Mathematical Modeling of Vibration Dampers of Vibration-Insulated Structures

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Abstract. Vibration dampers are installed on the machine foundations in order to reduce the vibration level. Such technological solutions are most expedient in the case of a harmonic load with a low instability of the vibration frequency. Unfortunately, dampers do not provide such a large reduction in the dynamic effect on the base, as vibration isolation, but in some cases their efficiency turns out to be quite sufficient with a relatively simple implementation and low manufacturing cost. The use of dynamic vibration dampers gives a great effect when an increased vibration of foundations occurs during the operation of equipment in metallurgical production, for example, when processing materials by pressure, reconstructing enterprises and replacing heavy equipment. During the operation of heavy forging equipment and manipulators for various purposes, the foundations of these devices can be considered as a rigid body. The model soil on which this foundation is installed can be considered a homogeneous elastic isotropic half-space. When calculating with such mathematical models, one can use solutions of the corresponding dynamic contact problems. A comparative analysis of the effectiveness of damping foundation vibrations using different foundation models, including the model of an elastic, homogeneous half-space and a system of semi-infinite rods, the modulus of elasticity of which increases with depth according to the quadratic law, shows a fairly close agreement.

1 Introduction

The classical equation of bending vibrations of a plate is applicable with sufficient accuracy as long as the bending wavelength is not less than five times the plate thickness [1]. For a rectangular plate loaded with lumped masses located periodically, the bending vibration equation has the form:

$$D\nabla^4 W + \rho h \frac{\partial^2 W}{\partial t^2} = h \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} \rho_{ij} \delta(x - -x_i) \delta(y - y_j) \frac{\partial^2 W}{\partial t^2}(x_i, y_j, t) \quad (1)$$

where $0 \le x \le a, 0 \le y \le b,\ x_i = \frac{ia}{m_1+1}, y_j == \frac{jb}{m_2+1}$,

wherein $i = 1,2, \dots m_1, j = 1,2, \dots m_2$.

$W(x, y, t)$ - deflection or deviation of the point (x, y) from the equilibrium position.

$D = \frac{Eh}{12(1-\nu^2)} -$ bending stiffness of the plate;

$\rho h = q -$ plate mass per unit surface.

This equation can be used if the distance from the edge of the plate to the point at which the vibrations are considered is greater than the thickness of the plate h, which in many cases can be considered as a constant value. [2- 5]

The boundary conditions for this problem can be formulated as follows:

1. For a supported edge, deflection and bending moment must be zero.

$$W = 0,\ \frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2} = 0 \text{ for } x = 0 \text{ } or \text{ } x == a,$$

or

$$W = 0,\ \frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2} = 0 \text{ for } y = 0 \text{ } or \text{ } y == b. \quad (2.1)$$

2. For the sealed edge

$$W = 0,\ \frac{\partial W}{\partial x} = 0, \text{for } x = 0 \text{ or } x = a,$$

$$W = 0,\ \frac{\partial W}{\partial y} = 0, \text{for } y = 0 \text{ or } y = b. \quad (2.2)$$

2 Equations and mathematics

Initial conditions:

$$W(x, y, 0) = \varphi(x, y), \qquad \frac{\partial W}{\partial t} == \psi(x, y). \quad (3)$$

An analytical solution to problem (1, 2, 3) is sought in the form

$$W(x,y,t)=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(a_{kn}cos\frac{k\pi x}{a}cos\frac{n\pi y}{b}+ +b_{kn}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}+c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b}+ +d_{kn}sin\frac{k\pi x}{a}sin\frac{n\pi y}{b})e^{i\omega_{kn}t}, \quad (4)$$

$$\nabla^4 W=\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\left(\frac{k\pi}{a}\right)^4+ 2\left(\frac{k\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2++\left(\frac{n\pi}{b}\right)^4)(a_{kn}cos\frac{k\pi x}{a}cos\frac{n\pi y}{b}+ +b_{kn}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}+c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b}+ +d_{kn}sin\frac{k\pi x}{a}sin\frac{n\pi y}{b})e^{i\omega_{kn}t}=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\left(\frac{k\pi}{a}\right)^2+ +\left(\frac{n\pi}{b}\right)^2\right)^2(a_{kn}cos\frac{k\pi x}{a}cos\frac{n\pi y}{b}++b_{kn}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}+ c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b}++d_{kn}sin\frac{k\pi x}{a}sin\frac{n\pi y}{b})e^{i\omega_{kn}t}, \quad (5)$$

$$\frac{\partial^2 W}{\partial t^2}=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\omega_{kn})^2(a_{kn}cos\frac{k\pi x}{a}cos\frac{n\pi y}{b}+ +b_{kn}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}+c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b}+ +d_{kn}sin\frac{k\pi x}{a}sin\frac{n\pi y}{b})e^{i\omega_{kn}t}. \quad (6)$$

$$\sum_{i=1}^{m_1}\delta(x-x_i)=2\sum_{m=1}^{\infty}\sum_{i=1}^{m}(cos\frac{m\pi x}{a}cos\frac{m\pi x_i}{a}+ +sin\frac{m\pi x}{a}sin\frac{m\pi x_i}{a}) \quad (7)$$

$$\sum_{j=1}^{m_2}\delta(y-y_j)=2\sum_{p=1}^{\infty}\sum_{i=1}^{p}(cos\frac{p\pi y}{b}cos\frac{p\pi y_j}{b}+ sin\frac{p\pi y}{b}sin\frac{p\pi y_j}{b}) \quad (8)$$

Substitute (4,5,6,7,8) into equation (1):

$$D\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2(a_{kn}cos\frac{k\pi x}{a}cos\frac{n\pi y}{b} + b_{kn}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}+c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b} + d_{kn}sin\frac{k\pi x}{a}sin\frac{n\pi y}{b})e^{i\omega_{kn}t} - \rho h\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\omega_{kn})^2(a_{kn}cos\frac{k\pi x}{a}cos\frac{n\pi y}{b}$$

$$+b_{kn}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}+c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b} + d_{kn}sin\frac{k\pi x}{a}sin\frac{n\pi y}{b})e^{i\omega_{kn}t} = 4h\sum_{i=1}^{m_1}\sum_{j=1}^{m_2}\rho_{ij}\sum_{m=1}^{\infty}\sum_{i=1}^{m}(cos\frac{m\pi x}{a}cos\frac{m\pi x_i}{a} + sin\frac{m\pi x}{a}sin\frac{m\pi x_i}{a})\sum_{p=1}^{\infty}\sum_{i=1}^{p}(cos\frac{p\pi y}{b}cos\frac{p\pi y_j}{b} + sin\frac{p\pi y}{b}sin\frac{p\pi y_j}{b})$$

$$\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\omega_{kn})^2(a_{kn}cos\frac{k\pi x_i}{a}cos\frac{n\pi y_j}{b}+ +b_{kn}cos\frac{k\pi x_i}{a}sin\frac{n\pi y_j}{b}++c_{kn}sin\frac{k\pi x_i}{a}cos\frac{n\pi y_j}{b}+ +d_{kn}sin\frac{k\pi x_i}{a}sin\frac{n\pi y_j}{b})e^{i\omega_{kn}t}. \quad (9)$$

We denote

$$D_{kn}(\omega_{kn})=D\cdot\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}(\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2- \rho h\omega_{kn}^2)\cdot e^{i\omega_{kn}t},$$

$$H_{kn}(\omega_{kn})=4\sum_{i=1}^{m_1}\sum_{j=1}^{m_2}\rho_{ij}h\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\omega_{kn}^2\cdot e^{i\omega_{kn}t}\cdot t. \quad (*)$$

Using these notation, equation (9) takes the following form

$$\frac{D_{kn}(\omega_{kn})}{H_{kn}(\omega_{kn})}\left(a_{kn}cos\frac{k\pi x}{a}cos\frac{n\pi y}{b}+ b_{kn}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}+c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b}+ +d_{kn}sin\frac{k\pi x}{a}sin\frac{n\pi y}{b}\right)= \sum_{i=1}^{m_1}\sum_{j=1}^{m_2}\sum_{m=1}^{\infty}(cos\frac{k\pi x}{a}cos\frac{m\pi x_i}{a}+ +sin\frac{m\pi x}{a}sin\frac{m\pi x_i}{a})\sum_{p=1}^{\infty}(cos\frac{p\pi y}{b}cos\frac{p\pi y_j}{b}+ +sin\frac{p\pi y}{b}sin\frac{p\pi y_j}{b})(a_{kn}cos\frac{k\pi x_i}{a}cos\frac{n\pi y_j}{b}+ +b_{kn}cos\frac{k\pi x_i}{a}sin\frac{n\pi y_j}{b}++c_{kn}sin\frac{k\pi x_i}{a}cos\frac{n\pi y_j}{b}+ d_{kn}sin\frac{k\pi x_i}{a}sin\frac{n\pi y_j}{b}) \quad (9.1)$$

After applying the summation operators and changing the places of the summation indices $k\to m$, $m\to k, n\to p, p\to n$, equation (9) takes the form:

$$D_{kn}(a_{kn}cos\frac{k\pi x}{a}cos\frac{n\pi y}{b} + b_{kn}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}+c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b} + d_{kn}sin\frac{k\pi x}{a}sin\frac{n\pi y}{b})=$$

$$=H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}(a_{mp}\,cos\frac{k\pi x}{a}cos\frac{n\pi y}{b}\cdot \cdot\sum_{i=1}^{\infty}cos\frac{k\pi x_i}{a}cos\frac{m\pi x_i}{a}\cdot\sum_{j=1}^{\infty}cos\frac{n\pi y_j}{b}cos\frac{p\pi y_j}{b} + b_{mp}cos\frac{k\pi x}{a}sin\frac{n\pi y}{b}\sum_{i=1}^{\infty}cos\frac{k\pi x_i}{a}cos\frac{m\pi x_i}{a} \sum_{j=1}^{\infty}sin\frac{n\pi y_j}{b}sin\frac{p\pi y_j}{b} + c_{kn}sin\frac{k\pi x}{a}cos\frac{n\pi y}{b}\sum_{i=1}^{\infty}sin\frac{k\pi x_i}{a}sin\frac{m\pi x_i}{a}$$

$\sum_{j=1}^{\infty} cos\frac{n\pi y_j}{b} cos\frac{p\pi y_j}{b} +$
$d_{kn} sin\frac{k\pi x}{a} sin\frac{n\pi y}{b} \sum_{i=1}^{\infty} sin\frac{k\pi x_i}{a} sin\frac{m\pi x_i}{a}$ (9.2)

Equating the terms of the equation from its left and right sides with the same combinations at cos and sin, we obtain the following system of equations

$$D_{kn}(a_{kn} cos\frac{k\pi x}{a} cos\frac{n\pi y}{b}) - H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} a_{mp} cos\frac{k\pi x}{a}\cdot$$

$$\cdot cos\frac{n\pi y}{b}\sum_{i=1}^{\infty} cos\frac{k\pi x_i}{a} cos\frac{m\pi x_i}{a} \cdot\sum_{j=1}^{\infty} cos\frac{n\pi y_j}{b} cos\frac{p\pi y_j}{b} = 0.$$

$$D_{kn}(b_{kn} cos\frac{k\pi x}{a} sin\frac{n\pi y}{b}) - H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} b_{mp} cos\frac{k\pi x}{a}$$

$$sin\frac{n\pi y}{b}\sum_{i=1}^{\infty} cos\frac{k\pi x_i}{a} cos\frac{m\pi x_i}{a}\cdot\sum_{j=1}^{\infty} sin\frac{n\pi y_j}{b} sin\frac{p\pi y_j}{b} = 0.$$

$$D_{kn}(c_{kn} sin\frac{k\pi x}{a} cos\frac{n\pi y}{b}) - H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} c_{mp} sin\frac{k\pi x}{a} cos\frac{n\pi y}{b}$$

$$\sum_{i=1}^{\infty} sin\frac{k\pi x_i}{a} sin\frac{m\pi x_i}{a}\cdot\sum_{j=1}^{\infty} cos\frac{n\pi y_j}{b} cos\frac{p\pi y_j}{b} = 0.$$

$$D_{kn}(d_{kn} sin\frac{k\pi x}{a} sin\frac{n\pi y}{b}) - H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} d_{mp} sin\frac{k\pi x}{a}$$

$$sin\frac{n\pi y}{b}\sum_{i=1}^{\infty} sin\frac{k\pi x_i}{a} sin\frac{m\pi x_i}{a}\cdot\sum_{j=1}^{\infty} sin\frac{n\pi y_j}{b} sin\frac{p\pi y_j}{b} = 0.$$

After a slight simplification, the system of equations will be reduced to the following form

$$D_{kn}(a_{kn}) - H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} a_{mp}\sum_{i=1}^{\infty} cos\frac{k\pi x_i}{a}$$

$$cos\frac{m\pi x_i}{a}\sum_{j=1}^{\infty} cos\frac{n\pi y_j}{b} cos\frac{p\pi y_j}{b} = 0.$$

$$D_{kn}(b_{kn}) - H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} b_{mp}\sum_{i=1}^{\infty} cos\frac{k\pi x_i}{a}$$

$$cos\frac{m\pi x_i}{a}\sum_{j=1}^{\infty} sin\frac{n\pi y_j}{b} sin\frac{p\pi y_j}{b} = 0.$$

$$D_{kn}(c_{kn}) - H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} c_{mp}\sum_{i=1}^{\infty} sin\frac{k\pi x_i}{a}$$

$$sin\frac{m\pi x_i}{a}\sum_{j=1}^{\infty} cos\frac{n\pi y_j}{b} cos\frac{p\pi y_j}{b} = 0.$$

$$D_{kn}(d_{kn}) - H_{kn}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} d_{mp}\sum_{i=1}^{\infty} sin\frac{k\pi x_i}{a}$$

$$sin\frac{m\pi x_i}{a}\sum_{j=1}^{\infty} sin\frac{n\pi y_j}{b} sin\frac{p\pi y_j}{b} = 0.$$

Since changes in the left and right sides of the equation should occur with the same frequencies, the modes of natural oscillations ξ_k are excited at the frequency of the acting force, i.e. the eigenfunctions satisfy the orthogonality conditions:

$$\sum_{i=1}^{N}\xi_k\,\xi_m \begin{cases} = 0, при\ k \neq 2r(n+1)\pm m,\ r = 0,1,2\ldots \\ \neq 0, при\ k = 2r(n+1)\pm m,\ r = 0,1,2\ldots \end{cases} \quad (**)$$

In this case, the following designations can be introduced:

$$C'_{km} = \sum_{i=1}^{\infty} sin\frac{k\pi x_i}{a} sin\frac{m\pi x_i}{a} = \\ = \begin{cases} -\frac{n+1}{2}, k = 2r(n+1) - m; r = 1,2\ldots \\ \frac{n+1}{2}, k = 2r(n+1) + m; r = 0,1,2\ldots \\ 0, k \neq 2r(n+1)\pm m, \quad r = 0,1,2\ldots \end{cases}$$

$$C'_{km} = \sum_{i=1}^{\infty} cos\frac{k\pi x_i}{a} cos\frac{m\pi x_i}{a} = \\ = \begin{cases} -\frac{n+1}{2}, k = 2r(n+1) - m; r = 1,2\ldots \\ \frac{n+1}{2}, k = 2r(n+1) + m; r = 0,1,2\ldots \\ 0, k \neq 2r(n+1)\pm m, \quad r = 0,1,2\ldots \end{cases}$$

$$\sum_{i=1}^{\infty} cos\frac{k\pi x_i}{a} sin\frac{m\pi x_i}{a} = 0,$$

$$\sum_{i=1}^{\infty} sin\frac{k\pi x_i}{a} cos\frac{m\pi x_i}{a} = 0.$$

In this case, the system of equations takes the form:

$$D_{kn}(a_{kn}) - \quad H_{kn} \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} a_{mp}\, C'_{km}C'_{knp} = 0.$$

$$D_{kn}(b_{kn}) - \quad H_{kn} \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} b_{mp}\, C'_{km}C'_{knp} = 0.$$

$$D_{kn}(c_{kn}) - \quad H_{kn} \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} c_{mp}\, C'_{km}C'_{knp} = 0.$$

$$D_{kn}(d_{kn}) - \quad H_{kn} \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} d_{mp}\, C'_{km}C'_{knp} = 0. \qquad (10)$$

We use the notation (*) for a new record of the system (10).

$$D_{kn}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2 - \rho h(\omega_{kn})^2)\delta_{km}\delta_{np}$$
$$- 4h\rho_{ij}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\omega_{kn}^2\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}C'_{km}C'_{knp}) \square$$

$$\square e^{i\omega_{kn}t}\begin{pmatrix}a_{mp}\\ b_{mp}\\ c_{mp}\\ d_{mp}\end{pmatrix} = 0\,. \qquad (11)$$

The characteristic equation of the problem has the form

$$D_{kn}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2 -$$
$$- \rho h(\omega_{kn})^2)\delta_{km}\delta_{np}$$
$$- 4h\rho_{ij}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\omega_{kn}^2\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}C'_{km}C'_{knp}) \square$$

$$\square e^{i\omega_{kn}t} = 0\,. \qquad (12)$$

From the characteristic equation we find the dependence of ω_{kn} on *k* and *n*.

$$(\omega_{kn})^2 == \frac{D_{kn}\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2}{\rho h\delta_{km}\delta_{np} - 4h\rho_{ij}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}C'_{km}C'_{np}}$$

$$(\omega_{kn})^2 == \frac{D_{kn}\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2}{h(\rho\delta_{km}\delta_{np} - 4\rho_{ij}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}C'_{km}C'_{np})}$$

In the obtained equalities, we use the condition (**):

$$(\omega_{kn})^2 = \\ = \frac{D_{kn}\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2}{h(\rho\delta_{km}\delta_{np} - 4\rho_{ij}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{m_1+1}{2}\frac{m_2+1}{2})},$$

where $k = 2r(m_1+1)+m,\ n = 2r(m_2++1)+p$.

$$(\omega_{kn})^2 = \frac{D_{kn}\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2}{h(\rho + 4\rho_{ij}(m_1+1)(m_2+1))},$$

where $k = 2r(m_1+1)-m,\ n = 2r(m_2++1)+p$ or $k = 2r(m_1+1)+m,\ n = 2r(m_2+1)-p$.

$$(\omega_{kn})^2 = \frac{D_{kn}\left(\left(\frac{k\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2}{h\rho}$$

where $k \neq 2r(m_1+1)+m,\ n \neq 2r(m_2++1)+p$, $k \neq 2r(m_1+1)-m,\ n \neq 2r(m_2+1)+p$ (13)

3 Results

A full spectrum of frequencies (13) of small transverse vibrations of a rectangular plate with lumped masses, located periodically, was obtained, provided that the thickness of the plate remains constant. It should be noted that ρ_{ij} is the density of each of the concentrated masses.

4 Conclusion

Based on the obtained results, the following conclusions were reached:

1. To find the frequencies of natural vibrations of plates loaded with concentrated masses, which are located periodically, the hyperbolic equation is used. As a result, we obtain the values of the natural frequencies of such a rectangular plate. Depending on the type of the given boundary conditions, it is possible to obtain various types of dependence of the natural vibration frequency with taking into account the geometry of the plate and its mechanical properties, for example, dependence on Poisson's ratio, or Young's modulus, etc. It is easy to see that the value of the numerical value of the frequencies strongly depends on the boundary conditions.
2. When solving dynamic problems of this kind, it is possible to use various boundary conditions, including boundary conditions for a supported or embedded edge. This allows us to investigate the differences in the number of obtained natural vibration frequencies of loaded plates.
3. The values of frequencies in problems of natural vibrations of plates loaded with lumped masses increase with decreasing thickness of the considered plate and do not depend on the geometry of lumped masses. It is

obvious that the considered lumped masses can be considered as points having a mass much greater than the mass of the surrounding continuous medium: elastic, viscoelastic, or plastic, at least in some δ-region around this lumped mass.

The use of modern methods of mathematical and functional analysis, the theory of functional series and Fourier series and the theory of functions of a complex variable to solve such problems, allows taking into account the geometric properties of complex structures. The use of methods for solving the equations of mathematical physics in combination with precise formulations of problems in the mechanics of a deformable solid body allows creating new technological solutions for which modern viscoplastic and composite materials can be used. Such materials are necessary for the implementation of new design solutions, since they have low material consumption [6, 7]. It is clear that modern structures created from modern viscoplastic and composite materials should work reliably in the elastic and viscoelastic regions under complex dynamic loads [8, 9].

Acknowledgements

This work was supported by the Ministry of Science and Higher Education of the Russian Federation under project 0707-2020-0034. This work was carried out using equipment provided by the Center of Collective Use of MSUT "STANKIN".

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