Methods of Mathematical Modeling of Automated Machine Systems of Multi-Nomenclature Production

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Abstract. Automated machine tool systems for multi-product manufacturing were designed for processing parts of a certain nomenclature, which includes separate groups of parts, united by the design, technological, organizational and economic properties. To reduce the number of options in the structure of an automated machine tool system, a target nonlinear design function was applied. Technological conditions were formulated in such a way as to allow synthesizing alternative structures of the designed automated system. At this stage, a matrix of relations was built and a formalized description of technological conditions was given. Specific and generalized technological conditions, which take into account the graph of relations and compatibility condition formulated for any pair of elements, were considered. The article discusses the generalized technological conditions obtained on the basis of the synthesis of compatibility and follow-up properties. The existence of generalized technological conditions for the elements of the automated system of the same name was assessed. The interaction of the elements of an automated system during processing, when the condition of the following and compatibility of technological parameters are met, was studied. By analyzing the functional and technological structure of an automated machine tool system for multi-product manufacturing, the key relationships between its main elements were determined, and the technological environment, a key subject of the system research, was identified.

1 Introduction

The development of integrated computer-aided design systems for automated machine-tool systems (CAD ACS) and specialized computer-aided design systems for individual stages of ACC design [1-3] is of great interest for the production of parts for multi-product manufacturing system.

In order to solve the problems of CAD ACC development, designers were first given the task of integrating automation systems and interconnecting systems, subsystems, elements of these systems at the stages of technological design, including the choice of computer technology [4, 5]. Although the above-mentioned tasks are solved on the basis of system design principles [6, 7], the coordination of the ready-to-use solutions at different (technological, algorithmic, computer, systemic) levels under the real conditions of ACC design is more difficult [8].

In many ways, production efficiency depends on the rational selection of equipment. At present, there does not exist an no methods for separating huge total sets of equipment and machine tools that are most expedient in terms of technical and economic indicators for processing parts of a certain nomenclature.

The design of an automated machine tool system is a complex multidimensional task that includes assessment, modeling, synthesis, analysis and optimization of project proposals. These tasks can be solved using only systematic approach, which accounts for fact that the specificity of complex objects must include both the features of their constituent elements and the nature of various relations and connections between them [9].

Typically, automated machine tool systems for diversified production are designed for processing parts of a certain nomenclature, which includes separate groups of parts, united by design, technological, organizational and economic properties. Therefore, defining a group of parts and nomenclature is an important task [10].Automated machine tool systems for diversified production, as a rule, were designed for processing parts of a certain nomenclature, which included separate groups of parts, united by design, technological, organizational and economic characteristics.

2 Nomenclature and its properties

Nomenclature is a set of parts that have similar design, functional, technological and organizational characteristics [11].

Formal description of a group of parts $D_{г}, г \in \{\Gamma\},$, where the nomenclature of parts $\{D^H\}$, processing of which is provided by the synthesized automated system:

$$\{D_1\}\cup\{D_2\}\cup\ldots\cup\{D_г\}\cup\ldots\cup\{D_M\}=\{D^H\}.$$

Let us d efine t he p roperties o f t he n omenclature, which we select from the subset of elements of the graph G = (X, O, T, I, D), which determine the structure of the automated machine to ol s ystem, i. e. Let u s d efine t he properties of the nomenclature, which we select from the subset o f e lements o f t he gr aph G = (X, O , T, I , D), which determine the structure of the automated machine tool system, i.e.

Nomenclature define properties th at distinguish subsets o f e lements o f a g raph G = (X, O , T , I , D), determining the structure of the automated machine tool, i.e.
we make the transition from *G = (X, O, T, I)* to *G * = (X ', O', T ', T')*, for which the condition:

$$\{X'\}\subseteq\{X\},\{T'\}\subseteq\{T\},\{O'\}\subseteq\{O\},\{I'\}\subseteq\{I\}\ .$$

The s et o f tr ansitions *{L}* and t he a ssociated machined surfaces of the set of parts *{Dg}* serve as initial parameters f or s ynthesizing t he s tructure o f an automated machine tool system and for which

$$P_i=\left\lfloor \underset{L}{\wedge}\pi_l^i\right\rfloor. \quad (1)$$

3 The matrix between the elements of an automated machine tool system

The p redicate m atrix of t he r elationship b etween t he elements $a_x, x\in\{X\}$ and $\pi_l, l\in\{L\}$ the automated machine t ool s ystem s ets t he t echnological capabilities of the equipment used:

$$M=\begin{cases} & \pi_1, & \pi_2, & \ldots, & \pi_l, & \ldots, & \pi_L \\ a_1 & y_{11}, & y_{12}, & \ldots, & y_{1l}, & \ldots, & y_{1L} \\ a_2 & y_{21}, & y_{22}, & \ldots, & y_{2l}, & \ldots, & y_{2L} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_x & y_{x1}, & y_{x2}, & \ldots, & y_{xl}, & \ldots, & y_{xL} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_X & y_{X1}, & y_{X2}, & \ldots, & y_{Xl}, & \ldots, & y_{XL} \end{cases} \quad (2)$$

From formula 1 f or a group of parts $d\in\{D_г\}$, it is obvious that:

$$d=\underset{N'}{\wedge}p_i\left\lfloor \underset{L}{\wedge}\pi_l^i\right\rfloor \quad (3)$$

for a group of parts $\{D_г\}$

$$\{D_г\}=\underset{M}{\wedge}d_j\left\{\underset{N'}{\wedge}p_i\left\lfloor \underset{L}{\wedge}\pi_l^i\right\rfloor\right\} \quad (4)$$

for part nomenclature $\{D^H\}$

$$\{D^H\}=\overset{M}{\underset{г=1}{\forall}}D_г\left[\underset{M}{\wedge}d_j\left\{\underset{N'}{\wedge}p_i\left\lfloor \underset{L}{\wedge}\pi_l^i\right\rfloor\right\}\right]. \quad (5)$$

For e ach tr ansition $\pi_l, l\in\{L\}$ (2), co nstruct a vector $y_l=(y_{1l},y_{2l},\ldots,y_{xl},y_{Xl})$. For each t ransition (2), construct a vector.

Its nonzero elements determine the equipment of the automated machine t ool s ystem $a_x, x\in\{X\}$, o n which this transition can be implemented:

$$\begin{aligned}\pi_1&=(y_{11},y_{21},\ldots,y_{x1},y_{X1})\\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ .\\ \pi_L&=(y_{1L},y_{2L},\ldots,y_{xL},y_{XL})\end{aligned} \quad (6)$$

We substituted vectors (6) into (1) for $P_i, i\in\{N'\}$

$$\begin{aligned}p_i=&\ \pi_1^i(y_{11},y_{21},\ldots,y_{x1},\ldots,y_{X1})\wedge\\ &\wedge\pi_2^i(y_{12},y_{22},\ldots,y_{x2},\ldots,y_{X2})\wedge\\ &\wedge\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ &\wedge\pi_l^i(y_{1l},y_{2l},\ldots,y_{xl},\ldots,y_{Xl})\wedge\\ &\wedge\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ &\wedge\pi_L^i(y_{1L},y_{2L},\ldots,y_{xL},\ldots,y_{XL}),\end{aligned}$$

and after the transformations we got:

$$p_i=\wedge\pi_l^i(y_{1l},y_{2l},\ldots,y_{xl},\ldots,y_{Xl})$$

or

$$p_i=\wedge\pi_l^i\left(\underset{X}{\forall}y_{xl}\right),$$

and for the detail

$$d_j=\underset{N'}{\wedge}p_i\left\lfloor \underset{L}{\wedge}\pi_l^i\left(\underset{X}{\forall}y_{xl}\right)\right\rfloor$$

For a group of parts $\{D_г\}$:

$$\{D_г\}=\underset{M'}{\wedge}d_j\left\{\underset{N'}{\wedge}p_i\left\lfloor \underset{L}{\wedge}\pi_l^i\left(\underset{X}{\forall}y_{xl}\right)\right\rfloor\right\}, \quad (7)$$

for part nomenclature $\{D^H\}$:

$$\{D^H\}=\overset{M}{\underset{ã=1}{\forall}}D_ã\left[\underset{M}{\wedge}d_j\left\{\underset{N'}{\wedge}p_i\left\lfloor \underset{L}{\wedge}\pi_l^i\left(\underset{X}{\forall}y_{xl}\right)\right\rfloor\right\}\right]. \quad (8)$$

If t he disjunction o f vectors $y_{xl}=0$ and i f $\underset{X}{\forall}y_{xl}=0$, then the given nomenclature of parts $\{D^H\}$ cannot be manufactured by this automated machine tool system according to (8).

We got a predicate, where $\forall d_j$ – is the merge quantifier with domain M:

$$\psi\{D^H\} = \underset{M}{\forall} d_j = \left\{ \underset{N'}{\wedge} p_i \left\lfloor \underset{L}{\wedge} \pi_l^i \left(\underset{X}{\forall} y_{xl} \right) \right\rfloor \right\} \quad (9)$$

The resulting predicate $\psi\{D^H\} = 1$, if for each part $d_j \in \{D^H\} \underset{X}{\forall} y^i{}_{xl} \neq 0$ there are many transitions $\pi_l, l \in \{L\}$ in the accepted equipment of the automated machine tool system $a_x, x \in \{X\}$, which is capable of processing any part d_j, $j = \overline{1, M}$.

The analysis of the model described in (9) shows that the number of variants of the structure of the automated machine tool system is large and therefore, to reduce the number of variants, it is possible to use the target nonlinear function of the MNP ACC design:

$$F \rightarrow \min[\{X\} \wedge \{T\} \wedge \{O\} \wedge \{I\}], \quad (10)$$

if

$$\psi\{D^H\} = 1 \quad (11)$$

The work [12] confirms the correctness and validity of function (10).

Minimize alternative structures of the automated system in order to obtain foreseeable set of options, can be isomorphic mapping relationships between the elements of ACC on the graph G *.

Thus, we get a dialectical relationship between the structures of the nomenclature of parts and an automated machine tool system.

For an automated machine tool system, it can be expressed through the properties of elements that implement the structure of the system (table 1). Based on the properties of the elements of an automated system, technological conditions (TC) can be formulated that will allow synthesizing alternative structures of the designed automated system, i.e.:

$$G^* = (X', T', O', I')^{\{TY\}} \Rightarrow G^{**} = (X'', T'', O'', I''),$$

where {TY} is a set of formalized technological conditions, which generates a graph of relations, on the basis of which a matrix of relations is built (Fig. 1) and a formalized description of technological conditions is given.

Let us consider technological conditions of two types: particular and generalized.

Consider the process conditions are of two types: partial and generalized.

Particular technological conditions are applied between elements of the set of the same name, and generalized conditions are applied between elements of unlike sets.

ACC element designation	X'	T'	C'	I'
X'	≡, ↔, ↦, ><	↔, ↦, ><	↔, ↦, ><	↔, ↦, ><
T'		≡, ↔, ↦, ><	↔, ↦, ><	-
O'			≡, ↔	↔, ↦, ><
I'		-		≡, ↔, ><

Fig. 1. Matrix of relations between ACC elements

At the same time, it is taken into account that dj is a complex part, which, in terms of its structural and technological characteristics and parameters, represents all the details of the group for which the ACC is designed. For example, the compatibility relationship between the workpiece and the ACC equipment is formally described:

$$\forall d_j \underset{X}{\forall} a_x \underset{S_d}{\exists} S_d \underset{S_x}{\exists} S_x \ni P\{(d_j[S_d^1 = shaft] \equiv a_x[S_x^1 = turning]) \wedge$$
$$\wedge (d_j[S_d^3] \geq a_x[S_x^2]) \wedge (d_j[S_d^4] \geq a_x[S_x^3]) \wedge (d_j[S_d^5] \leq a_x[S_x^4])\} \Rightarrow$$
$$\Rightarrow [d_j \leftrightarrow a_x] \quad (12)$$

Among the set of equipment on the basis of TU (12) for the part $d_j \in \{D^H\}$ under consideration, a subset of machines can be distinguished, according to their parameters, capable of ensuring the processing of this part in accordance with the specified quality parameters.

Thus, by (12), taking into account the graph of relations, we can formulate the compatibility conditions for any pair of elements.

Table 1. Matrix of properties of the main elements of the machine tool system (fragment)

ACC element	Element property	Property name notation	Property value notation
Machine tool (set *{X}*)	Processing precision	S_x^1	$[S_x^1]$
	Roughness of processing	S_x^2	$[S_x^2]$
		S_x^3	$[S_x^3]$
	Geometric parameters	S_x^4	$[S_x^4]$
	Processing method	S_x^5	$[S_x^5]$
	Power characteristics	S_x^6	$[S_x^6]$

Vehicle (set *{T}*)	A type	S_t^1	$\left[S_t^1\right]$
	Positioning accuracy	S_t^2	$\left[S_t^2\right]$
	Geometric parameters	S_t^3	$\left[S_t^3\right]$
	Power characteristics	S_t^4	$\left[S_t^4\right]$
	Lifting capacity	S_t^5	$\left[S_t^5\right]$
	Type of transported part or tool	S_t^6	$\left[S_t^6\right]$
Rigging (set *{O}*)	A type	S_0^1	$\left[S_0^1\right]$
	Processing precision	S_0^2	$\left[S_0^2\right]$
	Roughness of processing	S_0^3	$\left[S_0^3\right]$
	Geometric parameters	S_0^4	$\left[S_0^4\right]$
	Processing method Power characteristics	S_0^5	$\left[S_0^5\right]$
		S_0^6	$\left[S_0^6\right]$
	Basing scheme	S_0^7	$\left[S_0^7\right]$
Tool (set *{I}*)	A type	S_I^1	$\left[S_I^1\right]$
	Processing precision	S_I^2	$\left[S_I^2\right]$
	Roughness of processing	S_I^3	$\left[S_I^3\right]$
	Geometric parameters	S_I^4	$\left[S_I^4\right]$
	Processing method	S_I^5	$\left[S_I^5\right]$
	Cutting material	S_I^6	$\left[S_I^6\right]$
Surface to be treated (set *{N '}*)	A type	S_P^1	$\left[S_P^1\right]$
	Part material	S_P^2	$\left[S_P^2\right]$
	Processing precision	S_P^3	$\left[S_P^3\right]$
	Roughness of processing	S_P^4	$\left[S_P^4\right]$
	Geometric parameters	S_P^5	$\left[S_P^5\right]$
	Surface position	S_P^6	$\left[S_P^6\right]$
Part (set *{D}*)	A type	S_d^1	$\left[S_d^1\right]$
	Part material	S_d^2	$\left[S_d^2\right]$
	Processing precision	S_d^3	$\left[S_d^3\right]$
	Roughness of processing	S_d^4	$\left[S_d^4\right]$
	Geometric parameters	S_d^5	$\left[S_d^5\right]$
	Processing method	S_d^6	$\left[S_d^6\right]$
		S_d^7	$\left[S_d^7\right]$
	Basing scheme Weight	S_d^8	$\left[S_d^8\right]$

The technological conditions of compatibility between the part and the fixture can be represented as:

$$\forall d_j \underset{O}{\forall} pr_o \underset{Sd}{\exists} S_d \underset{S_o}{\exists} S_o \ni P\left\{\left(d_j\left[S_d^7\right] \equiv pr_o\left[S_o^7\right]\right)\wedge\right.$$
$$\wedge\left(d_j\left[S_d^5\right] - pr_o\left[S_o^4\right] \le k\right)\wedge\left(d_j\left[S_d^6\right] \equiv pr_o\left[S_o^5\right]\right)\left.\right\}\Rightarrow$$
$$\Rightarrow \left[d_j \leftrightarrow pr_o\right] \qquad (13)$$

where k is a coefficient that takes into account the economically justified conditions for using this device.
where k - factor that takes into account economically viable conditions for the use of this device.

The technological conditions given in (12), (13) have the transitivity property:

$$P\left[\left(d_j \leftrightarrow a_x\right)\wedge\left(d_j \leftrightarrow pr_o\right)\right] \Rightarrow \left(a_x \leftrightarrow pr_o\right) \qquad (14)$$

4 Simulation of an automated machine tool system

The following relation takes a special place among particular technological conditions, since it is decisive not only in the technological sequence of the arrangement of the elements of an automated machine tool system, but also in the direction of material flows, on which the structural structure of the entire automated system depends.

The ratio of repetition has a special place among private process conditions, as is determined not only in the automated sequencing machine tool arrangement of elements, but in the direction of material flow, on which the structural construction of the entire automated system.

The technological sequence of processing on the equipment $a_x, a_q, x, q, \in \{X\}$ is determined from the expression:

$$\underset{X}{\forall} a_x \underset{X}{\forall} a_q \underset{S_x}{\exists} S_x \underset{S_q}{\exists} S_q \ni P\left\{P\left(a_x, \vec{U}, a_q\right)\wedge\right.$$
$$\wedge\left(a_x\left[S_x^2\right] \ge a_q\left[S_q^2\right]\right)\wedge\left(a_x\left[S_x^3\right] \ge a_q\left[S_q^3\right]\right)\left.\right\}\Rightarrow$$
$$\Rightarrow \left\lfloor a_x \leftrightarrow a_q \right\rfloor \qquad (15)$$

The sequence in terms of the direction of material flows in an automated machine tool system can be determined from the technological condition connecting the elements of the transport system $b_t t \in \{T\}$ and the main technological equipment $a_x, x \in \{X\}$:

$$\underset{T}{\forall} b_t \underset{X}{\forall} ax \underset{S_t}{\exists} St \underset{S_x}{\exists} S_x \ni P\left\{P\left(b_t, \vec{U}, a_x\right)\wedge\right.$$
$$\wedge\left(b_t\left[S_t^3\right] \equiv a_x\left[S_x^4\right]\right)\wedge\left(b_t\left[S_t^4\right] \equiv a_x\left[S_x^6\right]\right)\left.\right\}\Rightarrow$$
$$\Rightarrow \left[b_t \mapsto a_x\right] \qquad (16)$$

The analysis showed that the generalized technological conditions were obtained on the basis of the synthesis of compatibility and succession properties, and since the compatibility condition contains an equivalence condition in an implicit form, then the verification of the existence of generalized technological conditions for the elements of the same name of an automated system is the establishment of the direction of material flows between them, and for dissimilar elements - technological adequacy.

The elements of the automated system interact during processing when the following condition (movement of material flows in the automated system) and the compatibility of technological parameters are met.

For two dissimilar objects, a generalized technological condition will be valid:

$$\underset{T}{\forall} b_t \underset{X}{\forall} ax \underset{S_t}{\exists} St \underset{S_x}{\exists} S_x \ni P\{(P(b_t,\vec{U},a_x)\wedge(b_t[S_t]\equiv a_x[S_x])\}\Rightarrow$$
$$\Rightarrow [b_t >< a_x] \qquad (17)$$

Expression (17), based on the previously defined graph *G(Z,U,P)*, allows us to build a generalized technological condition for several elements of an automated system capable of implementing a certain technological function, the consequence of which is transitivity:

$$[(pr_o >< a_x)\wedge(b_t \overline{><} a_q)] \qquad (18)$$

The truth of expression (18) provides the ability to choose the structure of an automated machine tool system, as shown in Fig. 2.

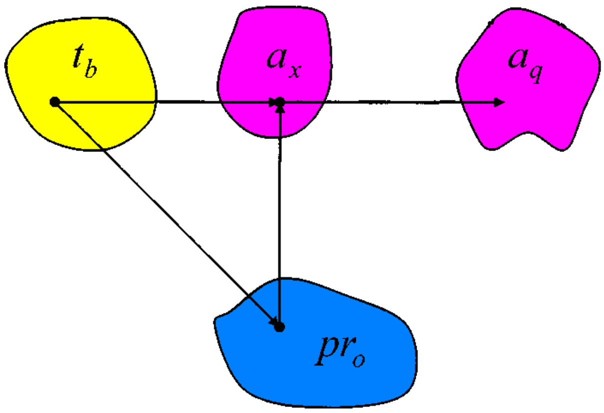

Fig. 2. The choice of elements of an automated system depending on technological conditions.

The resulting structure of the automated system satisfies condition (9), makes it possible to use various target design functions to obtain a quasi-optimal option, without contradicting (10).

By applying the investigated design procedures tailored to the specific optimization objective there is a possibility of a variety of options to get the structure of quasi-optimal automated machining system.

The sequence of mathematical modeling of the process of creating an automated machine tool system is shown in Fig. 3.

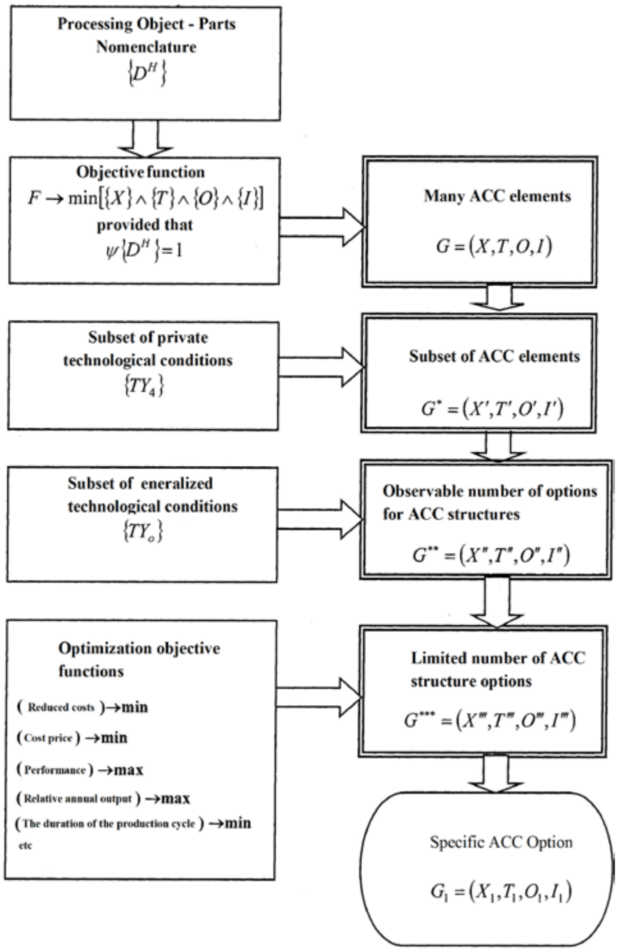

Fig. 3. Mathematical modeling of the process of designing an automated system.

5 Conclusion

The paper discusses the generalized technological conditions obtained on the basis of the synthesis of compatibility and follow-up properties, with the existence of generalized technological conditions for the elements of the automated system of the same name being accounted for simultaneously. The interaction of the elements of an automated system during processing, where the condition of compatibility of technological parameters is met, was investigated.

The analysis of the functional and technological structure of the automated machine tool system of multi-product manufacturing was carried out based on the relationships between its main elements, which describe the technological environment and are a subject of system research.

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