

Physics of warped dimensions and continuous spectra *

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Abstract. We study some features of a warped five-dimensional model that solves the hierarchy problem and exhibits a continuum of Kaluza-Klein (KK) modes with a mass gap at the TeV scale. We compute the propagators and spectral functions for massless bulk gauge bosons, and study how the continuum can be reached as the limit of a set of models with discrete spectrum. Finally, we study the low energy effective theory and provide explicit results for the Wilson coefficients.

1 Introduction

No clear deviations from the predictions of the Standard Model (SM) of electroweak (EW) and strong interactions have been found so far at past and present particle physics experiments [1]. In spite of that, the SM fails to describe a number of observational facts, such as dark matter and dark energy or the baryon asymmetry of the universe, and does not incorporate quantum gravity. So, it should be considered an effective theory, valid for energies below a certain cutoff scale. If this cutoff is near the Planck scale M_{Pl} , then the large hierarchy with respect to the electroweak scale M_{EW} requires fine tuning of the parameters in the ultraviolet (UV). This is known as the hierarchy problem, and is addressed by several extensions of the SM. Here, we are interested in models that generate hierarchical scales by a red-shift effect in a warped extra dimension [2]. In most of them, the extra dimension is compact and gives rise to a discrete tower of Kaluza-Klein (KK) states for each 5D field.

The lowest KK modes that do not correspond to SM fields are expected to have masses of the order of a few TeV, so they could in principle give rise to resonant signals at the LHC and future colliders. However, no such signals have been observed so far. These TeV KK modes would actually be more elusive to direct searches if their contribution to cross sections were not peaked, and hence difficult to distinguish from the background. Proposals in this sense include models with broad resonances [3], narrowly-spaced KK modes that cannot be resolved [4, 5] (see also [6] for a four-dimensional version) or, in the extreme case of

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vanishing spacing, a continuum of KK modes. Unparticle models with a conformal sector that gives rise to continuous spectra were proposed in [7]. When the unparticle stuff is charged under the SM gauge group, then the phenomenological viability of the model requires a mass gap of the order of the TeV [8]. This can be realized in extra-dimensional theories with scalar-gravity backgrounds with an admissible singularity at a finite proper distance [9, 10] (see [11] for a detailed analysis of stability). The phenomenology of this scenario has been explored in [12, 13] and, more recently, in [14–16]. Here, we discuss some basic features of this setup in the particular model proposed in [17], which allows for an exact analytical treatment.

2 The extra-dimensional model

Let us consider a 5D theory with a domain-wall warped geometry characterized by a metric

$$ds^2 = g_{MN} dx^M dx^N \equiv \bar{g}_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad \text{with} \quad \bar{g}_{\mu\nu} = e^{-2A(y)} \eta_{\mu\nu}, \quad (1)$$

where $y \in [y_0, y_s]$ parametrizes the extra dimension. The background we are interested in can be dynamically generated in a scalar-gravity theory with two three-branes at the boundary $y = y_0 = 0$ (UV brane) and at the point $y = y_1$ (IR brane), where $0 < y_1 < y_s \leq \infty$. The coordinate y is chosen such that $A(0) = 0$. The 5D action of the model reads [11]

$$S = \int d^5x \sqrt{|\det g_{MN}|} \left[-\frac{1}{2\kappa^2} R + \frac{1}{2} g^{MN} (\partial_M \phi) (\partial_N \phi) - V(\phi) \right] - \sum_{\alpha} \int_{B_{\alpha}} d^4x \sqrt{|\det \bar{g}_{\mu\nu}|} \lambda_{\alpha}(\phi) + S_{\text{GHY}} + S_{\text{SM}}, \quad (2)$$

where $V(\phi)$ is the bulk scalar potential, $\lambda_{\alpha}(\phi)$ are the UV ($\alpha = 0$) and IR ($\alpha = 1$) 4D brane potentials and $\kappa^2 = 1/(2M_5^2)$ with M_5 the 5D Planck scale. S_{GHY} is the standard Gibbons-Hawking-York boundary term, while S_{SM} refers to terms involving the SM fields. The values of y_1 and y_s are eventually determined by the dynamics of this action. The equations of motion (EoM) for ϕ and A can be written in terms of a superpotential $W(\phi)$, as [18]

$$\phi'(y) = \frac{1}{2} W'(\phi), \quad A'(y) = \frac{\kappa^2}{6} W(\phi), \quad (3)$$

while the brane potentials impose some boundary/jumping conditions on the branes.

A simple model that addresses the hierarchy problem was defined in Refs. [11, 15, 17] by the choice

$$W(\phi) = \frac{6k}{\kappa^2} (1 + e^{\nu\phi}). \quad (4)$$

A hierarchy of scales is generated dynamically: $k \lesssim M_5$ gives rise to $A(y_1) \approx \mathcal{O}(35)$, which implies $M_{\text{Pl}} \approx 10^{15} M_{\text{EW}}$ if the Higgs is localized at or close to y_1 . The spectrum of the KK modes of fields propagating in this background depends crucially of the value of the parameter ν . There is a critical value $\nu_c \equiv \kappa/\sqrt{3}$ such that: i) $\nu < \nu_c$ corresponds to ungapped continuum KK spectra; ii) $\nu = \nu_c$ corresponds to continuum KK spectra with a mass gap m_g ; and iii) $\nu > \nu_c$ leads to discrete KK spectra. In the following we will focus on the critical case $\nu = \nu_c$, in which there is a metric singularity at $y_s < \infty$. Moreover, we consider an approximation to the superpotential (4) that gives rise to a warp factor $A(y)$ [17]

$$A(y) \simeq ky \Theta(y_1 - y) + [ky_1 - \log(k(y_s - y))] \Theta(y - y_1), \quad (5)$$

where $\Theta(x)$ is the step function. This simplified metric allows to get exact analytical expressions for the propagators of 5D fields. The geometry between 0 and y_1 is AdS₅, as in

the Randall-Sundrum model [2], while the behaviour between y_1 and y_s is as in the linear dilaton model [4, 5, 16]. The natural scales measured by an observer on the IR brane is $\rho \equiv k e^{-A(y_1)} \sim \text{TeV}$, as required to explain the hierarchy, and continuity across y_1 imposes the relation $k(y_s - y_1) = 1$, with $y_s < \infty$ defined as the position of the singularity in (5). This turns out to imply $m_g \sim \rho$, and therefore opens possibility of observing the continuum at large colliders. Note that in conformal coordinates, $ds^2 = a(z)^2(dx^2 - dz^2)$, the singularity is located at infinite value of z . As a summary, the positions of the branes and the singularity in proper and conformal coordinates is given by

(UV brane)		(IR brane)		(IR singularity)
$y_0 = 0$	$<$	$y_1 = A(y_1)/k$	$<$	$y_s = y_1 + 1/k$
$z_0 = 1/k$	$<$	$z_1 = 1/\rho$	$<$	$+\infty$

3 Propagators

Next, we study the 5D propagators for fields propagating in this background. We work at the classical level (see [19] for a discussion of one-loop resummed propagators in a scenario with continuous spectrum). For definiteness we consider here the case of gauge bosons of an unbroken 5D symmetry. That is, we study the sector of S_{SM} with Lagrangian

$$\mathcal{L}_g = \int_0^{y_s} dy \left[-\frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} e^{-2A} \text{tr} \partial_y A_\mu \partial_y A_\mu \right]. \tag{6}$$

The boundary conditions for the A_μ field are Neumann at $y = 0$ and the requirement of regularity at y_s . We have already imposed the gauge condition $A_5 = 0$, which is compatible with that boundary condition. The type of spectrum for this field can be easily read from the Schrödinger form of the EoM, $-\partial_z^2 \tilde{A}_\mu(p, z) + V_A(z) \tilde{A}_\mu(p, z) = p^2 \tilde{A}_\mu(p, z)$, where we are using a mixed momentum-position representation in conformal coordinates and rescaled fields $\tilde{A}_\mu(p, z) = a(z)^{1/2} A_\mu(p, z)$ [10]. The effective Schrödinger potential for the metric in Eq. (5) is

$$V_A(z) = \frac{3}{4z^2} \Theta(z_1 - z) + \frac{\rho^2}{4} \Theta(z - z_1). \tag{7}$$

The fact that it asymptotes to a constant $\frac{\rho^2}{4}$ at large z indicates a continuous spectrum with mass gap $m_g = \rho/2$.

3.1 Continuum of gauge boson modes

To see this explicitly, we compute the 5D gauge boson propagators in the mixed representation, which are Green's functions of the EoM. They are given by

$$G_A^{\mu\nu}(y, y'; p) = \left[\eta^{\mu\nu} - (1 - \xi) \frac{p^\mu p^\nu}{p^2} \right] G_A(y, y'; p), \tag{8}$$

where ξ is a gauge parameter for the remaining gauge freedom and G_A obeys

$$p^2 G_A(y, y'; p) + \partial_y \left(e^{-2A(y)} \partial_y G_A(y, y'; p) \right) = \delta(y - y'). \tag{9}$$

The solution is required to satisfy the boundary conditions, with regularity enforced at Euclidean momenta. The explicit solution, given in [17], has an isolated simple pole at $s = 0$ and a branch cut along $[m_g^2, +\infty)$. Subtracting the pole part $G_A^0 \equiv \lim_{p \rightarrow 0} G_A(y, y'; p) = 1/(y_s p^2)$,

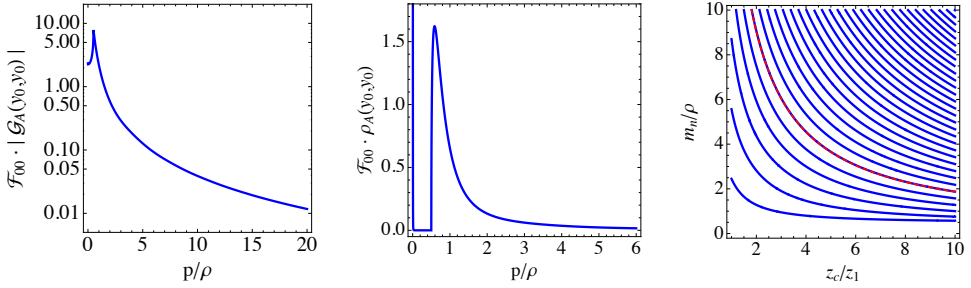


Figure 1. Plots of the UV-UV Green's function $\mathcal{F}_{00} \cdot |\mathcal{G}_A(y_0, y_0; p)|$ (left panel), spectral function $\mathcal{F}_{00} \cdot \rho_A(y_0, y_0; p)$ (middle panel), and spectrum of the lightest KK modes as a function of z_c/z_1 in the discretized model of Sec. 3.2 (right panel). For comparison with the numerical result (blue lines) in the right panel, we display the result for $n = 6$ by using the approximate analytical formula of Eq. (18) (dashed red line). We have used $A(y_1) = 35$ in all panels.

we define the continuum part of the propagator as $\mathcal{G}_A(y, y'; p) \equiv G_A(y, y'; p) - G_A^0$. We display in Fig. 1 (left) the behavior of $|\mathcal{G}_A(y_0, y_0; p)|$, normalized by the factor $\mathcal{F}_{00} \equiv \frac{\rho^2}{k}(ky_s)$ to make it almost invariant under changes of $A(y_1)$ [17]. A 5D spectral function can be defined by

$$\rho_A(y, y'; s) = -\frac{1}{\pi} \text{Im} G_A(y, y'; s + i0^+). \quad (10)$$

In our model it has the form $\rho_A(y, y'; s) = \frac{1}{y_s} \delta(s) + \eta_A(y, y'; s) \Theta(s - m_g^2)$, for a certain function η_A [17]. The pole at $s = 0$ produces the Dirac delta (an isolated zero mode), while the branch cut gives rise to η_A which is a continuous distribution above m_g . We plot $\mathcal{F}_{00} \cdot \rho_A(y_0, y_0; p)$ in Fig. 1 (middle). The Kramers-Kronig relations allow to recover the whole Green's function from its imaginary part, leading to the spectral representation

$$G_A(y, y'; s) = \int_0^\infty dm^2 \frac{\rho_A(y, y'; m^2)}{s - m^2 + i0^+}. \quad (11)$$

The spectral function can be related to the profiles of the KK modes and a 4D spectral density σ [20]:

$$\rho_A(y, y'; s) = \sigma(s) f_s(y) f_s^*(y'). \quad (12)$$

The profiles are normalizable (non-normalizable) for s in the discrete (continuous) spectrum.

Rather than writing the complete expression for σ , we write here the $s \gg \rho^2$ limit of the brane-to-brane spectral functions:

$$\rho_A(y_0, y_0; p^2) \underset{\rho \ll p}{\approx} \frac{1}{2[\log(p/k)]^2} \frac{k}{p^2}, \quad (13)$$

$$\rho_A(y_0, y_1; p^2) \underset{\rho \ll p}{\approx} \frac{1}{2\sqrt{\pi}} \frac{\sin(p/\rho) - \cos(p/\rho)}{\log(p/k)} \frac{k}{\rho^{1/2} p^{3/2}}, \quad (14)$$

$$\rho_A(y_1, y_1; p^2) \underset{\rho \ll p}{\approx} \frac{1}{2\pi} [1 - \sin(2p/\rho)] \frac{k}{\rho p}. \quad (15)$$

One might worry about the fact that $\rho_A(y, y'; s)$ with $y \neq y'$ is not positive for all s . In fact, there is no problem as the 5D spectral operator $\hat{\rho}_A$, with matrix elements $\langle y, p | \hat{\rho}_A | y', p \rangle = \rho_A(y, y'; p^2)$, is definite positive. Indeed, for any 5D function ϕ for which the following

integrals are convergent, we have

$$\begin{aligned} \langle \phi | \hat{\rho}_A | \phi \rangle &= \int dp dy dy' \phi^*(y, p) \rho_A(y, y'; p^2) \phi(y', p) \\ &= \int dp \sigma(p^2) \left(\int dy \phi^*(y, p) f_{p^2}(y) \right) \left(\int dy' f_{p^2}^*(y') \phi(y', p) \right) \\ &= \int dp \sigma(p^2) \left| \int dy \phi^*(y, p) f_{p^2}(y) \right|^2 \geq 0. \end{aligned} \tag{16}$$

The expression (12) also explains the fact, observed in [17, 21], that the non-vanishing formal eigenvalue $\lambda(s)$ of $\hat{\rho}_A(s)$ (note that $\hat{\rho}_A(s)$ has rank one due to its factorized form) is given by a divergent integral, when s belongs to the continuum spectrum:

$$\lambda(s) = \text{tr } \hat{\rho}_A = \int_0^{y_s} dy \rho_A(y, y; s) = \sigma(s) \int_0^{y_s} dy |f_s(y)|^2 \sim \infty, \quad s > m_g^2. \tag{17}$$

Actually, a finite integral would indicate that s belongs to the discrete spectrum! Three kinds of contributions to σ appear in the continuum model, i.e. $\sigma(s) = \frac{1}{\|f_0\|^2} \delta(s) + [\sigma_{\text{res}}(s) + \sigma_{\text{un}}(s)] \Theta(s - m_g^2)$, where $\sigma_{\text{res}}(s)$ is a continuous positive function vanishing at $s = m_g^2$, while $\sigma_{\text{un}}(s) \propto 1/(s - m_g^2)^{1/2}$ corresponds to a contribution from unparticles with a dimension $d_U = 3/2$ and a mass gap, see e.g. Ref. [19].

3.2 Gapped continuum spectrum as the limit of a discrete spectrum

The models with continuous spectra can be recovered as the zero-spacing limit of versions of them with discrete spectra [22]. This can be useful in understanding some properties of the continuum in terms of ordinary particles, or to build models with narrowly-spaced resonances with a similar phenomenology to the corresponding continuum model. For instance, for the model with superpotential (4) we could consider $\nu > \nu_c$ and then take the limit $\nu \rightarrow \nu_c$. To find a discrete version of the simplified model considered here, we simply end the extra dimension before the singularity y_s is reached. In conformal coordinates this amounts to restricting the coordinate z to a compact domain $[z_0, z_c]$, $z_c \geq z_1$ and imposing Neumann boundary conditions at the new boundary $z = z_c$ (Dirichlet conditions would work similarly). Adjusting z_c , we can smoothly evolve from a Randall-Sundrum model when $z_c = z_1$ to our continuum model when $z_c \rightarrow \infty$. The mass spectrum in the discretized version of the model of Eq. (5) has the approximate analytical behaviour

$$m_n^2 \simeq m_g^2 + \left(n - \frac{1}{4} \right)^2 \frac{\pi^2}{z_c^2}, \quad n = 1, 2, 3, \dots \tag{18}$$

This formula has a relative error $\lesssim 1\%$ which gets smaller for higher values of m_n^2 . We show in Fig. 1 (right) the spectrum of the lightest KK modes as a function of z_c/z_1 . Notice that the mass spacing Δm_n is governed by z_c , so that the eigenvalues pile-up as z_c increases. By using this approach, it is possible to recover the propagator and spectral functions in the continuum model as a limit of its discretized version:

$$G_{A(d)}(y, y'; s) \equiv \sum_n \frac{1}{\|f_n\|^2} \frac{f_n(y) f_n^*(y')}{s - m_n^2 + i0^+} \xrightarrow{z_c \rightarrow \infty} G_A(y, y'; s), \tag{19}$$

$$\rho_{A(d)}(y, y'; s) \equiv \sum_n \frac{f_n(y) f_n^*(y')}{\|f_n\|^2} \delta(s - m_n^2) \xrightarrow{z_c \rightarrow \infty} \rho_A(y, y'; s), \tag{20}$$

where $G_A(y, y'; s)$ and $\rho_A(y, y'; s)$ are given by Eqs. (11) and (12), respectively. In these expressions $f_n(y)$ is the eigenfunction for the n -th KK mode. A more detailed analysis will be provided in [20].

4 Low energy operators generated by the continuum

By integrating out the modes heavier than a certain scale μ , we can obtain a low-energy effective theory that contains only the light degrees of freedom with mass $m < \mu$. Let us consider the quark operators $(\bar{q}\gamma^\mu\lambda^a q)(\bar{t}\gamma_\mu\lambda_a t)$ induced by the exchange of gluon modes heavier than μ , where q and t stand for quarks localized on the branes or zero modes of quark bulk fields. The Wilson coefficient is in general given by a convolution

$$C_{qt}(\mu) \equiv \frac{1}{4\mu^2} \int dy dy' C_{qt}(y, y'; \mu) |f_0^q(y)|^2 |f_0^t(y')|^2, \quad (21)$$

with $f_0^{q,t}$ the profiles of the corresponding zero modes or the delta functions that localize the quarks.

The position-dependent coefficients $C_{qt}(y, y'; \mu)$ can be easily computed using the 5D propagators¹:

$$C_{qt}(y, y'; \mu) = \lim_{p \rightarrow 0} g_5^2 \mu^2 \left(G_A(y, y'; p) - \int_0^\mu ds \frac{\rho_A(y, y'; s)}{p^2 - s + i0^+} \right) = -g_4^2 y_s \mu^2 \int_{\mu^2}^\infty \frac{ds}{s} \rho_A(y, y'; s), \quad (23)$$

where g_4 is the 4D strong gauge coupling, related to the 5D one by $g_5 = \sqrt{y_s} g_4$. In the limit $\mu \gg m_g$ we can use Eqs. (13)-(15) to approximate

$$C_{qt}(y_0, y_0; \mu) \underset{m_g \ll \mu}{\approx} -g_4^2 \frac{ky_s}{2} \frac{\log(\mu/k) - 1}{[\log(\mu/k)]^3}, \quad (24)$$

$$C_{qt}(y_0, y_1; \mu) \underset{m_g \ll \mu}{\approx} -g_4^2 \frac{ky_s}{\sqrt{\pi}} \frac{\sin(\mu/\rho) + \cos(\mu/\rho)}{\log(\mu/k)} \left(\frac{\rho}{\mu} \right)^{1/2}, \quad (25)$$

$$C_{qt}(y_1, y_1; \mu) \underset{m_g \ll \mu}{\approx} g_4^2 \frac{ky_s}{2\pi} [\cos(2\mu/\rho) - 2\mu/\rho]. \quad (26)$$

We display in Fig. 2 the exact and approximate results of these coefficients $C_{qt}(y_\alpha, y_\beta; \mu)$ as a function of μ/ρ . The approximate analytical results of Eqs. (24)-(26) reproduce quite well the numerical exact results for $\mu/\rho > 2$. More relevant for today's indirect searches is the effective theory for energies sufficiently smaller than m_g , which contains no continuous degrees of freedom. This effective theory, corresponding to $\mu = m_g$, is obtained by integrating out all the KK gluons except the zero mode. The position-dependent Wilson coefficient is given by (23) with $\mu = m_g$ or, equivalently, by

$$C_{qt}(y, y'; m_g) = \lim_{p \rightarrow 0} g_5^2 m_g^2 \mathcal{G}_A(y, y'; p). \quad (27)$$

By using the low momentum behavior of the brane-to-brane Green's functions of the model of Eq. (5) (cf. Ref. [17]), the brane-to-brane Wilson coefficients are found out to be

$$C_{qt}(y_0, y_0; m_g) = -g_4^2 \frac{9}{16ky_s} \underset{1 \ll ky_s}{\approx} -g_4^2 \frac{9}{16A(y_1)}, \quad (28)$$

$$C_{qt}(y_0, y_1; m_g) = g_4^2 \frac{3(-3 + ky_s)}{16ky_s} \underset{1 \ll ky_s}{\approx} \frac{3}{16} g_4^2, \quad (29)$$

$$C_{qt}(y_1, y_1; m_g) = -g_4^2 \frac{9 + 2ky_s(-3 + ky_s)}{16ky_s} \underset{1 \ll ky_s}{\approx} -\frac{1}{8} g_4^2 A(y_1). \quad (30)$$

¹Notice that the result of Eq. (23) implies that $\bar{C}_{qt}(y, y'; \mu) \equiv C_{qt}(y, y'; \mu)/\mu^2$ obeys the evolution equation

$$\mu^2 \frac{\partial}{\partial \mu^2} \bar{C}_{qt}(y, y'; \mu) = g_4^2 y_s \cdot \rho_A(y, y'; \mu^2), \quad (22)$$

which is a first order differential equation that determines $\bar{C}_{qt}(y, y'; \mu)$ in terms of $\bar{C}_{qt}(y, y'; m_g)$ (see below).

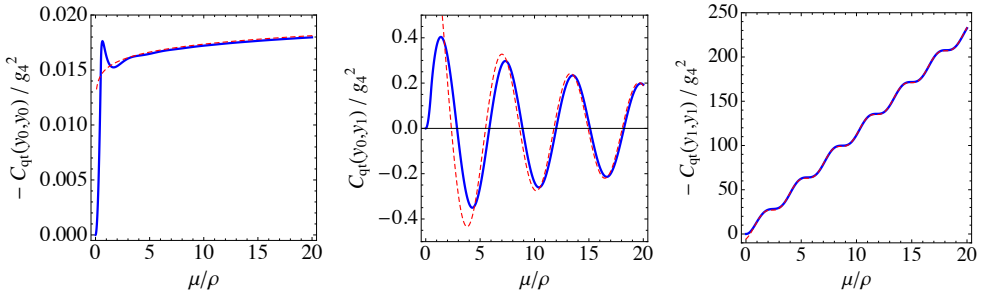


Figure 2. Wilson coefficients of the effective theory below the scale μ : $-C_{qt}(y_0, y_0; \mu)/g_4^2$ (left panel), $C_{qt}(y_0, y_1; \mu)/g_4^2$ (middle panel) and $-C_{qt}(y_1, y_1; \mu)/g_4^2$ (right panel); as functions of μ/ρ . The results of the numerical integration of Eq. (23) (blue lines) are compared with the analytic results of Eqs. (24)-(26) (dashed red lines). We have used $A(y_1) = 35$ in all panels.

The factors $A(y_1) = \log(k/\rho)$ produce a hierarchy of values of the Wilson coefficients, $|C_{qt}(y_0, y_0; m_g)| < |C_{qt}(y_0, y_1; m_g)| < |C_{qt}(y_1, y_1; m_g)|$. Finally, notice that $G_A(y, y'; p)$ has real values for $0 < p < m_g$, so that $C_{qt}(y, y'; m_g)$ is also real. An analysis similar to the one presented above has been provided in Ref. [23] within the linear dilaton model, leading to similar conclusions.

5 Conclusions

We have studied a warped extra-dimensional model that generates a hierarchy between the Planck scale and the electroweak scale by means of a warp factor. The geometry of the model is AdS_5 near the UV brane, and it behaves like a linear dilaton model towards the IR. The KK tower of massless gauge bosons (photon and gluon) has an isolated massless mode corresponding to a 4D SM gauge boson, and a continuum with mass gap $m_g \sim \text{TeV}$.

We have computed in this model the propagators and spectral functions for massless gauge bosons, and have related them to a continuum version of the KK expansion. We have shown that the continuum can be obtained as the limit of models defined on a compact interval of the conformal coordinate, which have only discrete spectrum. Finally, we have found the Wilson coefficients of low-energy operators generated by the continuous modes. Other features of this model not discussed in the present work include the appearance of broad resonances already at tree level, as well as the increase of the cross sections $\sigma(pp \rightarrow Q\bar{Q})$ with respect to the SM prediction [15, 17]. A study of its phenomenology at the LHC is in progress. Another interesting application of this scenario is the idea of continuum dark matter [24, 25].

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