

Equation of state for holographic nuclear matter as instanton gas *

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Abstract. In a holographic model, which was used to investigate the color superconducting phase of QCD, a dilute gas of instantons is introduced to study the nuclear matter. The free energy of the nuclear matter is computed as a function of the baryon chemical potential in the probe approximation. Then the equation of state is obtained at low temperature. Using the equation of state for the nuclear matter, the Tolman-Oppenheimer-Volkov equations for a cold compact star are solved. We find the mass-radius relation of the star, which is similar to the one for quark star. This similarity implies that the instanton gas given here is a kind of self-bound matter.

1 The Set-up

We start from the bottom-up holographic model which is proposed as Yang-Mills (YM) theory with color superconducting phase. It is given by the following gravitational theory [2–5].

$$S = \int d^{d+1}\xi \sqrt{-g} \mathcal{L}, \quad (1)$$

$$\mathcal{L} = \mathcal{L}_{\text{Gravity}} + \mathcal{L}_{\text{CSC}}, \quad (2)$$

$$\mathcal{L}_{\text{Gravity}} = \mathcal{R} + \frac{d(d-1)}{L^2}, \quad (3)$$

$$\mathcal{L}_{\text{CSC}} = -\frac{1}{4}F^2 - |D_\mu \psi|^2 - m^2|\psi|^2, \quad (4)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu \psi = (\partial_\mu - iqA_\mu)\psi. \quad (5)$$

It describes $d+1$ dimensional gravity coupled to a $U(1)$ gauge field, A_μ , and a charged scalar field, ψ . Here we consider the case of $d=5$. The charge q denotes the baryon number of the scalar ψ , and it is chosen as $q=2/N_c$ to represent the quark Cooper pair formation. The mass

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m is given to reproduce the corresponding conformal dimension of the diquark operator dual to the scalar field ψ with the charge $q = 2/N_c$. Here we put $1/2\kappa_6^2 = 1$, and the dimensionful $U(1)$ gauge coupling constant is set as unit. So the metric is dimension less and A_μ has dimension one as in the 3+1 dimensional case.

Here we add the sector of the $SU(N_f)$ gauge fields, which is used to build the nuclear matter through the instanton configuration, which generates the baryon number in QCD. The instanton configuration is here deformed since it is embedded in a higher dimensional curved space.

$$\mathcal{L} = \mathcal{L}_{\text{Gravity}} + \mathcal{L}_{\text{CSC}} + \mathcal{L}_V, \quad (6)$$

$$\mathcal{L}_V = -\frac{1}{4}\text{tr}F_{SU(N_f)}^2 \quad (7)$$

where $F_{SU(N_f)}^2$ denotes the 2-form $SU(N_f)$ gauge field. The coupling constant g_{YM}^2 is set as one. The mass dimension of the gauge field is taken to be one. Then, in order to pick up the coupling of the baryon number current and the instanton configuration, we add the Chern-Simons (CS) term,

$$S = \int d^{d+1}x \sqrt{-g} \mathcal{L} + S_{\text{CS}}. \quad (8)$$

Hereafter we proceed the analysis according to the probe approximation with $N_f = 2$.

1.1 Instanton in R^4

Before solving the equation of motion of our model, the ansatz for the $U(1)$ gauge field and the instanton configurations for $SU(2)$ gauge fields are given. The coordinates are denoted as $(\xi^0, \xi^1, \xi^2, \xi^3, \xi^4, \xi^5) = (x^0, x^1, x^2, x^3, w, z)$. Then, we make the ansatz for $U(1)$ gauge field as

$$A_b = A_b(z)\delta_b^0. \quad (9)$$

As for the $SU(2)$ vector fields, which are set as

$$\vec{f}_1 = F_{ab}^i \tau_i, \quad (10)$$

and the following ansatz are imposed,

$$(\vec{f}_1)_{ij} = Q(x^m - a^m, \rho)\epsilon_{ijk}\tau^k, \quad (\vec{f}_1)_{iz} = Q(x^m - a^m, \rho)\tau^i, \quad (11)$$

where ρ (a^m) denotes the instanton size (position), $\epsilon_{123z} = 1$, $i, j = 1, 2, 3$ and $m = 1, \dots, 4$, where $x^4 = z$. Then the $SU(N_f)$ vector part (7) is given for $N_f = 2$ as

$$\mathcal{L}_V = -\frac{3}{2}Q^2 \left[(g^{11})^2 + g^{11}g^{zz} \right]. \quad (12)$$

In order to see the baryon, Q is given as an instanton solution in flat 4D space $\{x^m\}$ [6, 7],

$$Q = \frac{2\rho^2}{((x^m - a^m)^2 + \rho^2)^2}. \quad (13)$$

Now let us make an ansatz for Q as follows:

$$Q^2 = \sum_i^{N_i} \frac{4\rho^4}{((x^m - a_i^m)^2 + \rho^2)^4}. \quad (14)$$

This corresponds to the dilute gas approximation of instantons. Then the size of instanton is determined by the variational principle so as to minimize the energy density.

Lastly, the CS term is given by

$$S_{CS} = \kappa_{CS} \epsilon^{m_1 \dots m_4} \int d^4x dz A_0 \text{tr}(F_{m_1 m_2} F_{m_3 m_4}). \quad (15)$$

2 Effective action and EoS for nuclear matter

Let us now evaluate the effective action of the instantons. To this end, the gravity background dual to the low temperature confinement phase is given as the following:

$$ds^2 = k^{2/5} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{4}{25} k^{-3/5} (k^{-2/5} dz^2 + z^2 d\theta^2), \quad (16)$$

where $k = 1 + z^2$. Then the matter action with embedded instantons is obtained as

$$\begin{aligned} S_{\text{matter}} &= \int d^6\xi \sqrt{-g} \left(-\frac{1}{4} F^2 - \frac{1}{4} \text{tr} F_{SU(2)}^2 \right) + S_{CS} \\ &= \int d^4x dz \left(\frac{1}{2} z k^{3/5} A_0'^2 - n z \frac{12}{25} \bar{q}^2 \left[k^{-4/5} + \frac{25}{4} k^{3/5} \right] + n n_0 A_0 \bar{q}^2 \right), \end{aligned} \quad (17)$$

where $A_0' = \partial_z A_0(z)$, and $n_0 = 48\kappa_{CS}$. In order to estimate this action, we solve the equation of motion of $A_0(z)$. It is obtained as

$$-\partial_z (z k^{3/5} A_0') + n n_0 \bar{q}^2 = 0. \quad (18)$$

Then we have

$$z k^{3/5} A_0' = \bar{d} = \frac{\pi^2}{6} n n_0 \frac{2z^3 + 3z\rho^2}{(z^2 + \rho^2)^{3/2}} + c. \quad (19)$$

Here, c denotes an integration constant of Eq. (18) over z . We take $c = 0$ to set as $\bar{d}(z=0) = 0$. Due to this condition, the matter action S_{matter} of (17) is written as

$$S_{\text{matter}} = \int d^4x \left\{ \mu \bar{Q} - \int dz \left(\frac{\bar{d}^2}{2z k^{3/5}} + n z \frac{12}{25} \bar{q}^2 \left[k^{-4/5} + \frac{25}{4} k^{3/5} \right] \right) \right\}, \quad (20)$$

where the chemical potential μ and the charge density $\bar{Q} \equiv \bar{d}(\infty)$ are given as

$$\mu = A_0(\infty) = \int_{z_1}^{\infty} dz \frac{\bar{d}}{z k^{3/5}} + A_0(z_1) = \int_{z_1}^{\infty} dz \frac{\bar{d}}{z k^{3/5}}, \quad (21)$$

$$\bar{Q} = \bar{d}(\infty) = \frac{\pi^2}{3} n n_0, \quad (22)$$

where z_1 is an arbitrary positive value and $A_0(z_1)$ is set as zero. which is the boundary condition in solving the differential equation of $A_0(z)$, Eq. (19).

Then the free energy density \mathcal{E} of the instanton system is given as

$$S_{\text{matter}} = - \int d^5\xi \mathcal{E}(\rho, \mu) \quad (23)$$

$$\mathcal{E}(\rho, \mu) = \int dz \left(\frac{\bar{d}^2}{2z k^{3/5}} + n z \frac{12}{25} \bar{q}^2 \left[k^{-4/5} + \frac{25}{4} k^{3/5} \right] \right) - \mu \bar{Q} \quad (24)$$

2.1 EoS for nuclear matter

By giving n, n_0, z_1 , which are the parameters of the present bottom-up model, $\mu(\rho)$ and $\mathcal{E}(\rho)$ are calculated according to the Eqs. (21) and (24) as the functions of ρ . Then a minimum point of $\mathcal{E}(\rho)$ is found at $\rho = \rho_{\min}$. Thus we can obtain $\mathcal{E}(\rho_{\min})$ and $\mu(\rho_{\min})$ for the given parameters n, n_0, z_1 .

Table 1. EoS of nuclear system for $n_0 = 1.5, z_1 = 0.1$

n	$\mu(\rho_{\min})$	$\mathcal{E}(\rho_{\min})$	ρ_{\min}	v_{\min}
0.005	0.0688	0.00031	0.01	2.09×10^{-8}
0.01	0.138	0.0000393	0.02	3.35×10^{-7}
0.015	0.209	-0.00022	0.03	1.70×10^{-6}
0.02	0.275	-0.00178	0.04	5.36×10^{-6}
0.03	0.411	-0.00829	0.06	2.75×10^{-5}
0.05	0.68	-0.0368	0.09	1.52×10^{-4}
0.1	1.335	-0.2075	0.13	9.2×10^{-4}

Table 1 shows a resultant example of such calculations for $n_0 = 1.5, z_1 = 0.1$. The regions of the calculations are restricted to the region for $\mu < 1.7$ [3]. On the other hand, the pressure p , which is given by $p = -\mathcal{E}$, of the instanton gas is negative for $\mu < \mu_c \sim 0.17$. So in this region, the gas is in an undesirable phase as a nuclear matter considered here.

Thus we find that the stable nuclear matter exists in the region, $1.7 > \mu > \mu_c \sim 0.17$. In this region of the nuclear matter, its dilute gas picture is reasonable since $v_{\min} \sim O(10^{-4}) \ll 1$, where $v_{\min} = \frac{4}{3}\pi\rho_{\min}^3 n$ indicates the volume which is occupied by instantons in a unit 3D volume.

Here we proceed the analysis, and we can arrive at the following approximate formula

$$p = a\mu(\mu - \mu_c), \quad (25)$$

where $a = 0.13$ and $\mu_c = 0.17$. Then using (25), the energy density is given at $T = 0$ as [8]

$$\epsilon = \mu q - p = \mu \frac{\partial p}{\partial \mu} - p = a\mu^2, \quad (26)$$

and the speed of sound is obtained as

$$C_s^2 = \frac{\partial p}{\partial \epsilon} = \frac{\partial p / \partial \mu}{\partial \epsilon / \partial \mu} = 1 - \frac{\mu_c}{2\mu}. \quad (27)$$

This leads to the following bound of C_s^2 as $\frac{1}{2} < C_s^2 < 1$. We notice that the lower bound $1/2$ is fairly large comparing to that of the ordinary nuclear matter. We should however notice that the constraint for C_s^2 comes from the formula (25), which is an approximate formula available at small μ . It is possible to improve (25) by adding correction terms of higher powers of μ . For example, by adding a term like μ^4 , we can find a new formula, $p = a'\mu(\mu - \mu'_c) + b'\mu^4$ with appropriate values for the parameters, a', μ'_c, b' . In this case, we can see that the maximum value of C_s is realized at μ smaller than 1 and its maximum value is suppressed from 1, then it decreases to $C_s = 1/\sqrt{3}$ in the large μ region. However, as we can see, the correction term of μ^4 becomes small in the small μ region. Then it is difficult to suppress largely the lower bound $1/2$. We find the small value of sound speed in region of

$0 < C_s^2 < 1/2$ for $\mu < \mu_c$, where $\mu_c (\neq \mu'_c)$ denotes the critical point and it satisfies $p(\mu_c) = 0$. In this region of μ , however, the pressure p is negative, then the instanton gas cannot make a nucleon matter like a star. This implies that our model can not cover the low density part of the normal nuclear matter, in which the sound velocity decreases to zero from its upper bound ($1/\sqrt{3}$). How to overcome this point is remained here as a future problem.

In this sense, the nuclear matter given here might be a special one. Our model is considered in a restricted region of density or μ , $\mu_c < \mu < 1.7$, where 1.7 denotes the transition point to the deconfined RN phase. Here, therefore, we proceed analysis by using a simple model with (25) and (26), then we arrive at the EoS of the nuclear matter given as the instanton gas. It is written as,

$$p = \epsilon - \sqrt{a\epsilon}\mu_c. \tag{28}$$

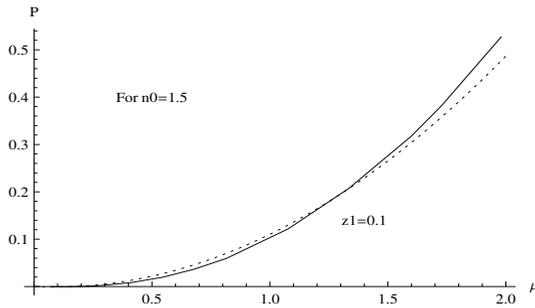


Figure 1. $p = -\mathcal{E}(\mu)$ versus μ at stable instanton size ρ_{\min} near phase transition point $\mu_c = 0.17$. The solid curve denotes the numerical calculations, and the dotted curve represents $p = 0.13\mu(\mu - \mu_c)$.

3 TOV equation and M - R relation of neutron star

The TOV equations for a star with the mass m and p at the radius r in the star are given as,

$$\frac{dp}{dr} = -G(\epsilon + p)\frac{m + 4\pi r^3 p}{r(r - 2Gm)}, \tag{29}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon. \tag{30}$$

Here, G denotes the gravitational constant. They can be solved by using the EoS given in (28) with the boundary condition, $p(r = 0) = p_c$ and $m(r = 0) = 0$. Solving these equations, we obtain the mass of the star, $M = m(R)$, and its radius R , where R is defined by $p(R) = 0$.

In order to obtain the numerical results with dimensionful quantities, we must adjust the scale parameters for each dimensionful quantities (\tilde{x}). To do so, the above TOV equations are rewritten by using the dimensionless quantities as [9]

$$\frac{d\tilde{p}}{d\tilde{r}} = -B(\tilde{\epsilon} + \tilde{p})\frac{\tilde{m} + 4\pi A\tilde{r}^3 \tilde{p}}{\tilde{r}(\tilde{r} - 2B\tilde{m})}, \tag{31}$$

$$\frac{d\tilde{m}}{d\tilde{r}} = 4\pi A\tilde{r}^2 \tilde{\epsilon}, \tag{32}$$

where

$$A = \frac{r_0^3 \epsilon_0}{m_0}, \quad B = \frac{Gm_0 \epsilon_0}{p_0 r_0}, \tag{33}$$

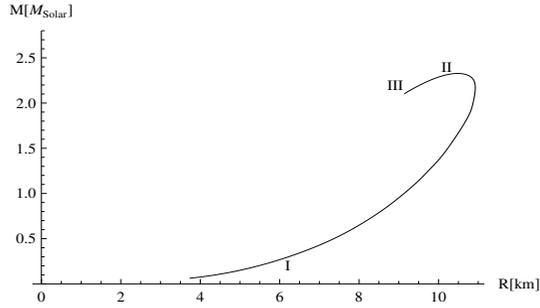


Figure 2. Plot of the neutron mass $M(R)$ (in unit of solar mass M_{\odot}) versus the radius R for $p = \epsilon - \sqrt{a\epsilon}\mu_c$ (or $p = a\mu(\mu - \mu_c)$), $\mu_c = 0.17$ and $a = 0.13$.

and the various variables (x) are replaced by the dimensionless quantities (\tilde{x}) by using its typical dimensionful value x_0 as $x = x_0\tilde{x}$. For example, we rewrite as $p = p_0\tilde{p}$.

Then the solutions of (31) and (32) with $A = B = 1$ are equal to the one of (29) and (30) with $G = 1$. Thus the solution of the latter equations are translated to the one of the former ones as shown in Fig. 2. It is given for $r_0 = 3.0$ km in natural unit. In this case, we find $m_0 = 2.03M_{\odot}$ ¹ and $\epsilon_0^{1/4} = 0.896$ GeV, which provides physical unit to the quantities given in the Table 1. For example, the energy density of the center in the star, whose mass is about two solar mass, is given by 0.00242 GeV⁴. This is about twice as large as the one of the normal nuclear matter.

4 Comparison with the observational data

There are some important observational data which can constrain the theoretical EoS. We now have the following at least three important constraints for the M - R relation;

1. Two solar mass ($2M_{\odot}$) neutron star observation from the Shapiro delay measurement [10, 11].
2. Radius constraint from GW170817 via the gravitational wave observation. The actual constraint is $9.0 < R < 13.6$ km for $M = 1.4M_{\odot}$ [12–14].
3. Upper bound of M for cold spherical neutron stars which is estimated from no detection of relativistic optical counterpart in the analysis of GW170817. The actual limit is estimated as the range $2.15 - 2.26M_{\odot}$ [15].

The maximum mass M_{\max} of our result overshoots the $2.15 - 2.26M_{\odot}$ constraint, but we can expect that there are some phase transitions at high density such as the chiral, deconfined, and color superconducting phase transitions which we do not consider in this study. Then, EoS can be softer than the present one at high density and thus M_{\max} should be smaller than the present value.

With our EoS, the M - R curve flows down to the left bottom, Such behavior is similar to the M - R curves obtained with EoS proposed in Ref. [16]; the actual behavior of the curves can be seen in Refs. [17, 18] denoted as ‘‘SQM1-3’’; these EoS which contain the quark matter make self-bounded neutron stars which do not have minimum masses. The tendency of the

¹The symbol M_{\odot} denotes the solar mass.

M - R curves in the small M region in our model may be modified when we suitably introduce "interactions" to the present model because we employ the dilute instanton gas approximation; the baryonic contribution can be expected to have relatively strong effects on EoS at low density. In addition, we here use the dilute gas of the instanton of the $SU(2)$ gauge field and thus the number of the flavor is considered as two. But since the core of the neutron star is very dense, the hyperon degree of freedom can join the game as the baryonic mode in addition to nucleons. In the present study, we consider the simple symmetric nuclear matter because the difference between the flavors and effects of the electrons are difficult to include in the model at present. Therefore, the inclusion of the effects such as the beta equilibrium and charge neutrality are our future work.

5 Summary and discussions

In this paper, we have studied cold nuclear matter and its equation of state (EoS) based on the six dimensional holographic model. The nuclear matter was introduced as a dilute gas of deformed instantons in the AdS soliton background. The instantons are electric charge neutral and they are made of the $SU(2)$ gauge fields and the action includes the Chern-Simons term as well as the kinetic term. Owing to the energy balance between these two terms in the action, the size of the instanton can be determined for fixed parameters of the model. As the result, EoS for nuclear matter, *i.e.*, pressure as a function of energy density, is obtained.

Then, we applied the EoS to solve the Tolman-Oppenheimer-Volkov equations for a compact star numerically and found the mass-radius (M - R) relationship. The curve is somewhat similar to the one for strange quark matter and we provided a certain interpretation for that. In our approach, it might be possible that the mass of an instanton is very heavy compared with an ordinary nucleon, which is a specific property in holographic QCD since the nucleon mass is proportional to the number of colors N_c . So far we have no definite answer and this is a future issue.

In order to improve our current study, several ingredients are taken into account. One is to go beyond the dilute gas approximation of instantons. The other is to study whether the system enjoys the phase transition from nuclear matter to (perhaps color superconducting) quark matter at higher baryon density. If such a phase transition is present, the EoS gets soften and the resultant M - R curve could be modified. Furthermore it will be interesting to extend our current study into the case with hyperon degrees of freedom. To this end, the holographic treatment of heavy-light meson system (for instance, see [19]) must be instructive. These will be the future issues.

Acknowledgments

We are grateful D. Blaschke and A. Schmitt for their useful comments during the online conference "A Virtual Tribute to Quark Confinement and the Hadron Spectrum" (vConf2021). It is also a pleasure to thank Masayuki Matsuzaki for useful discussions on the neutron stars and TOV equations.

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