Inclusive Hadroproduction of *P*-wave Heavy Quarkonia in pNRQCD

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Abstract. We compute NRQCD long-distance matrix elements that appear in the inclusive production cross sections of *P*-wave heavy quarkonia in the framework of potential NRQCD. The formalism developed in this work applies to strongly coupled charmonia and bottomonia. This makes possible the determination of color-octet NRQCD long-distance matrix elements without relying on measured cross section data, which has not been possible so far. We obtain results for inclusive production cross sections of χ_{cJ} and χ_{bJ} at the LHC, which are in good agreement with measurements.

1 Introduction

Hard processes involving heavy quarkonia are considered good probes of perturbative and nonperturbative aspects of QCD [1–4]. Especially, inclusive production cross sections of heavy quarkonia are studied extensively in hadron colliders. A good theoretical understanding of the quarkonium production mechanism is therefore important in deciphering QCD interactions from collider experiments.

The effective field theory nonrelativistic QCD (NRQCD) [5, 6] provides a factorization formalism for hard processes involving heavy quarkonia, which include inclusive production cross sections at large transverse momentum. NRQCD utilizes the separation of the scale of the heavy quark mass m from the scales mv and mv^2, where $v \ll 1$ is the relative velocity of the heavy quark Q and the antiquark $\bar{Q}$ inside the heavy quarkonium. The NRQCD factorization formula for inclusive production rate of a quarkonium Q can be written as

$$\sigma_{Q+X} = \sum_n \hat{\sigma}_{Q\bar{Q}(n)+X} \langle O^Q(n) \rangle, \tag{1}$$

where the $\hat{\sigma}_{Q\bar{Q}(n)+X}$ are short-distance cross sections (SDCSs) for inclusive production of a $Q\bar{Q}$ pair in a specific color and angular momentum state n, while the long-distance matrix elements (LDMEs) $\langle O^Q(n) \rangle$ correspond to the nonperturbative probabilities for the $Q\bar{Q}$ to evolve into the quarkonium Q. The LDMEs have known scalings in v, so that the sum (1) can be organized in powers of v, and is typically truncated at a desired accuracy in v. While the short-distance cross sections can be computed in perturbative QCD as series in the strong coupling α_s, the LDMEs must be determined nonperturbatively.

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While it has been known how to compute the color-singlet LDMEs, determinations of the color-octet LDMEs have usually been done by relying on measured cross section data, because it has not been known so far how to compute them from first principles. This approach led to inconsistent sets of LDME determinations that do not provide a comprehensive description of the important observables involving inclusive production of heavy quarkonia [7]. Therefore, it is much desirable to be able to compute color-octet LDMEs from first principles.

In the strongly coupled regime (characterized by the hierarchy $\Lambda_{\text{QCD}} \gg mv^2$), the potential NRQCD (pNRQCD) effective field theory [8–10] which is obtained by further integrating out the scale mv, provides expressions for the LDMEs for decays of heavy quarkonia into light particles in terms of quarkonium wavefunctions and gluonic correlators [11–13]. This greatly enhances the predictive power of the NRQCD factorization formalism, as the wavefunctions can be computed by solving the nonrelativistic Schrödinger equation (see, for example, refs. [14, 15]), and the gluonic correlators are universal quantities that do not depend on the heavy quark flavor or the radial excitation, and in principle can be computed in lattice QCD. It has been anticipated that a similar calculation could be done for the LDMEs for inclusive production cross sections. This formidable task has been accomplished for the first time in ref. [16] for the production of strongly coupled P-wave heavy quarkonia, and a general formalism for strongly coupled quarkonia of arbitrary quantum numbers has been established in ref. [17]. Similarly to the case of decay LDMEs, the results in refs. [16, 17] provide expressions for the production LDMEs in terms of quarkonium wavefunctions and universal gluonic correlators. These results greatly reduce the number of nonperturbative unknowns and enhances the predictive power of the NRQCD factorization formalism for inclusive production of heavy quarkonia.

2 Inclusive production of χ_{cJ} and χ_{bJ} in pNRQCD

The NRQCD factorization formula for the inclusive production rate of χ_{QJ} ($Q = c$ or b) at leading order in v is given by [6]

$$\sigma_{\chi_{QJ}} = (2J + 1) \left[\hat{\sigma}_{Q\bar{Q}(^3P_J^{[1]})} \langle O^{\chi_{Q0}}(^3P_0^{[1]}) \rangle + \hat{\sigma}_{Q\bar{Q}(^3S_1^{[8]})} \langle O^{\chi_{Q0}}(^3S_1^{[8]}) \rangle \right], \tag{2}$$

where the LDMEs are defined by [6, 18–20]

$$\langle O^{\chi_{Q0}}(^3P_0^{[1]}) \rangle = \langle \Omega | \frac{1}{3} \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma} \right) \psi \, \mathcal{P}_{\chi_{Q0}(\mathbf{P=0})} \, \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma} \right) \chi | \Omega \rangle, \tag{3a}$$

$$\langle O^{\chi_{Q0}}(^3S_1^{[8]}) \rangle = \langle \Omega | \chi^\dagger \sigma^i T^a \psi \Phi_\ell^{\dagger ab} \, \mathcal{P}_{\chi_{Q0}(\mathbf{P=0})} \, \Phi_\ell^{bc} \psi^\dagger \sigma^i T^c \chi | \Omega \rangle. \tag{3b}$$

Here, $|\Omega\rangle$ is the QCD vacuum, ψ and χ are Pauli spinor fields that annihilate and create a heavy quark and antiquark, respectively, $\mathbf{D} = \boldsymbol{\nabla} - ig\mathbf{A}$ is the gauge-covariant derivative, $\chi^\dagger \overleftrightarrow{\mathbf{D}} \psi = \chi^\dagger \mathbf{D}\psi - (\mathbf{D}\chi)^\dagger \psi$, $\mathbf{A}$ is the gluon field, $\boldsymbol{\sigma}$ is a Pauli matrix, T^a is a color matrix, and the operator $\mathcal{P}_{\chi_{Q0}(\mathbf{P})}$ projects onto states that include the quarkonium χ_{Q0} with momentum $\mathbf{P}$. The gauge-completion Wilson line defined by $\Phi_\ell = \mathcal{P} \exp\left[-ig \int_0^\infty d\lambda \ell \cdot A^{\text{adj}}(\ell\lambda)\right]$, where $\mathcal{P}$ is path ordering, A^{adj} is the gluon field in the adjoint representation, and ℓ is an arbitrary direction, is inserted to ensure the gauge invariance of the color-octet LDME [18–20]. We have used the heavy-quark spin symmetry to write eq. (2) in terms of the χ_{Q0} LDMEs [6]. An analogous formula holds for the production of h_Q.

The pNRQCD expression for the production LDMEs can be written as [17]

$$\langle \Omega | O^Q(n) | \Omega \rangle = \frac{1}{\langle \mathbf{P} = 0 | \mathbf{P} = 0 \rangle} \int d^3x_1 d^3x_2 d^3x_1' d^3x_2' \, \phi_Q^{(0)}(\mathbf{x}_1 - \mathbf{x}_2)$$
$$\times \left[-V_{O(n)}(\mathbf{x}_1, \mathbf{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2) \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_1') \delta^{(3)}(\mathbf{x}_2 - \mathbf{x}_2') \right] \phi_Q^{(0)*}(\mathbf{x}_1' - \mathbf{x}_2'), \tag{4}$$

where $V_{O(n)}$ is a contact term, and $\phi_Q^{(0)}(\mathbf{x}_1 - \mathbf{x}_2)$ is the unit-normalized quarkonium wavefunction at leading order in v. The wavefunction is an eigenfunction of the pNRQCD Hamiltonian, which is given at leading order in v by

$$-\frac{\nabla_1^2}{2m} - \frac{\nabla_2^2}{2m} + V^{(0)}(\mathbf{x}_1 - \mathbf{x}_2), \tag{5}$$

where $V^{(0)}$ is the static potential. The expression in eq. (4) is valid at leading order in v, up to corrections of relative order $1/N_c^2$, where N_c is the number of colors. We refer the readers to ref. [17] for details of the pNRQCD formalism for production LDMEs.

The contact term $V_{O(n)}$ can be computed in expansion in powers of $1/m$. At leading nonvanishing orders in $1/m$, the contact terms for the LDMEs that appear in eq. (2) are given by [17]

$$-V_{O(^3P_0^{[1]})}\Big|_{P-\text{wave}} = -\frac{1}{3}\sigma^i \otimes \sigma^j N_c \nabla_{\mathbf{r}}^i \delta^{(3)}(\mathbf{r})\nabla_{\mathbf{r}}^j, \tag{6a}$$

$$-V_{O(^3S_1^{[8]})}\Big|_{P-\text{wave}} = -\sigma^k \otimes \sigma^k N_c \nabla_{\mathbf{r}}^i \delta^{(3)}(\mathbf{r})\nabla_{\mathbf{r}}^j \frac{\mathcal{E}^{ij}}{N_c^2 m^2}, \tag{6b}$$

where $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$, and we neglect contributions that vanish in (4) when $\phi_Q^{(0)}(\mathbf{x}_1 - \mathbf{x}_2)$ is in a P-wave state. The tensor $\mathcal{E}^{ij}$ is defined by

$$\mathcal{E}^{ij} = \int_0^\infty dt\, t \int_0^\infty dt'\, t' \,\langle\Omega|\Phi_\ell^{\dagger ab}\Phi_0^{\dagger ad}(0;t)gE^{d,i}(t)gE^{e,j}(t')\Phi_0^{ec}(0;t')\Phi_\ell^{bc}|\Omega\rangle, \tag{7}$$

where $E^{i,a}(t) = E^{i,a}(t,\mathbf{0}) = G^{i0,a}(t,\mathbf{0})$ is the chromoelectric field, $G^{\mu\nu,a}T^a = G^{\mu\nu}$ is the gluon field strength tensor, and $\Phi_0(t;t') = \mathcal{P}\exp\left[-ig\int_t^{t'} d\tau A_0^{\text{adj}}(\tau,\mathbf{0})\right]$ is a Schwinger line.

By using the results for the contact terms $V_{O(n)}$ and eq. (4), we obtain the following expressions for the production LDMEs in pNRQCD [16, 17]

$$\langle O^{\chi_{Q0}}(^3P_0^{[1]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2, \tag{8a}$$

$$\langle O^{\chi_{Q0}}(^3S_1^{[8]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\frac{\mathcal{E}}{9N_c m^2}, \tag{8b}$$

where $R^{(0)}(r)$ is the radial wavefunction, and $\mathcal{E} = \frac{3}{N_c}\delta^{ij}\mathcal{E}^{ij}$ is a gluonic correlator. We expect corrections to these expressions to be suppressed by v^2. Since v^2 is comparable to $1/N_c^2$ for bottomonia, while v^2 is larger than $1/N_c^2$ for charmonia, we take the uncertainties in the pNRQCD expressions of the LDMEs to be of relative order v^2.

The correlator $\mathcal{E}$ has a logarithmic scale dependence at one loop level, which is given by

$$\frac{d}{d\log\Lambda}\mathcal{E}(\Lambda) = 12C_F\frac{\alpha_s}{\pi} + O(\alpha_s^2), \tag{9}$$

where $C_F = (N_c^2 - 1)/(2N_c)$. This implies the following evolution equation for the LDMEs

$$\frac{d}{d\log\Lambda}\langle O^{\chi_{Q0}}(^3S_1^{[8]})\rangle = \frac{4C_F\alpha_s}{3N_c\pi m^2}\langle O^{\chi_{Q0}}(^3P_0^{[1]})\rangle. \tag{10}$$

This agrees with the known one-loop evolution equation derived from perturbative calculations in NRQCD [6]. The consistency with NRQCD factorization at two-loop level has been discussed in ref. [17].

Because the operator definition for $\mathcal{E}$ involves only gluon fields, it is independent of the radial excitation or the flavor of the heavy quark. Hence, the ratio of the color-singlet and color-octet LDMEs at leading order in v given by

$$\frac{m^2 \langle O^{\chi_{Q0}}(^3S_1^{[8]}) \rangle}{\langle O^{\chi_{Q0}}(^3P_0^{[1]}) \rangle} = \frac{\mathcal{E}}{9N_c}, \tag{11}$$

is universal for all P-wave quarkonium states. This, in turn, implies that a determination of $\mathcal{E}$ leads to determination of both color-singlet and color-octet production LDMEs for all P-wave charmonium and bottomonium states.

2.1 Inclusive hadroproduction of χ_{cJ}

We now present the phenomenological results for hadroproduction rates of $\chi_{cJ}(1P)$ ($J = 1$, 2) at the LHC. We employ the SDCSs computed at next-to-leading order accuracy in α_s from ref. [21], and take $\Lambda = 1.5$ GeV for the $\overline{MS}$ scale associated with the color-octet LDME.

Since a lattice QCD calculation of $\mathcal{E}$ has not yet been done, we determine $\mathcal{E}$ by comparing the cross sections computed from eq. (2) with the measured cross section ratio $r_{21} = (d\sigma_{\chi_{c2}(1P)}/dp_T)/(d\sigma_{\chi_{c1}(1P)}/dp_T)$ at the LHC, where p_T is the transverse momentum of the χ_{cJ}. Note that the ratio is independent of the radial wavefunction. We obtain [17]

$$\mathcal{E}(\Lambda = 1.5 \text{ GeV})|_{\text{NLO}} = 1.17 \pm 0.05. \tag{12}$$

Since the fixed-order calculations of the SDCSs may contain large logarithms of p_T/m, resummation of the logarithms can have significant effects on their shapes in p_T. If we use the SDCSs from ref. [22, 23] which include resummed leading logarithms of p_T/m at leading power (LP) in m/p_T, we obtain [17]

$$\mathcal{E}(\Lambda = 1.5 \text{ GeV})|_{\text{LP+NLO}} = 4.48 \pm 0.14. \tag{13}$$

We compare the pNRQCD result for $r_{21} \times B_{\chi_{c2}(1P)}/B_{\chi_{c1}(1P)}$ obtained in ref. [17] as a function of the transverse momentum $p_T^{J/\psi}$ of the J/ψ produced in decays of χ_{cJ}, where $B_{\chi_{cJ}(1P)} = \text{Br}[\chi_{cJ}(1P) \rightarrow J/\psi\gamma] \times \text{Br}(J/\psi \rightarrow \mu^+\mu^-)$, with measurements at the LHC in figure 1.
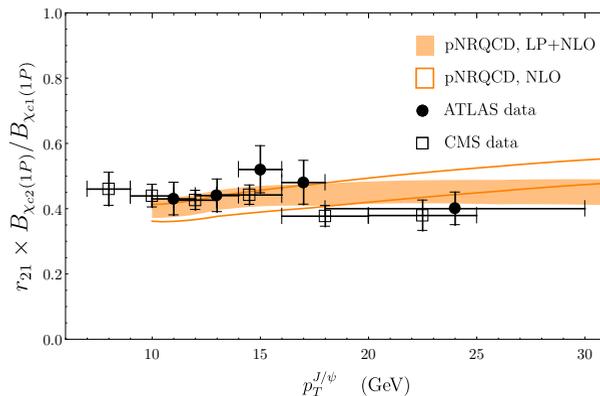

Figure 1. pNRQCD results for the cross section ratio $r_{21} \times B_{\chi_{c2}(1P)}/B_{\chi_{c1}(1P)}$ at the $\sqrt{s} = 7$ TeV LHC in the rapidity region $|y| < 0.75$ compared to CMS [24] and ATLAS [25] measurements. From ref. [17].

Based on the results for $\mathcal{E}$ that we obtain, we can compute absolute cross sections of χ_{cJ} at the LHC. We use $|R^{(0)\prime}(0)|^2 = 0.057$ GeV^5, which is determined in ref. [17] from the two-photon widths of χ_{c1} and χ_{c2} at one-loop level and measurements in ref. [26]. We show the pNRQCD results for the hadroproduction cross sections at the LHC from ref. [17] compared to ATLAS [25] measurements in figure 2.
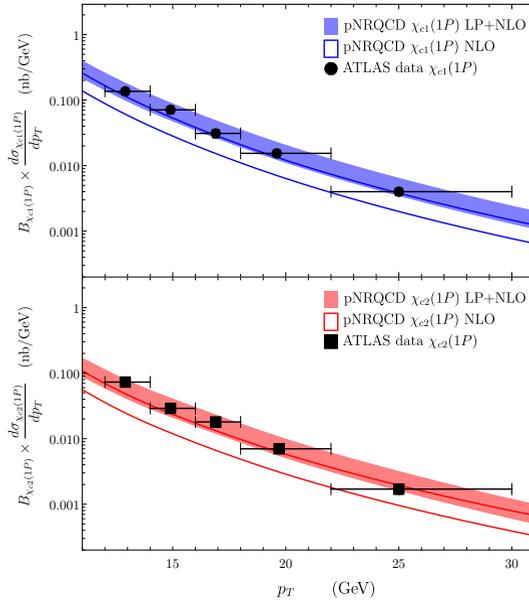

Figure 2. pNRQCD results for the production cross sections of $\chi_{c1}(1P)$ and $\chi_{c2}(1P)$ at the $\sqrt{s} = 7$ TeV LHC in the rapidity region $|y| < 0.75$ compared to ATLAS [25] measurements. From ref. [17].

Finally, we consider the polarization of the J/ψ from decays of $\chi_{c1}(1P)$ and $\chi_{c2}(1P)$. The polarization parameter $\lambda_\theta^{\chi_{cJ}}$ is defined by $\lambda_\theta^{\chi_{cJ}} = (1 - 3\xi_{\chi_{cJ}})/(1 + \xi_{\chi_{cJ}})$, where $\xi_{\chi_{cJ}}$ is the fraction of longitudinally produced J/ψ from decays of $\chi_{cJ}(1P)$. We compare in figure 3 the pNRQCD results for $\lambda_\theta^{\chi_{c1}}$ and $\lambda_\theta^{\chi_{c2}}$ in ref. [17] with the experimental constraints from CMS [27].

2.2 Inclusive hadroproduction of χ_{bJ}

We now compute the hadroproduction rates of $\chi_{bJ}(nP)$ at the LHC, where $J = 1, 2$ and $n = 1$, 2, and 3. Similarly to the case of χ_{cJ}, we employ the SDCSs computed at next-to-leading order accuracy in α_s from ref. [21], and take $\Lambda = 4.75$ GeV for the $\overline{\text{MS}}$ scale associated with the color-octet LDME. We compute $\mathcal{E}(\Lambda = 4.75$ GeV$)$ from $\mathcal{E}(\Lambda_0 = 1.5$ GeV$)$ by using the one-loop renormalization-group improved formula

$$\mathcal{E}(\Lambda) = \mathcal{E}(\Lambda_0) + \frac{24 C_F}{\beta_0} \log \frac{\alpha_s(\Lambda_0)}{\alpha_s(\Lambda)}, \tag{14}$$

where $\beta_0 = 11 N_c/3 - 2n_f/3$ and $n_f = 4$ is the number of active quark flavors. We take the average of eq. (12) and eq. (13) to obtain $\mathcal{E}(\Lambda_0 = 1.5$ GeV$) = 2.8 \pm 1.7$. From this we can compute the ratio r_{21} of the $\chi_{b1}(1P)$ and $\chi_{b2}(1P)$, which is independent of the radial wavefunction. We compare the pNRQCD result for the ratio r_{21} from ref. [17] as a function of the transverse momentum $p_T^{\Upsilon(1S)}$ of the $\Upsilon(1S)$ from decays of χ_{bJ} with LHCb [28] and CMS [29] measurements in figure 4.
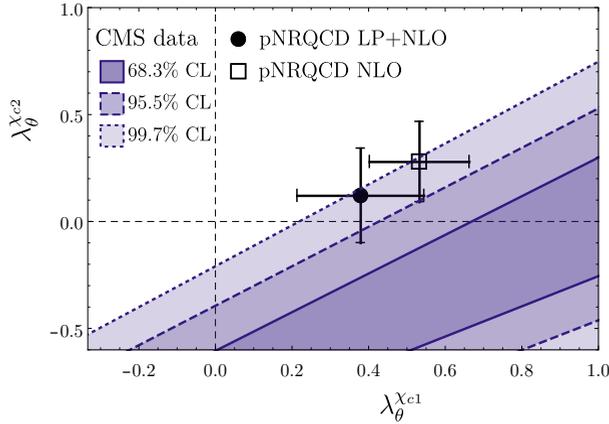
Figure 3. pNRQCD results for the polarization parameters $\lambda_\theta^{\chi_{cJ}}$ for $J = 1$ and 2 at the $\sqrt{s} = 7$ TeV LHC averaged over 8 GeV $< p_T^{J/\psi} < 30$ GeV compared with the experimental constraints from CMS [27]. From ref. [17].
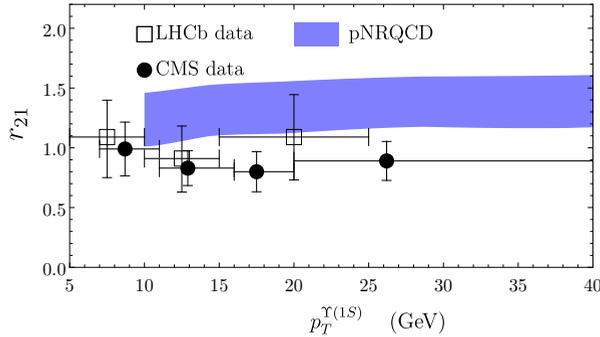

Figure 4. pNRQCD results for the cross section ratio r_{21} for the $\chi_{b1}(1P)$ and $\chi_{b2}(1P)$ states at the $\sqrt{s} = 7$ TeV LHC compared with LHCb and CMS data. From ref. [17].

In order to compute absolute cross sections of $\chi_{bJ}(nP)$, we take the values for $|R^{(0)\prime}(0)|^2$ from the averages of potential-model calculations considered in ref. [13], which are given by $|R_{1P}^{(0)\prime}(0)|^2 = 1.47$ GeV^5, $|R_{2P}^{(0)\prime}(0)|^2 = 1.74$ GeV^5, and $|R_{3P}^{(0)\prime}(0)|^2 = 1.92$ GeV^5. The results obtained in ref. [17] for the absolute cross sections of $\chi_{b1}(nP)$ and $\chi_{b2}(nP)$ for $n = 1$, 2, and 3 are shown in the left panel of figure 5. In order to compare with available measurements from LHCb [30], we compute the feeddown fractions $R_{\Upsilon(n'S)}^{\chi_b(nP)}$ for $n \geq n'$, which are the fractions of $\Upsilon(n'S)$ produced from decays of $\chi_b(nP)$. The fractions $R_{\Upsilon(n'S)}^{\chi_b(nP)}$ are computed from the $\chi_{cJ}(nP)$ production cross sections multiplied by the branching ratios $\mathrm{Br}[\chi_{cJ}(nP) \rightarrow \Upsilon(n'S) + \gamma]$, divided by the the inclusive production rate of $\Upsilon(n'S)$. We take the direct $\Upsilon(n'S)$ production cross sections computed in ref. [31], from which we compute the inclusive production rates by adding the feeddown contributions from decays of $\chi_b(nP)$ and $\Upsilon(n''S)$, where $n \geq n'$ and $n'' \geq n' + 1$. We show the results for $R_{\Upsilon(n'S)}^{\chi_b(nP)}$ at the $\sqrt{s} = 7$ TeV LHC from ref. [17] compared with LHCb [30] data in the right panel of figure 5.
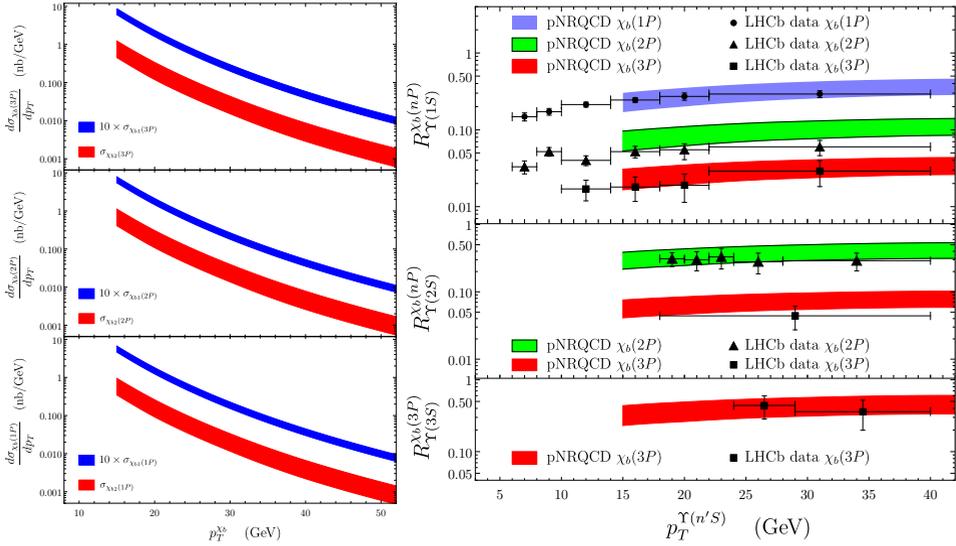
Figure 5. Left panel : pNRQCD results for the inclusive production rates of $\chi_{bJ}(nP)$ ($J = 1$, 2 and $n = 1$, 2, and 3) at the $\sqrt{s} = 7$ TeV LHC in the rapidity range $2 < y < 4.5$. Right panel : pNRQCD results for the feeddown fractions $R^{\chi_b(nP)}_{\Upsilon(n'S)}$ compared to LHCb [30] data. From ref. [17].

3 Summary and outlook

We reviewed recent results for inclusive hadroproduction of *P*-wave heavy quarkonia in potential NRQCD. By working in the strong coupling regime, expressions for the NRQCD LDMEs for inclusive production of heavy quarkonia have been obtained in terms of quarkonium wavefunctions and universal gluonic correlators. This greatly reduces the number of nonperturbative unknowns, thanks to the universal nature of the gluonic correlators. In the case of *P*-wave heavy quarkonia, the LDMEs that appear in NRQCD factorization formulas at leading order in v can be determined from the quarkonium wavefunctions and a single gluonic correlator $\mathcal{E} = \frac{3}{N_c}\delta^{ij}\mathcal{E}^{ij}$, where the tensor $\mathcal{E}^{ij}$ is defined in eq. (7). Based on this result, the hadroproduction cross sections of χ_{cJ} and χ_{bJ} have been computed in refs. [16, 17] by using a determination of $\mathcal{E}$ from measurements of the ratio of χ_{c1} and χ_{c2} cross sections at the LHC. The phenomenological results are in reasonable agreement with available LHC data.

The formalism developed in ref. [17] for the LDMEs can be applied to inclusive production rates of any strongly coupled heavy quarkonium state. It would especially be interesting to compute in potential NRQCD the production LDMEs for *S*-wave quarkonia such as J/ψ, $\psi(2S)$, and Υ, which may shed light on the long-standing puzzle of J/ψ polarization and help understand the η_c production mechanism [32–36].

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