

BIC in waveguide arrays

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Abstract. We propose a simple theoretical model based on the coupled-mode theory which allows to calculate the spectral properties and transmittance of the one-dimensional waveguide structures. The model was verified on the common coupled-waveguide array in which the existence of the symmetry-protected bound state in the continuum (BIC) was confirmed experimentally by Plotnik et al. [Phys. Rev. Lett. 107, 28-31 (2011)]. The method can be extended to topologically nontrivial lattices to explore the properties of the BICs protected by time-reversal symmetry.

1 Introduction

Bound state in the continuum (BIC) represents a fascinating exception to the conventional wisdom as despite it lies within the continuum and coexists with the extended state, it remains confined and does not radiates away; for recent reviews see, e.g., [1]-[3]. BICs exist in a wide range of material system due to various confinement mechanisms which are fundamentally different from the discrete levels of the conventional bound states, which lie outside the continuum, for example the bound state of an atom, the guided modes of an optical fiber below the light line and the defect states in a bandgap. In this contribution, we focus on BICs that may occur in coupled photonic lattices. We present a simple theoretical model based on the coupled-mode theory (CMT) which allows to calculate the spectral properties and transmittance of the one-dimensional waveguide structures. The model was applied to a perturbed coupled-waveguide array in which the existence of the symmetry-protected BICs was confirmed experimentally by Plotnik et al. [4]. We examine the properties of the bound modes induced by the perturbation and show how the perturbation affects the transmittance of the structure. The approach can be extended to topologically nontrivial lattices to explore the conditions under which the robust topological BIC states protected by time-reversal symmetry exist.

2 Theoretical model

We consider a 1D waveguide array – see Fig. 1 – that supports a continuum of extended states propagating along x -axis, which are even along perpendicular direction (y), along which the two defects are attached above and below this array. The evolution of the light in the coupled-waveguide structure depicted in Fig. 1 is described within

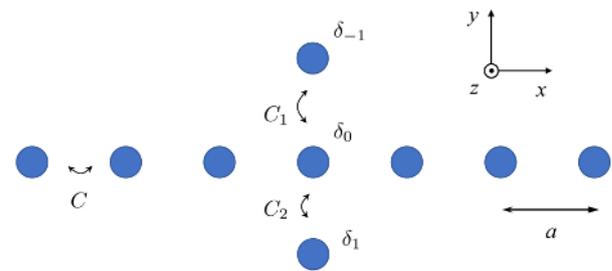


Figure 1. A coupled-waveguide system consisting of a 1D array and two additional waveguides above and below the array.

CMT by a set of coupled equations for the slowly varying modal amplitudes ψ_m on the m -th waveguide ($m \in \mathbb{Z}$) in the periodic array along x -direction which reads as

$$i \frac{d\psi_m}{dz} = C (\psi_{m-1} + \psi_{m+1}), \quad m \neq 0 \quad (1)$$

$$i \frac{d\psi_0}{dz} = \delta_0 \psi_0 + C (\psi_{-1} + \psi_1) + C_1 \varphi_{-1} + C_2 \varphi_1 \quad (2)$$

$$i \frac{d\varphi_{-1}}{dz} = \delta_{-1} \varphi_{-1} + C_1 \psi_0 \quad (3)$$

$$i \frac{d\varphi_1}{dz} = \delta_1 \varphi_1 + C_2 \psi_0, \quad (4)$$

where C is the coupling coefficient in the array, $\varphi_{\pm 1}$ are the modal amplitudes on the waveguides attached below and above the array, $\delta_{\pm 1}$ are the variations of their propagation constants with respect to the propagation constant in the unperturbed array, δ_0 is the variation of the propagation constant in the 0-th waveguide in the periodic array and $C_{1,2}$ are the coupling coefficients between upper and lower additional waveguides attached to 0-th waveguide in the periodic array.

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Instead of solving the whole set of the Eqs. (1)-(4), we first consider separately a one-dimensional periodic array of waveguides in the x -direction and a finite array in the vertical direction along y -axis, consisting of the 0-th waveguide in the periodic array and two additional waveguides above and below the array. By analysing the solutions of both subsystems we prove that asymmetric state belonging to the discrete spectrum of the vertical array is the solution of the set of the Eqs. (1)-(4), provided the system is vertically symmetric $\delta_1 = \delta_{-1}$. Due to the symmetry mismatch between this asymmetric state and the continuum, the asymmetric mode is decoupled from the Bloch modes of the periodic array and thus it forms BIC, provided its propagation constant lies in the continuum. To prove its existence numerically, we employ the set of the Eqs. (1)-(4) to derive the analytical expressions for the transmittance of the whole system and demonstrate the appearance of the peaks of the transmittance which arise when vertical asymmetry is introduced by imposing the condition $\delta_{-1} = -\delta_1$.

3 Results

In order to inspect the spectral properties of the waveguide array depicted in Fig. 1, we at first solve the Eqs. (1)-(2) for an unperturbed periodic array, i.e., we assume vanishing coupling coefficients $C_{1,2} = 0$ and perfect periodicity $\delta_0 = 0$. We seek the solution in the form of the plane wave $\psi_m = A \exp(-ik_x am - i\epsilon z)$, where ϵ stands for change of the propagation constant (which corresponds to the energy in the quantum theory) and a is the period of the lattice and we obtain the following dispersion relation:

$$\epsilon = 2C \cos(k_x a). \quad (5)$$

By using the latter equation we evaluate the real and imaginary components of the Bloch wavenumber k_x , which are shown in Fig. 2.

In the second step, we find the states in the vertical waveguide, i.e., we solve the Eqs. (2)-(4) when $C = 0$, where we also neglect the variations of the propagation constants associated with attached waveguides, i.e., $\delta_{\pm 1} = 0$ and $\delta_0 = 0$. The discrete spectrum has the form $(\varphi_{-1} \ \psi_0 \ \varphi_1)^T = (A_{-1} \ A_0 \ A_1)^T \exp(-i\epsilon z)$, which corresponds to three different modes, namely two symmetric (S-modes), which exist when $A_0 \neq 0$

$$\epsilon_{S_{1,2}} = \pm \sqrt{C_1^2 + C_2^2} \quad (6)$$

$$A_{-1} = \frac{C_1}{\epsilon_{S_1}} A_0 \quad (7)$$

$$A_1 = \frac{C_2}{\epsilon_{S_2}} A_0, \quad (8)$$

and one asymmetric (A-mode), which exists when $A_0 = 0$

$$\epsilon_A = 0 \quad (9)$$

$$C_1 A_{-1} = -C_2 A_1. \quad (10)$$

The eigenvalues of the symmetric modes are indicated by full circles in Fig. 2, the eigenvalue of the asymmetric

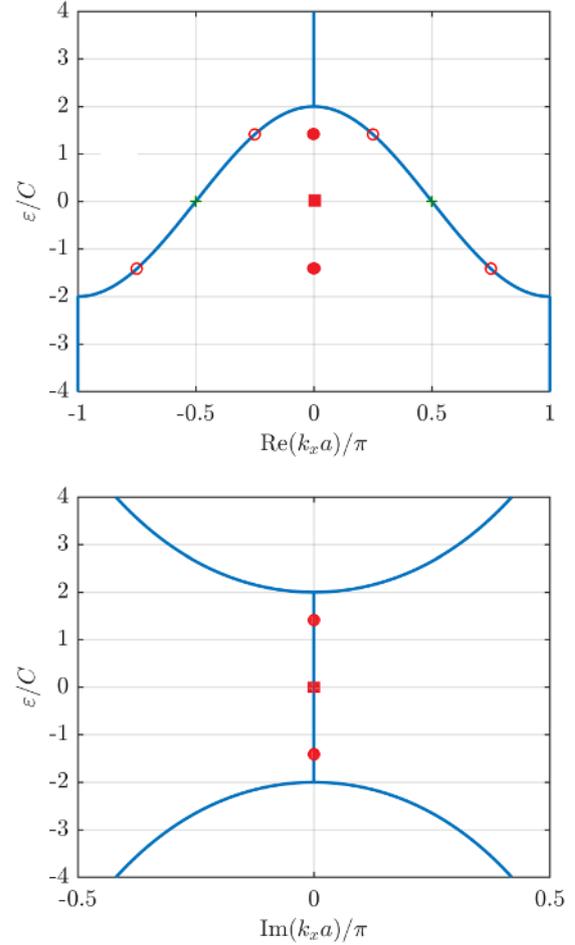


Figure 2. The real and imaginary parts of the band structure of 1D coupled waveguide array shown in Fig. 1. The positions of the states in the vertical waveguide are also indicated: S-modes (full circles) and A-mode (full square). The parameters are $C_1 = C_2 = C$ and $\delta_{-1} = \delta_1 = 0$.

mode is denoted by full square. The eigenvalues belonging to the continuum with the ϵ/C corresponding to ϵ_A and $\epsilon_{S_{1,2}}$ which form the pairs of the counter-propagating waves with $k_x a = \pm\pi/2$ and $k_x a = \pm\pi/4$; $k_x a = \pm 3\pi/4$ are indicated by crosses and open circles, respectively in Fig. 2 (the upper panel). We note, that the values $\epsilon_{S_{1,2}}$ of the symmetric modes can lie within or outside the continuum depending on the ratio $C_{1,2}/C$, while the asymmetric A-mode lies always within the continuum.

In the third step, we consider the whole coupled system shown in Fig. 1, which is described by the Eqs. (1)-(4). Now, for the moment, we assume the trivial vertical symmetry $\delta_{-1} = \delta_1 = 0$. The localized A-mode given by the Eqs. (9)-(10) is also the solution of the whole system as $\psi_m = 0$, i.e., the A-mode cannot couple to propagating modes of the periodic array and thus it is a true BIC. Generalization of the result for the case, when $\delta_{\pm 1} \neq 0$, is straightforward: the localized A-mode can occur only if $\delta_{-1} = \delta_1$; instead of the Eq. (9), its energy is given by $\epsilon_A = \delta_{-1} = \delta_1$ and the Eq. (10) remains

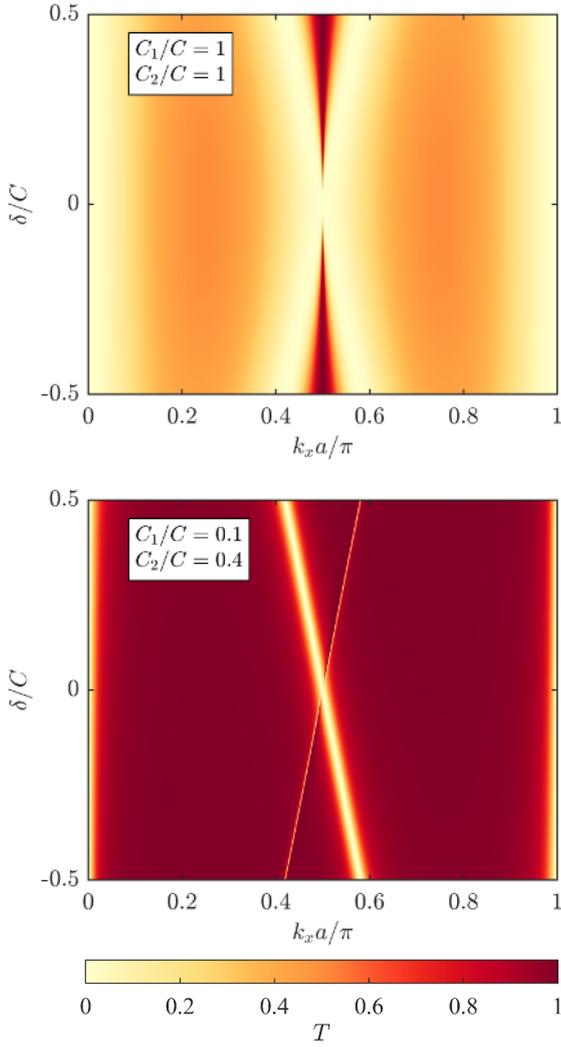


Figure 3. The transmittance T of the coupled-array system shown in Fig. 1 as a function of the Bloch wavenumber k_x and the asymmetry parameter $\delta = -\delta_{-1} = \delta_1$. The coupling coefficients C_1 and C_2 are indicated in the insets.

valid. Therefore, the asymmetric A-mode is a BIC provided $-2C < \delta_{-1} = \delta_1 < 2C$.

To examine the effect of a BIC on the characteristics of the structure we have calculated its transmittance and reflectance. We follow the approach described in Ref. [5] and assume the incident and reflected plane wave

$$\psi_m = e^{-ik_x a m - i\epsilon z} + r e^{ik_x a m - i\epsilon z}, \quad m \leq 0 \quad (11)$$

and the transmitted plane wave

$$\psi_m = t e^{-ik_x a m - i\epsilon z}, \quad m \geq 0. \quad (12)$$

After substituting the Eqs. (11) and (12) into the coupled equations (1)-(4) one obtains the analytical expressions for the amplitude transmittance t and reflectance r as

$$t = r + 1 = \frac{2C \sin(k_x a)}{2C \sin(k_x a) + i\mu}, \quad (13)$$

where

$$\mu = \delta_0 + \frac{C_1^2}{\epsilon - \delta_{-1}} + \frac{C_2^2}{\epsilon - \delta_1}. \quad (14)$$

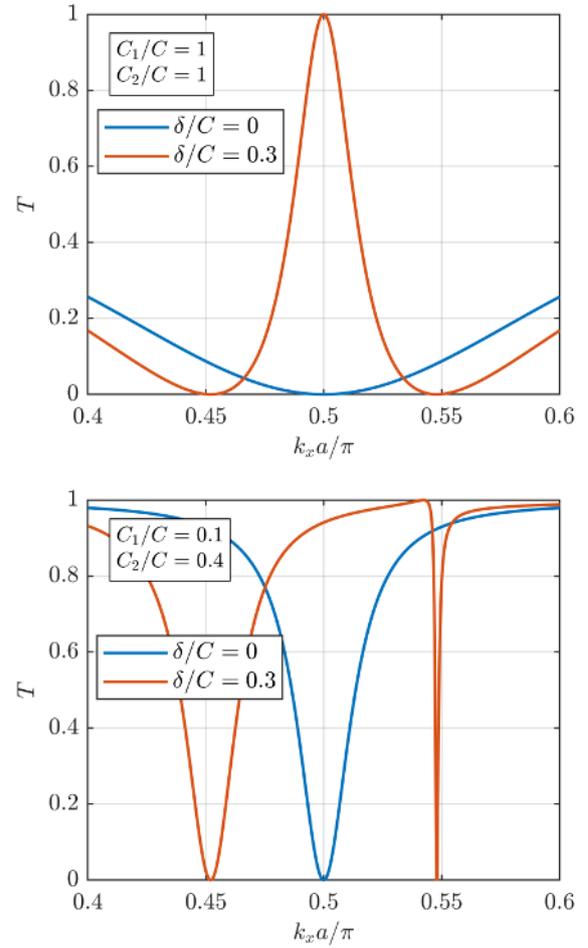


Figure 4. The transmittance spectra T for the parameters of Fig. 3 corresponding to the system with preserved (blue line) and broken (red line) symmetry.

The relation for the transmission given by the Eq. (13) for the coupled array system coincides with Fano scattering function [6] which reads as

$$\sigma(\epsilon) = \frac{(\epsilon + q)^2}{\epsilon^2 + 1}, \quad (15)$$

where ϵ is dimensionless energy and q is the shape parameter which determines the asymmetry of the profile. As a matter of fact, the intensity transmittance T defined as $T = |t|^2$ can be written in the form

$$T = \frac{\alpha_k^2}{\alpha_k^2 + 1}, \quad (16)$$

where $\alpha_k = 2C \sin(k_x a) / \mu$ plays the role of the dimensionless energy ϵ .

One can see that the transmittance T shown in Figs. 3 and 4 vanishes when $2C \sin(k_x a) = 0$, which happens at $k_x a = 0$ and $k_x a = \pm\pi$ corresponding to the edges of the continuum band, where the group velocity tends to zero. The transmittance also vanishes at $\epsilon = 0$ due to diverging parameter $|\mu| \rightarrow \infty$ when $\delta = 0$ in the Eq. (13), see also the Eq. (14), and corresponds to BIC.

Breaking the vertical symmetry yields coupling of the A-state to the continuum and mediates the leakage of light into periodic array. It follows from our numerical experiments that the parameter δ_0 does not affect significantly BIC-induced features in the transmittance. Therefore, we set $\delta_0 = 0$ hereafter. Similarly, it is a “strength” of the asymmetry between δ_1 and δ_{-1} rather than actual values of the parameters that has the most prominent effect on the mentioned features. Therefore, in the following we focus on the class of asymmetric structures where vertical asymmetry is introduced through the parameter δ which is defined as $\delta = \delta_1 = -\delta_{-1}$. When the vertical symmetry is broken, μ becomes finite, coupling of the asymmetric mode to the continuum becomes non-negligible and the existence of the quasi-BIC state manifests itself as the peak $T = 1$ in the transmittance. The peak can be assigned to the constructive interference of the two counter-propagating modes with $k_x a = \pm\pi/2$ in the case $C_1 = C_2$ (see the upper panels in Figs. 3 and 4, $k_x a = \pi/2$) and excited localized A-mode given by the Eqs. (9) and (10). When $C_1 \neq C_2$, position of the peak moves with δ (see the lower panels in Figs. 3 and 4). The shape of the resonances is affected by the ratio C_1/C_2 : when $C_1 = C_2$, the peak is Lorentzian and corresponds to the diverging asymmetry parameter $|q| \rightarrow 0$ – see upper panels of Figs. 3 and 4, while for $C_1 \neq C_2$, for example $C_1 = 0.1C$ and $C_2 = 0.4C$, the resonance possesses asymmetric Fano-like line shape – see lower panels of Figs. 3 and 4. The peak is always accompanied by two minima, at which the whole system becomes totally reflective, both of them arise due to the vanishing denominators in the second and third term in the Eq. (14) leading to an infinite scattering potential at the poles $\varepsilon = \delta_{\pm 1}$, i.e., $|\mu| \rightarrow \infty$ in the Eq. (13).

Within the continuum the ratio $C_{1,2}/C$ determines the fraction of the energy density transferred through the periodic chain of the waveguides. When $C = C_1 = C_2$ the transmittance has maxima $T = 0.5$ at $k_x a = \pm m\pi/4$ and

$k_x a = \pm 3m\pi/4$, while for smaller values of the coupling coefficients the structure becomes totally transparent as it is shown in lower panels of Figs. 3 and 4.

4 Conclusions

We developed a theoretical model which allows to characterize spectral properties of the 1D photonic lattices consisting of a periodic chain of waveguides with two additional waveguides attached above and beneath the array and to analyze various kinds of BICs with broken vertical symmetry. The method can be extended to topologically nontrivial lattices to explore properties of the optical BIC pseudospin states.

Acknowledgments

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