EPJ Web of Conferences **250**, 02006 (2021) https://doi.org/10.1051/epjconf/202125002006
DYMAT 2021

An alternative method for the measurement of mechanical properties at intermediate strain rates: a numerical study

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Abstract. A feasibility study of an innovative apparatus for dynamic characterization of materials at intermediate strain rates is presented. The working principle is based on the Split Hopkinson bar, but the wave propagation occurs through properly sized springs. The system is designed to generate and transmit tension or compression waves having a low propagation speed, in order to reduce the specimen strain rate at the impact. At first, a simplified theory is presented for the estimation of longitudinal wave speed in springs as a function of the main engineering parameters of the coil dimensioning. Then, a preliminary sizing of the apparatus is proposed based on basic considerations of wave propagation theory. Finally, a numerical model of a compression test is presented as a proof-of-concept.

1 Introduction

Nowadays, one of the most widely used method for the characterization of the dynamic behaviour of materials is based on the split Hopkinson pressure bar (SHPB) [1, 2]. In this machine, a stress wave propagates along slender bars and dynamically deforms a specimen. By measuring the strain signals in the bars, it is possible to determine indirectly the dynamic properties of the tested material [3, 4]. The control of test strain rate is primarily achieved by adjusting the bar particle velocity on the input side and by specimen length.

Typically, Hopkinson bars are made of metallic materials, such as steel, aluminium and titanium, but also polymeric [5, 6]. Since these materials are very rigid, the pressure wave speed is elevated and, consequently, the particle velocity is high as well. Moreover, the available stroke during dynamic testing depends on both the length of the preloaded bar and the intensity of the preload. This means that long preloaded bars are required for performing tests at moderate strain rate without increasing the transmitted load.

To cover the range between quasi-static and high strain rate regimes, several measurement apparatuses have been developed, but none appears to be the dominant one [7]. These are driven by three main methods: servo-hydraulic drive, linear acceleration of masses or angular acceleration of masses. In particular, servo-hydraulic machines [8] have a similar architecture

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to those used for quasi-static testing but adapt the loading system to increase the specimen strain rate. On the other hand, drop towers [9] are machines in which the mechanical action is provided by the impact of a mass on a plate to which one end of the specimen is clamped. The mass is accelerated by gravitation only or with the assistance of a preloaded spring energy. Moreover, flywheel systems [10] are experimental apparatuses that exploit the circumferential velocity of a rotating disk combined with the high inertia of the disk itself to exert dynamic loading on the specimen. In some cases, the abovementioned apparatuses are modified by combining the loading system with Hopkinson output bar to improve the force measurement quality. Furthermore, modified versions of the SHPB have also been developed to make it suitable for intermediate strain rates; however, the long test time determined a significant output bar length, as in [11]. Most of these machines have to deal with measurement issues, such as large record oscillations, inertial effects, and short duration of mechanical loading.

In this paper, an innovative machine concept – the "Hopkinson Spring" – for dynamic characterization of materials at intermediate strain rates is presented. The alternative method of dynamic driving is based on the generation and transmission of slow longitudinal waves which propagate along properly sized springs. The apparatus design is inspired by the Split Hopkinson bar system, and its performance has been then simulated in a compression test applied to a polymer.

2 Theoretical background

2.1 Longitudinal wave propagation in helical springs

A helical spring can be considered, in first approximation, as a one-dimensional pseudo-continuous elastic system characterized by a linear relationship between force and displacement. Therefore, the macroscopic longitudinal behaviour of the spring can be described with the same theory applied to solid bars. According to Wittrick's theory [12], the propagation speed of longitudinal waves in springs can be expressed using the following Equation (1):

$$c = h_0\sqrt{\frac{K}{m}} \tag{1}$$

where K is the spring elastic constant, h_0 is the height of the spring in its relaxed state and m is the total mass of the spring.

Equation (1) can be developed to explicit the wave propagation speed as a function of the engineering characteristics of helical springs. Given a spring of medium diameter D with wire diameter d and helix angle α, the elastic constant K can be expressed by Equation (2), according to the classical linear elasticity theory of torsion of beams, whereas its mass is calculated from Equation (3):

$$K = \frac{d^4 E\pi \sin\alpha}{16(1+\nu)D^2 h_0} \tag{2}$$

$$m = \rho\frac{\pi d^2}{4}\frac{h_0}{\sin\alpha} \tag{3}$$

where E and ν are Young modulus and Poisson ratio of the spring material.

Substituting (2) and (3) into (1), it results in Equation (4) for the longitudinal wave speed in helical springs:

$$c = \frac{d}{D}\sin\alpha\sqrt{\frac{E}{4\rho(1+\nu)}} \tag{4}$$

An interesting point about Equation (4) is that the wave propagation speed in springs depend both on material properties (as well as for bars), but also on geometric characteristics, such as helix angle and spring index ($C = D/d$). Therefore, it is possible to adjust the propagation speed of longitudinal waves.

2.2 Spring-bar analogy

An equivalence between the spring and an analogous solid bar with the same diameter D is here assumed to be valid. Their mechanical behaviour is considered similar too. Therefore, both the spring and the bar must be characterized by the same longitudinal wave speed c and particle velocity v. The analogous bar density ρ' can be calculated with the respect of the bar volume, as in Equation (5):

$$\rho' = \frac{m}{V'} = \rho \frac{\pi d^2}{4} \frac{h_0}{\sin\alpha} \frac{4}{\pi D^2} \frac{1}{h_0} = \left(\frac{d}{D}\right)^2 \frac{\rho}{\sin\alpha} \quad (5)$$

Thus, the particle velocity v induced by the perturbation of the stress wave can be expressed according to Equation (6):

$$v = \frac{\sigma'}{\rho' c} = \frac{F}{\pi \frac{D^2}{4}} \left(\frac{D}{d}\right)^3 \sqrt{\frac{4(1+\nu)}{E\rho}} \quad (6)$$

where F is the actual force in the spring and σ' is the analogous bar stress.

3 Overall apparatus dimensioning

Equation (4) shows that the wave speed in helical springs is much lower than in bars, mainly due to the small helix angle used to manufacture them, but also due the spring index. For example, in a steel spring with angle $\alpha = 6°$ and index $C = 4.75$ the longitudinal waves travel at 1/100th of the speed at which they would propagate in bars.

Reasoning in terms of a Split Hopkinson bar-like apparatus, the single components (Pre-stressed, Input, and Output) can be designed much shorter than bars, ensuring a significant stress wave duration. According to the literature, a good loading wave duration for intermediate strain rate tests is around 10 ms. Here, assuming the design length of the Pre-stressed spring to be 400 mm with the above geometric parameters, a stress wave duration of 16 ms is derived by the well-known calculation $T = 2h_0/c$.

Since the strain state in a spring is not constant through the wire cross section, it is more complex to process such signals than in the Hopkinson bar. However, the existing advanced non-contact measurement techniques allow to measure the specimen strain directly, without the need for indirect processing of bar signals. For these reasons, the input spring was sized minimizing its length just enough to guarantee the entire generated stress wave transmission. Regarding the Output spring, as a consequence of the results of preliminary numerical simulations, the length has been imposed to be a bit lower than the one necessary to the propagation of the whole stress wave generated by the Pre-stressed spring.

For what concerns the size of the wire diameters, the choice has depended on the maximum required impact velocity on the specimen. In the design of this apparatus, a maximum impact velocity around 2 m/s and 20 kN of maximum transmitted force have been chosen.

Table 1 reports the main springs characteristics.

Table 1. Hopkinson Spring characteristics.

	Pre-stressed	**Input**	**Output**
D [mm]	190	200	168
C	4.75	4	4.2
α [°]	6	8	8
h_0 [mm]	400	660	380
c [m/s]	49.3	78.0	74.3
T [ms]	16.2		10.2

4 Numerical model and results

The Hopkinson Spring concept has been numerically simulated to investigate further theoretical insights that simplified one-dimensional theory cannot capture. The three steel springs were sized in accordance with the geometric parameters of Table 1. One half inactive coil was added to each end of the springs to provide a flat surface for better load transfer between the Pre-stressed and Input springs and at the specimen interfaces. Moreover, the ends of the Input and Output springs pointing towards the specimen have been plugged by a 5 mm thick plate to allow spring-specimen contact.

A cylindrical specimen of diameter $D_s = 7$ mm and length $L_s = 8$ mm was chosen to simulate a compression test. The specimen material is an ABS structural polymer which has been dynamically characterized by Johnson-Cook model [13].

Since specimen strain measurement is assumed to be performed by non-contact measurement techniques, the average nodal displacement results at its ends are considered for strain and strain rate calculations. On the other hand, force measurement is obtained indirectly by interposing a load cell (a bar of diameter $D_{bar} = 18$ mm and length $L_{bar} = 220$ mm) between the specimen and the Output spring. In this case it is assumed that the specimen and load cell are in dynamic equilibrium for the whole test duration since the wave propagation speed in the load cell is much higher than in the springs and the specimen is deformed slowly.

A numerical simulation has been performed on a FE model with an explicit solver, in which a tension preload force of 10 kN is applied to the Pre-stressed spring and then instantaneously released. The simulation results have been stored every 0.01 ms, for a total of 35 ms of test history. The time history of the forces in the springs, specimen, and load cell have been measured through cutting planes perpendicular to the longitudinal axis. The details of the numerical model are shown in Figure 1.

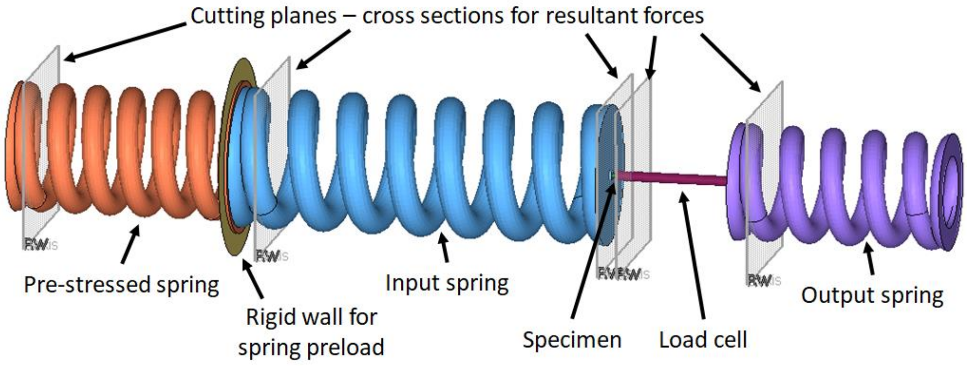

Fig. 1. Numerical model of the Hopkinson Spring.

The results of the numerical model have been analysed to verify the proposed theoretical approach. Figure 2a shows the force curves in the springs at the cutting plane positions of Figure 1. It can be noted that, although the instantaneous preload release, the rise of the mechanical waves is less pronounced compared to those in traditional SHPB systems. This can be attributed to the size of the springs, which cannot be approximated to one-dimensional bodies. However, the proposed dimensioning formulas can roughly predict the constant part of wave signals, such as the stress wave period in Table 1, or the axial velocity in Input. In the example of Figure 2b, the velocity has been calculated through Equation (6) with the average force shown in Figure 2a.

Finally, regarding the measurement of the specimen mechanical properties, it can be affirmed that the measured force on the load cell corresponds precisely to the specimen resistance (Figure 3a), while the strain calculation confirms the test feasibility at intermediate strain rates (Figure 3b).

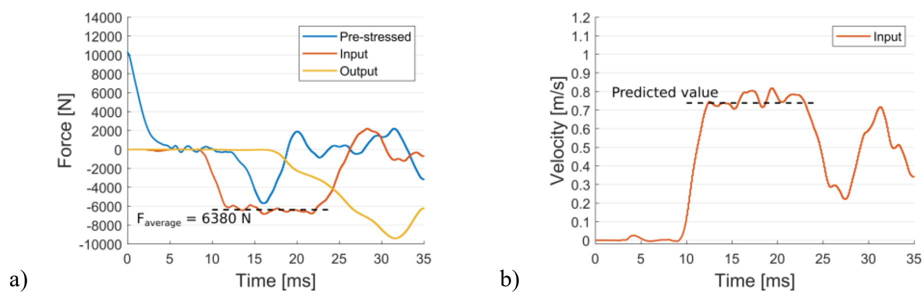

Fig. 2. a) Cross section resultant forces in the springs; b) Particle velocity in Input spring.

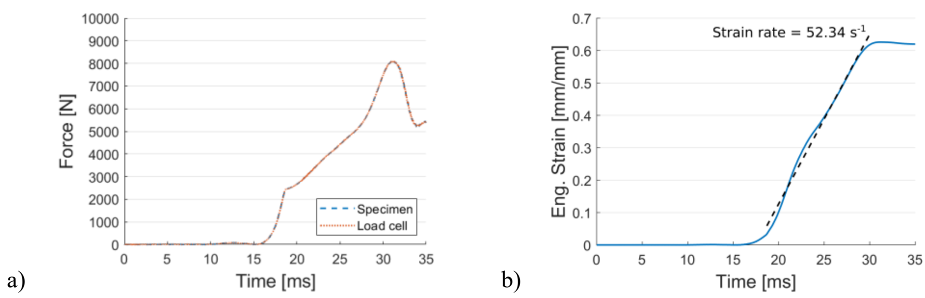

Fig. 3. a) Dynamic equilibrium between specimen and load cell; b) Test strain and average strain rate.

5 Conclusions

In this paper, the feasibility of an innovative apparatus for performing mechanical tests on materials at intermediate strain rates has been presented. The proposed solution differs from the state of art both in the dynamic actuation and in the specimen strength measurement aspects. The working principle is based on the Split Hopkinson bar, but exploits the propagation of slow mechanical waves in springs. A simple theoretical description allows for the design of the Hopkinson Spring by varying the springs' geometric and material characteristics. The performance of the apparatus has been numerically verified.

Patent application

The concept of this innovative apparatus has been submitted for international patent application.

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