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Single-spin asymmetry $A_T^{\sin(2\phi-\phi_S)}$ in $\pi^- p$ Drell-Yan process within TMD factorization

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Abstract. We study the single-spin asymmetry $A_T^{\sin(2\phi-\phi_S)}$ in the pion-induced Drell-Yan process within the transverse momentum dependent factorization (TMD factorization). The asymmetry can be expressed as the convolution of the Boer-Mulders function and the transversity function. We numerically estimate the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ at the COMPASS kinematics with the model results for the pion meson distributions from the light-cone wave function approach and the available parametrization for the proton distributions. We also include the TMD evolution formalism both proton and pion parton distribution functions by using two different parametrizations on nonperturbative Sudakov form factor. We find that the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ as functions of x_p, x_π, x_F and $q_\perp$ is qualitatively consistent with the recent COMPASS measurement.

1 Introduction

The T-odd Boer-Mulders function $h_1^\perp$ represents the transversely polarization asymmetry of quarks inside an unpolarized hadron, which manifests the novel structure of hadrons. In particular, a chiral-odd Boer-Mulders function convoluted with the transversity distribution h_1 can be used to describe the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ with ϕ_S the azimuthal angle of target transverse spin. Recently, the first measurement on the $\sin(2\phi - \phi_S)$ asymmetry has been performed by the COMPASS [1], which adopted a pion beam to collide on the transversely polarized nucleon target. Although no clear tendency is observed on the $\sin(2\phi - \phi_S)$ asymmetry due to relatively large statistical uncertainties, it indeed indicates that the asymmetry has a negative sign and substantial size.

In this work, we numerically estimate the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ at the COMPASS kinematics within the TMD factorization [2]. In the calculation, we adopt the model result for the distribution of the pion meson from the light-cone wave function approach, and we choose the available parametrization for the transversity distribution of the proton. Furthermore, we take into account the scale dependence of the pion Boer-Mulders function and the proton transversity function, for which we consider two different parameterizations on the nonperturbative Sudakov form factor.

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2 The TMD evolution of distribution functions

In the $\boldsymbol{b}_\perp$ space, the evolution of the TMD distribution function $\tilde{F}(x, b; \mu, \zeta_F)$ is encoded in Collins-Soper (CS) equation [2] and the renormalization group equation. The general solution for the energy dependence of $\tilde{F}$ can be obtained by solving those equations ($b = |\boldsymbol{b}_\perp|$)

$$\tilde{F}(x, b, Q) = \mathcal{F} \times e^{-S(Q,b)} \times \tilde{F}(x, b, \mu_i), \tag{1}$$

where $\mathcal{F}$ is the factor related to the hard scattering, $S(Q, b)$ is the Sudakov form factor. Hereafter, we set $\mu = \sqrt{\zeta_F} = Q$ and express $\tilde{F}(x, b; \mu = Q, \zeta_F = Q^2)$ as $\tilde{F}(x, b; Q)$ for simplicity. Eq. (1) demonstrates that the distribution $\tilde{F}$ at an arbitrary scale Q can be determined by the same distribution at an initial scale μ_i through the evolution encoded by the exponential form $\exp(-S(Q, b))$.

In order to access the information of $\tilde{F}(x, b; \mu, \zeta_F)$ in the whole b region, we adopt a convenient way to take into account the evolution behavior of $\tilde{F}(x, b; Q)$ in the large b region by including a nonperturbative Sudakov-like form factor $S_{\rm NP}$. Combining the perturbative part and the nonperturbative part, one has the complete result for the Sudakov form factor appearing in Eq. (1)

$$S(Q, b) = S_{\rm P}(Q, b) + S_{\rm NP}(Q, b). \tag{2}$$

with the boundary of the two parts set by the $b_{\rm max}$ to guarantee b_* is always at the perturbative region. In literature there are different prescription [3, 4, 8] for b_*. In the perturbative region, TMD PDFs at a fixed scale $\tilde{F}(x, b, \mu)$ can be expressed as the convolution of the perturbatively calculable coefficients C and the collinear counterparts of TMD PDFs $\tilde{F}_{q/H}(x, b; \mu) = \sum_i \int_x^1 \frac{d\xi}{\xi} C_{q\leftarrow i}(x/\xi, b; \mu) F_{i/H}(\xi, \mu)$. $F_{i/H}(\xi, \mu)$ is the corresponding collinear distribution at the dynamic scale μ which can be related to b_* by $\mu = c/b_*$, with $c = 2e^{-\gamma_E}$ and the Euler Constant $\gamma_E \approx 0.577$. $C_{q\leftarrow i}(x/\xi, b; \mu) = \sum_{n=0}^{\infty} C^{(n)}_{q\leftarrow i}(\alpha_s/\pi)^n$ is the perturbatively calculable coefficient function. Therefore, in the small b region, we can also express the Boer-Mulders function of the pion at a fixed energy scale μ as $\widetilde{h}^{\alpha\perp}_{1,q/\pi}(x, b; \mu) = (\frac{-ib^\alpha_\perp}{2}) T^{(\sigma)}_{q/\pi,F}(x, x; \mu)$, where the hard coefficients are calculated up to LO, and the collinear function $T^{(\sigma)}_{q/\pi,F}(x, x; \mu)$ is the chiral-odd twist-3 quark-gluon-quark correlation function and is related to the first transverse moment of the Boer-Mulders function $h^{\perp(1)}_{1,q/\pi}$ by $T^{(\sigma)}_{q/\pi,F}(x, x; \mu) = 2M_\pi h^{\perp(1)}_{1,q/\pi}$. Note that the perturbative part $S_{\rm P}(Q, b)$ can be expanded as α_s/π series, for which we take up to the accuracy of next-to-leading-logarithmic (NLL) order.

The nonperturbative part of the Sudakov form factor S_{NP} can not be calculated perturbatively, it is usually extracted from experimental data. There are several different parametrizations on S_{NP}, in this paper, we will discuss two of them. One is the SIYY parametrization [5],

$$S^{f_{1,q/p}}_{\rm NP}(Q, b) = \frac{g_1}{2} b^2 + \frac{g_2}{2} \ln\frac{b}{b_*} \ln\frac{Q}{Q_0}, \tag{3}$$

$$S^{f_{1,q/\pi}}_{\rm NP} = g^\pi_1 b^2 + g^\pi_2 \ln\frac{b}{b_*} \ln\frac{Q}{Q_0}, \tag{4}$$

where $S^{f_{1,q/p}}_{\rm NP}$ and $S^{f_{1,q/\pi}}_{\rm NP}$ correspond to the nonperturbative Sudakov form factors of the unpolarized TMD distribution of the proton and the pion, respectively. Here, b_* follows the choice in Refs. [4] and the parameters are fitted from the NN and $\pi^- N$ Drell-Yan process datas at the initial energy scale $Q_0^2 = 2.4$ GeV^2 yielding $g_1 = 0.212^{+0.006}_{-0.007}$, $g_2 = 0.84^{+0.040}_{-0.035}$, $g^\pi_1 = 0.082 \pm 0.022$, $g^\pi_2 = 0.394 \pm 0.103$ [6]. Note that the nonperturbative Sudakov form factor $S_{\rm NP}$ for quarks from one proton and antiquarks from another proton satisfies [7]

$$S^q_{\rm NP}(Q, b) + S^{\bar{q}}_{\rm NP}(Q, b) = S_{\rm NP}(Q, b). \tag{5}$$

The other parametrization is the Bacchetta-Delcarro-Pisano-Radici-Signori (BDPRS) parametrization [8],

$$\tilde{f}_1^a(x,b^2;Q^2) = f_1^a(x;\mu^2)e^{-S(\mu^2,Q^2)}e^{\frac{1}{2}g_K(b)\ln(Q^2/Q_0^2)}\tilde{f}_{1\text{NP}}^a(x,b^2), \tag{6}$$

where $g_K = -g_2b^2/2$. $\tilde{f}_{1\text{NP}}^a(x,b^2)$ is the intrinsic nonperturbative part of the PDFs, which is parameterized as

$$\tilde{f}_{1\text{NP}}^a(x,b^2) = \frac{1}{2\pi}e^{-g_1\frac{b^2}{4}}(1-\frac{\lambda g_1^2}{1+\lambda g_1}\frac{b^2}{4}), \tag{7}$$

with

$$g_1(x) = N_1\frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}. \tag{8}$$

Here, the fitted results of the parameters are $\alpha = 2.95$, $\sigma = 0.17$, $\lambda = 0.86\ \text{GeV}^{-2}$, $g_1(\hat{x}) = 0.28\ \text{GeV}^2$, with $N_1 \equiv g_1(\hat{x})$, $\hat{x} = 0.1$. The b_* prescription in this parametrization follow the choice in Refs. [8].

2.1 Formalism of the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ in Drell-Yan process

In the pion-induced Drell-Yan process $\pi^-(P_\pi) + p^\uparrow(P_p) \longrightarrow \gamma^*(q) + X \longrightarrow l^+(\ell) + l^-(\ell') + X$, the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ is defined as [9]:

$$A_T^{\sin(2\phi-\phi_S)}(x_1,x_2,Q) = \frac{F_T^{\sin(2\phi-\phi_S)}(x_1,x_2,Q)}{F_U^1(x_1,x_2,Q)}. \tag{9}$$

The denominator can expressed as the convolution of the unpolarized distribution functions from each hadron $F_U^1 = \mathcal{C}[f_{1,q/\pi}f_{1,\bar{q}/p}]$, while the numerator is expressed as the convolution of the pion Boer-Mulders distribution and the proton transversity distribution $F_T^{\sin(2\phi-\phi_S)} = -\mathcal{C}[\frac{\boldsymbol{h}\cdot\boldsymbol{k}_{a\perp}}{M_\pi}h_{1,q/\pi}^\perp h_{1,\bar{q}/p}]$ with $\boldsymbol{h} = \hat{\boldsymbol{q}} \equiv \boldsymbol{q}_\perp/|\boldsymbol{q}_\perp|$. After dealing with the convolution and performing the Fourier transformation, we can get the following expressions for the structure functions [10]

$$F_T^{\sin(2\phi-\phi_S)} = -\frac{1}{N_c}\sum_q e_q^2\int_0^\infty \frac{db}{4\pi}b^2 J_1(q_\perp b)h_{1,q/p}(x_p,\mu_b)T_{\bar{q}/\pi,F}^{(\sigma)}(x_\pi,x_\pi,\mu_b)e^{-\left(S_{\text{NP}}^{f_{1,q/p}}+S_{\text{NP}}^{f_{1,q/\pi}}+S_{\text{P}}\right)} + (q\leftrightarrow\bar{q}),$$
$$F_U^1 = \frac{1}{N_c}\sum_q e_q^2\int_0^\infty \frac{bdb}{2\pi}J_0(q_\perp b)f_{1,q/\pi}(x_\pi,\mu_b)f_{1,\bar{q}/p}(x_p,\mu_b)e^{-\left(S_{\text{NP}}^{f_{1,q/p}}+S_{\text{NP}}^{f_{1,q/\pi}}+S_{\text{P}}\right)} + (q\leftrightarrow\bar{q}). \tag{10}$$

As there is no extraction on the Boer-Mulders function of the pion meson, we apply a model result based on the light-cone wave function of the pion meson from Ref. [11] at the model scale $\mu_0^2 = 0.25\text{GeV}^2$. For consistency, we apply the unpolarized distribution function of the pion meson $f_{1\pi}(x)$ using the same model. We also apply the result from the light-cone constituent quark model in Ref. [12] for comparison.

For the collinear distributions of the proton, we resort to existing parameterizations, i.e., we adopt the NLO set of the CT10 parametrization (central PDF set) for the unpolarized distribution function $f_1(x)$ of the proton, and we choose the transversity distribution extracted from SIDIS data via the TMD evolution formalism [4].

Furthermore, we apply the QCDNUM evolution package [13] to perform the evolution of $f_{1,q/\pi}$ from the model scale μ_0 to another energy. As for the energy evolution of the twist-3

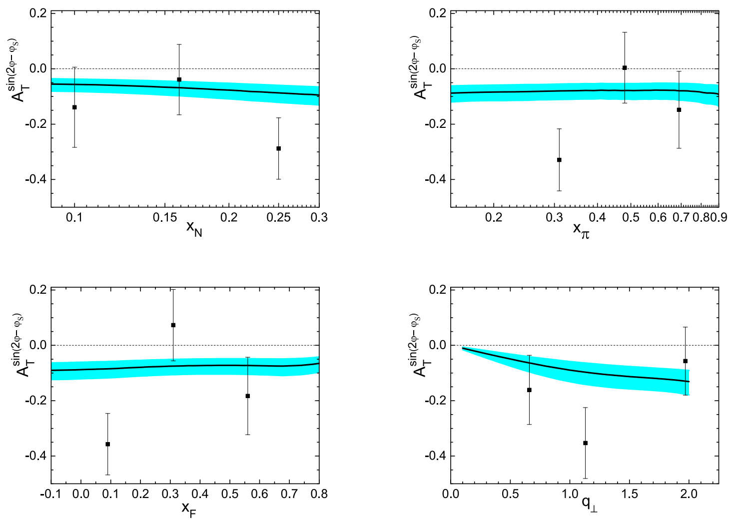

Figure 1. The asymmetry $A_T^{\sin(2\phi-\phi_S)}$ in the $\pi^- N^\uparrow$ Drell-Yan process calculated from the SIYY parametrization on the nonperturbative form factor. The four figures correspond to the asymmetries as functions of x_N (upper left), x_π (upper right), x_F (lower left) and $q_\perp$ (lower right). The solid lines show the results from the central values of the parameters, while the shaded area show the uncertainty bands determined by the uncertainties of the parameters. The solid squares represent the COMPASS data for comparison.

collinear correlation function $T_{q,F}^{(\sigma)}$, we follow the same choice in Ref. [11]. Similarly, we also include the kernel in Ref. [4] to evolve the transversity distribution of the proton.

With all the ingredients above, we calculate the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ in the pion-induced transversely polarized Drell-Yan process at the COMPASS kinematics and compare with data [1]. In Fig. 1, we plot our numerical results of the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ using the SIYY parametrization [Eqs. (3) and (4)] for the nonperturbative Sudakov form factor and the pion Boer-Mulders function from Ref. [11]. The solid curves correspond to the results calculated from the central values of the parameters, while the shaded area shows the uncertainty band determined by the uncertainties of the parameters. To make the TMD factorization valid in the kinematic region, the integration over the transverse momentum $q_\perp$ is performed in the region of $0 < q_\perp < 2$ GeV [14]. In the figure the solid squares show the experimental data measured by the COMPASS Collaboration [15], with the error bars corresponding to the sum of the systematic error and the statistical error.

As shown in Fig. 1, in all the cases the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ in the $\pi^- P$ Drell-Yan from our calculation is negative, in agreement with most of the data from COMPASS. Our estimate also shows that the asymmetry changes slightly with the change of x_N, x_π, or x_F, and the magnitude of the x_N-, x_π-, and x_F-dependent asymmetries is around 0.05 to 0.10. For the $q_\perp$ asymmetry, we find that its magnitude is about 0.05 to 0.15 and moderately increases with increasing $q_\perp$ in the region $q_\perp < 2$ GeV.

To study the impact of different parameterizations of the nonperturbative part on the asymmetry, in Fig. 2, we present the result using the BDPRS evolution formalism [Eqs. (6) and (7)] by the dashed lines. The solid lines show the results in Fig. 1 (central results) for comparison. We find that, in the case of $q_\perp$-dependent asymmetry, the result from the BD-

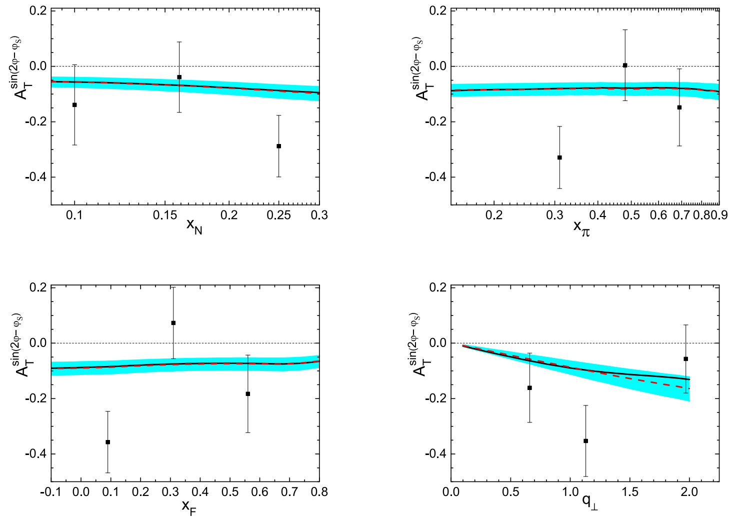

Figure 2. Similar to Fig. 1, but the asymmetry calculated from the BDPRS parametrization on the nonperturbative form factor. The dashed lines plot the central results, while the solid lines are the central results in Fig. 1 for comparison.

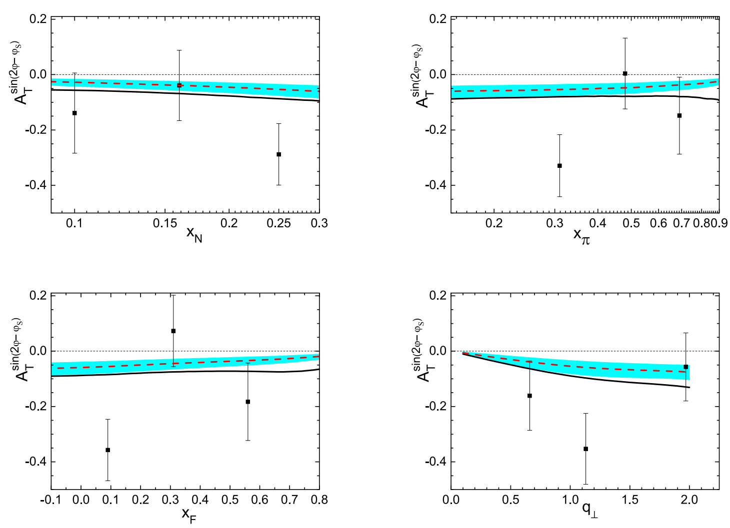

Figure 3. Similar to Fig. 1, but the asymmetry calculated from the Boer-Mulders function of the pion in a light-cone constituent model [12]. The dashed lines plot the central results, while the solid lines are the central results in Fig. 1 for comparison.

PRS parametrization is qualitatively different from the result from the SIYY parametrization, particularly in the region $q_\perp \in [1.5, 2]$ GeV; while for the x_N-, x_π-, and x_F-dependent asymmetries the results from the two evolution formalisms are consistent.

To study the effect of different pion distribution functions on the numerical calculation of $A_T^{\sin(2\phi-\phi_S)}$, in Fig. 3 (the dashed lines), we adopt the pion Boer-Mulders function obtained from the light-front constituent quark model [12] and the SIYY parametrization to perform the calculation. The solid lines correspond to the results in Fig. 1 (central results) for comparison. We find that the sign of the asymmetries from the model results for pion distributions in Ref. [12] are still negative, while their magnitudes are generally smaller than those in Fig. 1.

2.2 Conclusion

In this work, we applied the TMD factorization approach to study the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ in $\pi^- p$ Drell-Yan process at the kinematical region of the COMPASS. The asymmetry arises from the convolution of the pion Boer-Mulders function and the proton transversity distribution. In the calculation, we applied two different approaches (SIYY and BDPRS parametrizations) to perform the TMD evolution of pion and proton distributions for comparison. Their main difference is the treatment on the nonperturbative part of evolution, while the perturbative part is the same and has been kept at NLL accuracy in this work. As the nonperturbative part associated with the pion Boer-Mulders function is still unknown, we assume that it has the same form as that the unpolarized distribution function. The hard coefficients associated with the corresponding collinear functions in the TMD evolution formalism are kept at leading-order accuracy. We then calculated the asymmetry $A_T^{\sin(2\phi-\phi_S)}$ with the available parametrization for the proton distribution and two different model results from the light-cone wave function approach for pion meson distribution. we find that the asymmetry is sensitive to the choice of the pion distribution function, while different choice of the TMD evolution formalism will only on the nonperturbative TMD evolution only affect the shape of the $q_\perp$-dependent of the asymmetry. We also find that the corresponding asymmetry $A_T^{\sin(2\phi-\phi_S)}$ is consistent with the COMPASS measurement in sign and magnitude. Furthermore, our study may provide a framework to access the Boer-Mulders function of the pion and the corresponding nonperturbative Sudakov form factor through transversely polarized $\pi^- p$ data.

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