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On polarization characteristics of highly focused fields concentrated along the axis

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Abstract. After having proven that an uncertainty relation holds for the on-axis power content of highly-focused fields, in this Communication we explore and discuss the consequences of such a relation concerning the polarization state characterizing the fields that satisfy it.

1 Introduction

When referring to the concept of structured light, one finds in the literature a wide variety of works concerning the generation and application of custom light fields (see, for instance, Ref. [1] and references therein). The phenomenon arises whenever a vector light beam is focused by a high numerical aperture objective. In such cases, the electric field acquires a component along the optical axis, which eventually turns into a longitudinal component [2]. Accordingly, tightly focused vector fields will be able to exhibit complex and versatile polarization topologies.

The study of the field in the focal region of an optical system with high numerical aperture has received much attention in the last two decades, because of its potential applications in microlithography, image processing, diffractive optics, adaptive optics, or holographic data storage [2–19]. Furthermore, the analysis of the on-axis intensity of highly focused optical fields has also proven to be of much practical interest, because it allows us to design structures such that their behavior along the axis can be modulated [7, 9, 11, 14–16, 20].

Within this scenario, recently [21] a novel and convenient measure for the size of the region where the axial power content mainly concentrates has been proposed on the basis of an uncertainty principle. From it, it is also possible to determine the functional form of the incident beam that warrants the equality in the uncertainty relationship. Interestingly, this incident field has not a unique polarization structure.

In this Communication, we present some results about the polarization features of the focused field generated from such paraxial incident beams, characterized by different polarization states. This analysis includes global polarization features [22]. Furthermore, we investigate and discuss the influence on the polarization and transverse structure of the focused field as the σ-parameter is changed (note that this parameter contains information about the lower bound of the uncertainty principle). To be self-contained, in next section some basic definitions are introduced, while the main finding are discussed in Sec. 3

2 Basic definitions

To better understand the point of our contribution, let us first introduce a series of basic definitions. Thus, consider a monochromatic beam at the entrance pupil of an aplanatic focusing system with a high numerical aperture. Following the theory of vector field propagation, the electric field at the focal region can be written as

$$\mathbf{E}(r,\phi,z) \ = \ A \int_0^{\theta_0}\int_0^{2\pi} \mathbf{E}_0(\theta,\varphi) e^{ikr\sin\theta\cos(\phi-\varphi)} \times e^{-ikz\cos\theta} \sin\theta d\theta d\varphi, \qquad (1)$$

in terms of the so-called Richards-Wolf integral [23]. In this expression, (r,ϕ,z) denote the cylindrical coordinates at the focal area, with θ and φ being the polar and azimuthal angles at the reference Gaussian sphere, respectively. Furthermore, in (1) the input vector angular spectrum reads as

$$\mathbf{E}_0 = \sqrt{\cos\theta}\left[(\mathbf{E}_s \cdot \mathbf{e}_1)\,\mathbf{e}_1 + (\mathbf{E}_s \cdot \mathbf{e}_2')\,\mathbf{e}_2\right], \qquad (2)$$

where $\mathbf{E}_s$ is the transverse beam distribution at the entrance pupil of the optical system, while the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are defined as

$$\mathbf{e}_1 = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix}, \qquad \mathbf{e}_2 = \begin{pmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ \sin\theta \end{pmatrix}. \qquad (3)$$

These unit vectors point along the azimuthal and radial directions, respectively, while $\mathbf{e}_2' = (\cos\varphi, \sin\varphi, 0)$ denotes

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the projection of $\mathbf{e}_2$ onto the entrance pupil plane. As for the other parameters in (1), A is a constant, k is the wave number, and $\theta_0 = \max\{\theta\}$ is the semi-aperture angle, with the numerical aperture being $\mathrm{NA} = \sin\theta_0$.

Taking into account the field given by (2), the on-axis intensity distribution reads as

$$I(0,z) = |\mathbf{E}(0,z)|^2 = \left| \int_0^{\theta_0} \tilde{\mathbf{F}}(\theta) e^{-ikz\cos\theta} \sin\theta d\theta \right|^2, \quad (4)$$

where we introduce a new vector field, defined as

$$\tilde{\mathbf{F}}(\theta) \equiv A \int_0^{2\pi} \mathbf{E}_0(\theta,\varphi) d\varphi. \quad (5)$$

After recasting this new field in terms of a new variable $\alpha \equiv \cos\theta$ (with $\alpha_0 = \cos\theta_0$) and the electric field in terms of a dimensionless variable $\bar{z} \equiv kz$, the intensity (4) can be rewritten as

$$I(0,z) = |\mathbf{E}(0,\bar{z})|^2 = \left| \int_{\alpha_0}^{1} \tilde{\mathbf{F}}(\alpha) e^{-i\bar{z}\alpha} d\alpha \right|^2. \quad (6)$$

This expression highlights a dual relationship between the electric field, $\mathbf{E}(0,\bar{z})$, and the new field $\tilde{\mathbf{F}}(\alpha)$, with $\bar{z}$ and α playing the role of canonically conjugate variables.

3 Results

As it has recently been shown [21], the dispersion in the variable α of new vector field $\tilde{\mathbf{F}}(\alpha)$ and the axial power content satisfy an uncertainty relation. Upon the basis of this uncertainty principle, it has been proposed a novel and adequate measure for the size of the region where such an axial power content mainly concentrates. The functional form for the incident beam that satisfies the equality of such an uncertainty relation is given by

$$\tilde{\mathbf{F}}(\alpha) = e^{-\frac{\sigma}{2}\frac{(\alpha-\bar{\alpha})^2}{(1-\alpha_0)^2}} \mathbf{u}, \quad (7)$$

where $\mathbf{u}$ is a unitary constant vector, σ an arbitrary real, positive parameter, and

$$\bar{\alpha} = \frac{1}{I_0} \int_{\alpha_0}^{1} \alpha |\tilde{\mathbf{F}}(\alpha)|^2 d\alpha, \quad (8)$$

with

$$I_0 = \int_{\alpha_0}^{1} |\tilde{\mathbf{F}}(\alpha)|^2 d\alpha. \quad (9)$$

By inspecting the quantity (7), it is seen that, interestingly, incident fields with different polarization states give rise to the same on-axis irradiance distribution.

In particular, here we consider two polarization states for incident paraxial field, namely, linear (l) and circular (c). That is, in Eq. (2) we insert the expression

$$\mathbf{E}_{s,j}(\theta) = g(\theta)\mathbf{u}_j, \quad (10)$$

with $j = l, c$, and where

$$\mathbf{u}_l = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \mathbf{u}_c = \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad (11)$$

and

$$g(\theta) = \frac{F(\theta)}{\sqrt{\cos\theta}\,(1+\cos\theta)}. \quad (12)$$

In the latter expression, $F(\theta)$ denotes the modulus of the vector field defined in (7) – also remember that $\alpha = \cos\theta$, as defined earlier on, in Sec. 2.

If we consider the linearly polarized incident field, at the focal plane ($z = 0$) we find

$$\mathbf{E}_{0,l}(r,\phi) = \begin{pmatrix} D_0(r) + \cos 2\phi\, D_2(r) \\ \sin 2\phi\, D_2(r) \\ 2i\cos\phi\, D_1(r) \end{pmatrix} \quad (13)$$

where the following definitions have been introduced for simplicity:

$$D_0(r) = \int_0^{\theta_0} \sqrt{\cos\theta}\, g(\theta)(1+\cos\theta) J_2(kr\sin\theta) \sin\theta d\theta, \quad (14a)$$

$$D_1(r) = \int_0^{\theta_0} \sqrt{\cos\theta}\, g(\theta) \sin\theta J_1(kr\sin\theta) \sin\theta d\theta, \quad (14b)$$

$$D_2(r) = \int_0^{\theta_0} \sqrt{\cos\theta}\, g(\theta)(1-\cos\theta) J_2(kr\sin\theta) \sin\theta d\theta. \quad (14c)$$

As it can be noticed, the polarization state of the field (13) is fully nonuniform over the whole focal plane, since it depends at each point on ϕ. Therefore, the cylindrical symmetry around the optical axis is broken. We also notice that, at the focus ($r = 0$), the field is linearly polarized for any value of the σ-parameter. At any other point, the field is elliptically polarized, in general. Finally, the electric field at each point ($r \neq 0, \phi \neq \pi/2$) is contained within a plane defined by the normal

$$\mathbf{N}_l(r,\phi) = 2D_1(r)\cos\phi \begin{pmatrix} \sin 2\phi D_2(r) \\ -D_0 + \cos 2\phi D_0 \\ 0 \end{pmatrix}. \quad (15)$$

That is, the normal $\mathbf{N}_l$ is always contained within the XY-plane.

If we now consider the incident circular field, we find the following expression for the field at the focal plane:

$$\mathbf{E}_{0,c}(r,\phi) = \begin{pmatrix} D_0(r) + e^{2i\phi} D_2(r) \\ iD_0(r) - ie^{2i\phi} D_2(r) \\ 2ie^{i\phi} D_1(r) \end{pmatrix}. \quad (16)$$

where the same definitions (14) also hold. In this case, the field polarization is also totally nonuniform at the focal plane, although the polarization state is independent of ϕ, hence being the same along concentric rings around the optical axis. At the focus, $r = 0$, the field is circularly polarized for any value of the σ-parameter, while the polarization state in any other point is, in general, elliptical. Finally, it is also seen that the electric field is contained within the plane defined by the normal

$$\mathbf{N}_c(r,\phi) = \begin{bmatrix} 2D_1(r)\sin\phi \left(D_0(r) + D_2(r)\right) \\ -2D_1(r)\cos\phi \left(D_0(r) + D_2(r)\right) \\ D_0^2(r) - D_2^2(r) \end{bmatrix}, \quad (17)$$

with a z-component independent of ϕ.

4 Concluding remarks

As it is mentioned above, the purpose of this Communication is to present some results about the polarization features of the focused field generated from paraxial incident beams with different polarization states. In particular, we have considered linearly and circularly polarized incident fields, and have investigated the influence on the polarization and transverse structure of the focused field in terms of the σ-parameter, which contains information about the lower bound of the uncertainty principle introduced in Ref. [22]. In both cases, as discussed in Sec. 3, a series of common features are observed (although with some particularities proper of each polarization state). First, the field polarization at the focal plane is fully nonuniform, although circularly polarized fields preserve a rotational symmetry, which linearly polarized ones do not. Second, at the focus, the field preserves the polarization of the incident field for any value of the σ-parameter. Finally, the fields are contained within planes defined by the corresponding normal; in the case of linearly polarized incident fields this normal is perpendicular to the XY-plane, while for circularly polarized fields there is a z-component, although it is independent of ϕ, thus in compliance with the associated rotational symmetry (around the z-axis, for a given value of r).

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