases} \quad (3)$$

Thus, the impact of the external information environment on the mind of an individual can be described as the impact on particles (modeling information images) in a potential well. Let us record the equations for the case in Fig. 2.

The Schrödinger equation outside the potential well for the information image (IR) will look as:

$$\frac{d^2\varphi_{out}(x)}{dx^2} - \frac{2m}{\hbar^2}(U_0 - E)\varphi_{out} = 0 \quad (4)$$

Where,

φ_{out} is a wave function outside the potential well.
E is the total energy of an IR.
m is mass (complexity) of IR.
$\hbar$ is Planck constant.

By introducing the interaction coefficient k_1,

$$k_1 = \sqrt{\frac{2m}{\hbar^2}(U_0 - E)}$$

We obtain,

$$\frac{d^2\varphi_{out}(x)}{dx^2} - {k_1}^2\varphi_{out} = 0 \quad (5)$$

For region 1, the solutions of this equation will look as:

$$\varphi_1(x) = A_1 e^{k_1 x} + B_1 e^{-k_1 x}$$

$$\varphi_3(x) = A_3 e^{k_1 x} + B_3 e^{-k_1 x} \quad (6)$$

For the wave function to be limited (which is impossible otherwise for real information images), B_1 и $A_3 = 0$.

For region 2, located in the well (i.e. in the human mind), it will look as:

$$\frac{d^2\varphi_{out}(x)}{dx^2} + \frac{2m}{\hbar^2}(E)\varphi_{out} = 0 \quad (7)$$

Here, the interaction coefficient k_2 will be,

$$k_2 = \sqrt{\frac{2m}{\hbar^2}E} \quad (8)$$

And the solution will look as:

$$\varphi_2(x) = C\sin(k_2 x + \propto)$$

Thus, the wave functions for an information image in the mind of an individual will have a form:

$$\varphi_1(x) = A_1 e^{k_1 x}$$

$$\varphi_2(x) = C\sin(k_2 x + \propto)$$

$$\varphi_3(x) = B_3 e^{-k_1 x} \quad (9)$$

The external impact will be recorded by disturbance V(x), which is added to the general form of the Schrödinger equation:

$$\frac{d^2\varphi(x)}{dx^2} - \frac{2m}{\hbar^2}(U(x) + V(x) - E)\varphi = 0 \quad (10)$$

Disturbance can be set in different ways, depending on its type, for example, by normal distribution or other methods.

This model should become the foundation for experimental research and practical testing of the modeling results in the future.

5 Conclusion

Thus, in this paper, we presented the fundamentals of the mathematical model of the human mind based on the mathematical apparatus of quantum mechanics (primarily, the potential wells and elements of virtual particles in the problem statement), defined the task of modeling the cognitive activity of the human brain based on classical mathematical models and physical analogies, gave a brief overview of the theory of information representations and its foundations, highlighted the general regularities and the phenomena connected with the behavior of information representations of the human mind and virtual particles and proposed options of potential wells for modeling cognitive brain functions.

This allowed us to propose a foundation for a mathematical model, which will be further tested experimentally (based on external information impact on the individual and his psychophysiological reactions).

The results of these studies can be used to create predictive models of various cognitive disorders (diseases) and to be used in diagnostics. First of all, this concerns the problems of perception of information, the processing of complex images (connecting different categories of information, for example, color and sound), etc.

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