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A Virtual Tribute to Quark Confinement and the Hadron Spectrum (vConf21)

Medium evolution of a static quark-antiquark pair in the large N_c limit

Miguel Ángel Escobedo[1,*]

[1]Instituto Galego de Física de Altas Enerxías (IGFAE), Universidade de Santiago de Compostela. E-15782, Galicia, Spain

Abstract. We study the transitions between the different color states of a static quark-antiquark pair, singlet and octet, in a thermal medium. This is done non-perturbatively exploiting the infinite mass limit of QCD. This study is interesting because it can be used for future developments within the framework of Effective Field Theories (EFTs) and because it can be combined with other techniques, like lattice QCD or AdS/CFT, to gain non-perturbative information about the evolution of quarkonium in a medium. We also study the obtained expressions in the large N_c limit. This allows us to learn lessons that are useful to simplify phenomenological models of quarkonium in a plasma. More details can be found in [1].

1 Introduction

One of the most interesting probes of the medium created in heavy-ion collisions is heavy quarkonium. There are several features that make this particle interesting in the context of quark-gluon plasma studies. The mass of heavy quarks is large. Therefore, a heavy quark pair can only be created at the beginning of the collision in a perturbative process not affected by the medium. However, the interaction with the medium does modify the probability that a bound state is formed and its lifetime. Therefore, measuring R_{AA} we can extract information about the medium.

There are three mechanism that modify the number of quarkonium states inside of a quark-gluon plasma: colour screening, inelastic collisions with medium particles and recombination. It is desirable to have a formalism in which the three mechanism can be described consistently. Moreover, we need a quantum mechanical description. The reason is that, when thermal modifications are a leading order effect, the question we wish to answer is whether a bound state can exists in such conditions or not.Note that bound state formation (as for example the hydrogen atom) is a problem that requires applying quantum mechanics.

A promising formalism that meets these requirements is that of open quantum systems. In this framework, we consider a *universe* made of heavy quarks and a environment, that is made of light quarks and gluons. The state of this *universe* is determined by its density matrix. This matrix evolves following an unitary evolution, however, it is a very complex object that contains information about all the degrees of freedom of the *universe*. It is convenient to work with a simpler object called the reduced density matrix. It is obtained from the full density matrix after performing a trace over the environment degrees of freedom. The reduced

*e-mail: miguelangel.escobedo@usc.es

density matrix contains all the information needed to determine the state of the heavy quarks, however, its evolution is not unitary any more. In general, the time evolution of the reduced density matrix is ruled by the so-called master equation.

The master equation for the evolution of quarkonium in a medium has been derived in two different limits. In perturbation theory [2, 3] and in the limit $\frac{1}{r} \gg T$ [4]. There are other derivations in the literature that do not discuss the Non-Abelian nature of QCD, which is one of the focus of this work. Once the master equation is known, it is not an easy task to apply it to obtain phenomenological results due to the high computational cost. However, the use of Monte Carlo methods has improved the situation in recent years [5, 6].

Our motivation in this work is to gain information about the master equation in more general settings. More specifically, we ask ourselves if it is possible to know something about the master equation in the case $\frac{1}{r} \sim T$ beyond the perturbative regime. We expect that the needed non-perturbative medium information can be encoded in expectation values of gauge invariant operators. These operators might be computed using lattice QCD or other non-perturbative techniques in the future. Even if this computation is not possible at the moment, to obtain these expectation values could be useful to get non-perturbative insights from the large N_c limit. A first step in this direction is to study the static limit, defined as the limit in which the heavy quark mass is infinite.

2 Non-relativistic effective field theories

In this section we review one of the main tools we are going to use in our investigation, Non-relativistic effective field theories (NREFTs). Heavy quarks have a mass m that is much larger than Λ_{QCD} and the temperature of the medium. Other relevant energy scales, are the inverse of the typical radius $\frac{1}{r} \sim mv$ (with $v \ll 1$) and the binding energy $E \sim mv^2$. Therefore, quarkonium is a system in which several well-separated energy scales are present. In these situations it is useful to apply EFTs. The large separation between energy scales can break the convergence of naive perturbation theory or make lattice computations very costly. Using EFTs we can reorganize the computation in such a way that we only deal with one energy scale at each step, solving the previously mentioned problems. In the case of heavy quarks, we can integrate the physics at the scale m to go from QCD to non-relativistic QCD (NRQCD) [7, 8]. This can be done pertubatively and ignoring medium effects, since $m \gg \Lambda_{QCD}, T$. In NRQCD, heavy quarks are not represented by a bispinor field. Instead, we have a spinor field for heavy quarks and another spinor field for the antiquarks. One can also integrate out the scale $\frac{1}{r}$ to go from NRQCD to potential NRQCD (pNRQCD) [9]. The pNRQCD Lagrangian at $T = 0$ is

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S + O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)Tr(O^\dagger\mathbf{r}g\mathbf{E}S + S^\dagger\mathbf{r}g\mathbf{E}O) + \frac{V_B(r)}{2}Tr(O^\dagger\mathbf{r}g\mathbf{E}O + O^\dagger O\mathbf{r}g\mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q\,. \quad (1)$$

The degrees of freedom of the heavy quarks and antiquarks are reorganized in a singlet S and octet O field. The pNRQCD Lagrangian is manifestly gauge invariant. This simplifies the connection with quantities that can be computed using lattice QCD. If $\frac{1}{r} \gg T$, we can use the previous Lagrangian as a starting point. If this is not the case, then the matching between NRQCD and pNRQCD has to be done taking into account the presence of the medium.

Let us now remark an interesting property of the matching at $T = 0$. We can match NRQCD to pNRQCD in the static limit. In this limit the matching can be done non-perturbatively since the propagation of a heavy quark is described by a time-like Wilson line. Using this we can relate all Wilson coefficients to expectation values of gauge invariant operators. Once the we have obtained the pNRQCD Lagrangian in the static limit, we can

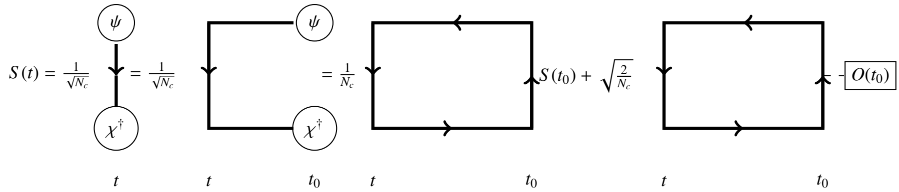

Figure 1. Backwards evolution of the singlet. We are using a birdtrack notation, more details in [1]

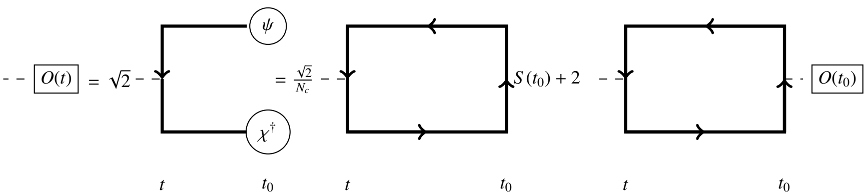

Figure 2. Backwards evolution of the octet.

apply it to study situations in which the mass is not infinite, as for example real quarkonium. The reason is that each term has a well-defined power counting in terms of $\frac{1}{m}$. We expect that this nice property is also true at finite temperature, although more developments are needed to match NRQCD to pNRQCD in this case. Therefore, we move in the next section to study the evolution of a pair of static quarks in the medium.

3 The static limit

We use the NRQCD Lagrangian in the static limit

$$\mathcal{L}_{NRQCD} = \mathcal{L}_{QCD} + \psi^\dagger iD_0\psi + \chi^\dagger iD_0\chi \,. \tag{2}$$

We study the evolution of operators that transform like singlets or octets made of static quark fields, they are called interpolating fields in the pNRQCD literature [9]. We expect that these fields are close to the singlet and octet fields of pNRQCD in the limit of small r, however the precise relation needs to be worked out. From now on we are going to use a slight abuse of language and identify the singlet and octet with their corresponding interpolating field. In order to work out the evolution of a static quark-antiquark pair we need to take into account that the evolution of a static quark is given by a time-like Wilson line and make use of the Fierz identity. The evolution of the singlet is represented in fig. 1 and that of the octet in fig. 2. Using these results we can obtain the probability to find a singlet or octet at time t knowing that there was one singlet or octet at time t_0. In [1] we found the expectation value that correspond to each of the four cases. An unusual characteristic which they share is that they mix fields in different branches of the Schwinger-Keldysh contour. Even though a non-pertubative study of these expectation values is beyond the scope of this work, we can obtain some information from studying them in the large N_c limit.

3.1 Singlet to singlet transition

The probability to find a singlet at time t assuming that there is a singlet at time t_0 is proportional to

$$Tr(W^{\dagger}_{SS}(\mathbf{R},\mathbf{r};t,t_0)W_{SS}(\mathbf{R},\mathbf{r};t,t_0)\rho_l)\,, \tag{3}$$

where W_{SS} is a Wilson loop of the kind that defines the static potential. In the large N_c limit diagrams mixing gluons attached to W_{SS} and gluons attached to $W^{\dagger}_{SS}$ are suppressed. Therefore,

$$Tr(W^{\dagger}_{SS}(\mathbf{R},\mathbf{r};t,t_0)W_{SS}(\mathbf{R},\mathbf{r};t,t_0)\rho_l) = |Tr(W_{SS}(\mathbf{R},\mathbf{r};t,t_0)\rho_l)|^2\ . \tag{4}$$

This result suggests that, in the large N_c limit, the survival probability of the singlet can be studied using an effective non-Hermitian Hamiltonian.

3.2 Singlet to octet transition

For this particular transition the analysis of the consequences of the large N_c limit allows to relate it to two different expectation values. One of them is proportional to the singlet to singlet transition while there is another new term appearing in this transition. This structure is reminiscent of the fact that the probability to find an octet is equal to the probability to find anything minus the probability to find a singlet.

3.3 Octet to singlet transition

This observable has a structure similar to the singlet to octet transition. However, there is an important quantitative difference. This transition is suppressed in the large N_c limit by a factor $\frac{1}{N_c^2}$.

3.4 Octet to octet transition

In this case we observe that diagrams that connect the quark with the antiquark are suppressed by a power of $\frac{1}{N_c^2}$. This means that in the large N_c limit the octet behaves like an uncorrelated pair of particles. In this sense, the large N_c limit seems to justify the molecular hypothesis often used in the literature to solve the Boltzmann equation [10], at least in the low density limit in which this study is performed.

3.5 Qualitative picture

We can distinguish two different cases. The first case is the one in which the density of singlets D_s is similar to the density of octets D_o

$$D_s(t) = Sd(t-t_0)D_s(t_0)\,, \tag{5}$$

$$D_o(t) = (Q(t-t_0) - Sd(t-t_0))D_s(t_0) + Qd(t-t_0)D_o(t_0)\,. \tag{6}$$

The singlet decays into octets following an evolution that can be encoded with an effective Hamiltonian. The octet evolves like an uncorrelated pair of heavy quarks sourced by the decay of singlets. This situation is maintained until the density of singlets decreases so much that we can consider that $D_s \sim \frac{1}{N_c^2}$. Then, the power counting changes and the evolution equations at leading order in the large N_c limit are

$$D_s(t) = Sd(t-t_0)D_s(t_0) + \frac{Iq(t-t_0) - Sd(t-t_0)}{N_c^2 - 1} D_o(t_0)\,, \tag{7}$$

$$D_o(t) = Qd(t-t_0)D_o(t_0)\,. \tag{8}$$

Now, in order to compute the density of singlets, it is important to consider the decay of octets into singlets. Regarding the octet, we can neglect the decay of singlets and just consider that the octets evolve like uncorrelated pairs of quarks.

4 Conclusions

We tried to justify that the study of the static limit can be relevant to understand the physics of *real* heavy quarkonium. We have studied all the possible transitions between singlets and octets in the large N_c limit and wrote the corresponding expectation value of gauge invariant operators, which may be computed using non-perturbative techniques. They have the special feature that they involve the two branches of the Schwinger-Keldysh contour.

We studied the consequences that we can obtain from using the large N_c limit. Although we have not discussed it explicitly, these consequences are compatible with previous computations using perturbation theory or pNRQCD in the $\frac{1}{r} \gg T$ limit. The evolution of the singlet can be encoded in an effective Hamiltonian (that is not necessarily Hermitian). The decay of octets into singlets can be ignored, at least as long as the density of singlets is not very low. The octet propagates like an uncorrelated pair of particles. This suggests that the large N_c limit might be used to justify the molecular chaos hypothesis, however, to confirm this would need a study considering large densities of heavy quarks.

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