Recent progress on few-body hypernuclei

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Abstract. Few-body Λ hypernuclei provide valuable information towards understanding strange matter. Recent experimental progress by the STAR Collaboration at the RHIC facility and by the ALICE Collaboration at the LHC has been matched by theoretical progress reviewed here: (i) lifetimes of the hypertriton $^3_\Lambda$H, $^3_\Lambda$n if particle-stable, $^4_\Lambda$H and $^4_\Lambda$He and their charge symmetry breaking, and (ii) the onset of $\Lambda\Lambda$ hypernuclear binding.

1 Introduction

Single- and double-Λ hypernuclei provide unique extension of nuclear physics into strange hadronic matter [1]. Experimental data on Λ and $\Lambda\Lambda$ hypernuclei are unfortunately poorer both in quantity and quality than the data available on normal nuclei. Nevertheless, the few dozen Λ separation energies B_Λ of single-Λ hypernuclei ($^A_\Lambda$Z) determined across the periodic table from A=3 to 208, and the three $\Lambda\Lambda$ hypernuclei ($^A_{\Lambda\Lambda}$Z) firmly established so far [2], provide useful testground for the role of strangeness in dense hadronic matter, say in neutron star matter. Particularly meaningful tests of hyperon-nucleon and hyperon-hyperon strong-interaction models are possible in light Λ and $\Lambda\Lambda$ hypernuclei, $A \leq 6$, which the topics reviewed below are concerned with.

2 $^3_\Lambda$H lifetime

Measurements of the $^3_\Lambda$H lifetime in emulsion or bubble-chamber experiments during the 1960s and early 1970s gave conflicting and puzzling results. Particularly troubling appeared a conference report by Block *et al.* claiming a lifetime of $\tau(^3_\Lambda$H)=(95^{+19}_{-15}) ps [3], to be compared with a free Λ lifetime τ_Λ=(236±6) ps [4] measured in the same He bubble chamber (BC). However, the He BC experiment [5] concluding that era, coauthored by the same Block, reported a value of $\tau(^3_\Lambda$H)=(246^{+62}_{-41}) ps, in agreement with a Λ lifetime of (263±2) ps. Given a weakly bound Λ, $B_\Lambda(^3_\Lambda$H)=0.13±0.05 MeV from emulsion studies [6], it was anticipated that $\tau(^3_\Lambda$H)$\approx\tau_\Lambda$. Recent measurements of $\tau(^3_\Lambda$H) in relativistic heavy ion collision experiments renewed interest in the $^3_\Lambda$H lifetime problem. The first round of results from STAR [7] and ALICE [8] suggested that $\tau(^3_\Lambda$H) is shorter than τ_Λ by as much as (30±17)%. Whereas STAR's most recent published lifetime, from Au-Au collisions at $\sqrt{s_{NN}}$=7.7 to 200 GeV, is even shorter than that: $\tau(^3_\Lambda$H)=142^{+24}_{-21}±29 ps [9], ALICE latest published lifetime, from Pb-Pb collisions at $\sqrt{s_{NN}}$=5.02 TeV, is close to τ_Λ: $\tau(^3_\Lambda$H)=242^{+34}_{-38}±17 ps [10], or 254±15±17 ps as reported in ICHEP 2020 [11]. The latest news is that this STAR-ALICE apparent divergence has been

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resolved by new lifetime measurements in Au-Au collisions at $\sqrt{s_{\mathrm{NN}}}$=3 GeV reported by STAR at SQM2021: $\tau(^3_\Lambda \mathrm{H})$=231±29±37 ps [12].

Table 1. Calculated hypertriton weak decay rates $\Gamma(^3_\Lambda \mathrm{H_{g.s.}})$ in units of the free-Λ decay rate Γ_Λ, using $^3_\Lambda \mathrm{H}(\frac{1}{2}^+)$ wavefunctions that satisfy $B_\Lambda(^3_\Lambda \mathrm{H})$=0.13±0.05 MeV from emulsion studies [6], and two-body branching ratios $R_3 = \Gamma(^3_\Lambda \mathrm{H} \to \pi^- +^3\mathrm{He})/\Gamma(^3_\Lambda \mathrm{H} \to \pi^- +\mathrm{all})$. The underlined R_3 value is the BC world-average experimental value [5].

Source	Method	πFSI	R_3	$\Gamma(^3_\Lambda \mathrm{H})/\Gamma_\Lambda$
RD (1966) [13]	closure-Λpn	no	–	1.14
Congleton (1992) [14]	closure-Λd	no	0.33±0.02	1.15
Kamada (1998) [15]	Faddeev-Λpn	no	0.379	1.06
GG (2019) [16]	closure-Faddeev-Λpn	no	0.362	1.11±0.01
HH (2020) [17]	πEFT(LO)-Λd	no	0.37±0.05	0.98±0.15
GG (2019) [16]	closure-Faddeev-Λpn	yes	0.357	1.23±0.02
POGFG (2020) [18]	πEFT(LO) $\Lambda pn+\Sigma NN$	yes	0.35±0.04	$1.38^{+0.18}_{-0.14}$

A taste of what Theory has to say about the hypertriton lifetime is demonstrated by a representative selection of $^3_\Lambda \mathrm{H_{g.s.}}(\frac{1}{2}^+)$ lifetime calculations assembled in Table 1. Its upper part lists works that disregard pion final-state interaction (πFSI). The closure approximation was used in Refs. [13, 14, 16], whereas the other two works [15, 17] accounted microscopically for the outgoing nucleon phase space and FSI. The lifetimes $\tau(^3_\Lambda \mathrm{H})$=1/$\Gamma(^3_\Lambda \mathrm{H})$ derived from this upper part are shorter than τ_Λ by less than 13%. In contrast, the two works listed in the lower part of the table suggest $^3_\Lambda \mathrm{H}$ lifetimes shorter than τ_Λ by more than 20%, owing primarily to πFSI where outgoing-pion plane waves are superseded by realistic pion distorted waves. This enhances $\Gamma(^3_\Lambda \mathrm{H})$ by about 10% [16], or more realistically by 15% [18]. Interestingly, the recent πEFT work [18] is the only one that bothered to address the small ΣNN components, of order $\lesssim$1% probability, in the ΛNN dominated $^3_\Lambda \mathrm{H}$. A relatively large reduction of $\Gamma(^3_\Lambda \mathrm{H})$ by $\approx$10% was found, owing to interference between the $\Sigma \to N\pi$ and $\Lambda \to N\pi$ weak-decay amplitudes. This effect was disregarded by Kamada *et al.* [15], thereby rendering their widely cited lifetime questionable.

The $^3_\Lambda \mathrm{H}$ lifetime results shown in Table 1 hold generally for $^3_\Lambda \mathrm{H}$ wavefunctions corresponding to given B_Λ values within the emulsion $B_\Lambda(^3_\Lambda \mathrm{H})$=0.13±0.05 MeV interval. The B_Λ dependence of $\tau(^3_\Lambda \mathrm{H})$ was studied in two of these works, with almost B_Λ-independent lifetime calculated in LO πEFT [17] and a rather strong B_Λ dependence LO πEFT [18], as shown in Fig. 1. The three red points in the figure correspond to three distinct values of $B_\Lambda(^3_\Lambda \mathrm{H})$ reached by considering ultraviolet (UV) cutoff Λ_{UV} values smaller than Λ_{UV}=1200 MeV, beginning at which convergence is assured. This amounts to slightly varying the πEFT short-range input. The resulting B_Λ dependence of the computed two-body decay rate $\Gamma(^3_\Lambda \mathrm{H} \to ^3\mathrm{He}+\pi^-)$ is satisfied by the blue points, all for Λ_{UV}=1200 MeV, obtained by varying systematically within allowed uncertainties some of the NN and YN chiral fit data [19]. Of the three red points, the middle one is that highlighted in Table 1 while the left one corresponds to $B_\Lambda(^3_\Lambda \mathrm{H})$=0.069 MeV and $\tau(^3_\Lambda \mathrm{H})$=234±27 ps, a lifetime compatible with both ALICE and STAR new preliminary lifetime values [11, 12].

Given the strong correlation found in the πEFT calculation [18] between $B_\Lambda(^3_\Lambda \mathrm{H})$ and $\tau(^3_\Lambda \mathrm{H})$, a $^3_\Lambda \mathrm{H}$ lifetime as close to τ_Λ supports ALICE preliminary value $B_\Lambda(^3_\Lambda \mathrm{H})$=0.05±0.06±0.10 MeV [20] over STAR's published value 0.41±0.12±0.11 MeV [21]. As demonstrated in Ref. [18], going to as high values of $B_\Lambda(^3_\Lambda \mathrm{H})$ would lead to considerably shorter values of $\tau(^3_\Lambda \mathrm{H})$, which are compatible perhaps with STAR's published value [9], but are incompatible with STAR's new preliminary value [12].
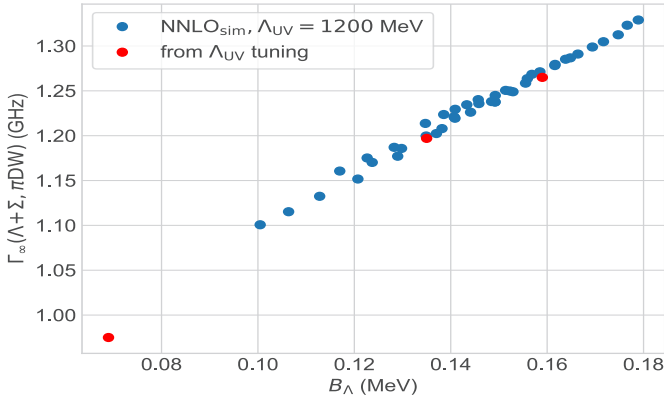
Figure 1. $\Gamma(^3_\Lambda H \to \pi^- + ^3He)$ vs. $B_\Lambda(^3_\Lambda H)$ in πEFT [19]. The three red points are from Ref. [18], see text.

3 $^3_\Lambda$n lifetime

$^3_\Lambda$n was conjectured by the HypHI GSI Collaboration [22] to be particle stable, while unstable unanimously in recent theoretical calculations [23–25]. In $^3_\Lambda$n decays induced by $\Lambda \to p + \pi^-$, where $^3_\Lambda$n neutrons are spectators, the $^3_\Lambda$n $\to (pnn) + \pi^-$ weak decay rate is given in the closure approximation essentially by the $\Lambda \to p + \pi^-$ free-space weak-decay rate, whereas in $\Lambda \to n + \pi^0$ induced decays the production of a third low-momentum neutron is suppressed by the Pauli principle, so this $^3_\Lambda$n weak decay branch may be disregarded up to perhaps a few percents. Hence $\Gamma(^3_\Lambda n)/\Gamma_\Lambda \approx 1.114 \times 0.641 = 0.714$ [16], where the factor 1.114 follows from a difference between recoil energies in the $^3_\Lambda$n and Λ phase space factors, and the factor 0.641 is the free-space $\Lambda \to p + \pi^-$ fraction of the total $\Lambda \to N + \pi$ weak decay rate. This corresponds to a $^3_\Lambda$n lifetime estimate of $\tau(^3_\Lambda n) \approx 368$ ps, considerably longer than $181^{+30}_{-24} \pm 25$ ps or $190^{+47}_{-35} \pm 36$ ps deduced from the $nd\pi^-$ and $t\pi^-$ alleged $^3_\Lambda$n decay modes [22, 26], and thereby questioning the conjectured stability of $^3_\Lambda$n.

4 $^4_\Lambda$H and $^4_\Lambda$He lifetimes

A preliminary STAR lifetime value for $^4_\Lambda$H was reported in this meeting: $\tau(^4_\Lambda H)=218\pm8\pm12$ ps [12]. Do we understand this value in simple terms? A back-of-the-envelope estimate gives the following approximate expressions for $^4_\Lambda$H (Z=1) and $^4_\Lambda$He (Z=2) decay rates relative to the free Λ decay:

$$\Gamma(^4_\Lambda Z)/\Gamma_\Lambda \approx (1 + \eta(\bar{q})) \times (\alpha_Z \times 0.7 + 1 \times 0.3) + 0.20, \tag{1}$$

where $\eta(\bar{q}) \approx 0.50 \pm 0.05$ is an exchange contribution to the dominant s-wave pionic weak decay rate, $\alpha_Z = \frac{2}{3} (\frac{1}{3})$ for Z=1 (Z=2) by applying the $\Delta I = \frac{1}{2}$ rule to the π^4He two-body decay modes, $R_4=0.7$ is the pionic two-body decay branching ratio [27], with $1-R_4=0.3$ standing for the pionic multi-body part. Additional factors arising from recoil kinematics enhancement and p-wave decay suppression largely cancel out. Finally, the factor 0.20 stands for the observed $\Lambda N \to NN$ non-mesonic (n.m.) decay fraction $\Gamma_{n.m.}/\Gamma_\Lambda$ [28, 29]. The resulting $\tau(^4_\Lambda Z)=1/\Gamma(^4_\Lambda Z)$ lifetime estimates

$$\tau_{th}(^4_\Lambda H) = 195 \pm 10 \text{ ps}, \quad \tau_{th}(^4_\Lambda He) = 263 \pm 13 \text{ ps}, \tag{2}$$

with a 5% assigned theoretical uncertainty, are in good agreement with $\tau_{\text{exp}}(^4_\Lambda\text{H})=193\pm25$ ps and with $\tau_{\text{exp}}(^4_\Lambda\text{He})=250\pm19$ ps as measured at KEK [28] and confirmed, for $^4_\Lambda\text{He}$, at BNL [29]. A preliminary value $\tau_{\text{exp}}(^4_\Lambda\text{H})=180\pm7$ ps (stat. only) was derived in a test run of E73 at J-PARC [30].

5 Charge symmetry breaking

A special feature of the $^4_\Lambda\text{H}$–$^4_\Lambda\text{He}$ mirror hypernuclei is the particularly strong charge symmetry breaking (CSB) reflected in their spectra, as shown in Fig. 2. Although one-pion exchange (OPE) does not contribute directly to the ΛN strong interaction owing to isospin invariance, it does contribute as pointed out by Dalitz and von Hippel (DvH) through a CSB potential $V_{\text{CSB}}^{\text{OPE}}$ generated by admixing the SU(3) octet $\Lambda_{I=0}$ and $\Sigma^0_{I=1}$ hyperons in the physical Λ hyperon [34]. For the $^4_\Lambda\text{H}$–$^4_\Lambda\text{He}$ g.s. levels built on the mirror ^3H–^3He cores, and using $A=4$ wavefunctions generated within a LO πEFT no-core shell-model calculation [35], the regularized Yukawa tail of $V_{\text{CSB}}^{\text{OPE}}$ gives rise to

$$\text{OPE (DvH):} \quad \Delta B_\Lambda^{J=0} \approx 175 \pm 40 \text{ keV}, \quad \Delta B_\Lambda^{J=1} \approx -50 \pm 10 \text{ keV}, \tag{3}$$

where $\Delta B_\Lambda^J \equiv B_\Lambda^J(^4_\Lambda\text{He}) - B_\Lambda^J(^4_\Lambda\text{H})$. Remarkably, the large $\Delta B_\Lambda^{J=0}$ OPE CSB (central plus tensor) contribution to the splitting of the $^4_\Lambda\text{H}$–$^4_\Lambda\text{He}$ mirror g.s. levels roughly agrees with the observed value $\Delta B_\Lambda^{J=0}=233\pm92$ keV shown in Fig. 2 which is considerably larger than the ≈ 70 keV CSB part of the Coulomb-dominated $\Delta B(^3\text{H}$–$^3\text{He})=764$ keV in the mirror core nuclei, driven apparently by short-range ρ^0-ω mixing. Considerably smaller hypernuclear CSB contributions were found in ΛNNN calculations by Coon *et al.* [36]: $\Delta B_\Lambda^{J=0}(\pi^0\eta + \rho^0\omega) \approx -20$ keV, $\Delta B_\Lambda^{J=0}(\pi^0\eta + \rho^0\omega) \approx -10$ keV.
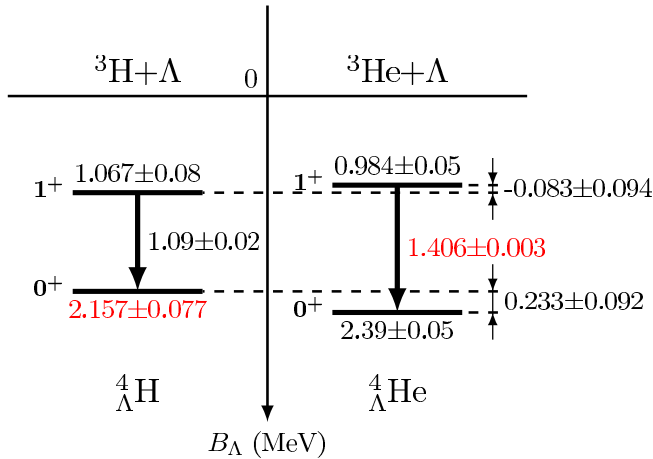

Figure 2. $A = 4$ hypernuclear level scheme. Recent measurements of $^4_\Lambda\text{He}(1^+)$ excitation energy [31] and of $^4_\Lambda\text{H}(0^+_{\text{g.s.}})$ binding energy [32] are marked in red. CSB splittings are shown to the right of the $^4_\Lambda\text{He}$ levels. Preliminary values from STAR are $\Delta B_\Lambda^{J=0}=0.13\pm0.13\pm0.07$ MeV, $\Delta B_\Lambda^{J=1}=-0.19\pm0.13\pm0.07$ MeV [33].

6 Onset of $\Lambda\Lambda$ hypernuclear binding

Reliable data on $\Lambda\Lambda$ hypernuclei are scarce. the Nagara emulsion event [37, 38] identified unambiguously as $^6_{\Lambda\Lambda}\text{He}$, with $\Delta B_{\Lambda\Lambda}(^6_{\Lambda\Lambda}\text{He})=B_{\Lambda\Lambda}(^6_{\Lambda\Lambda}\text{He})-2B_\Lambda(^5_\Lambda\text{He})=0.67\pm0.17$ MeV [38], is the lightest

particle-stable $\Lambda\Lambda$ hypernucleus found so far. Are there lighter particle-stable $\Lambda\Lambda$ species? This question has been addressed recently in a LO πEFT calculation [39], sketched below, and in a full coupled-channel πEFT calculation at NLO [40], both concluding that the onset of $\Lambda\Lambda$ hypernuclear binding is given by the isodoublet $^{5}_{\Lambda\Lambda}$H–$^{5}_{\Lambda\Lambda}$He.

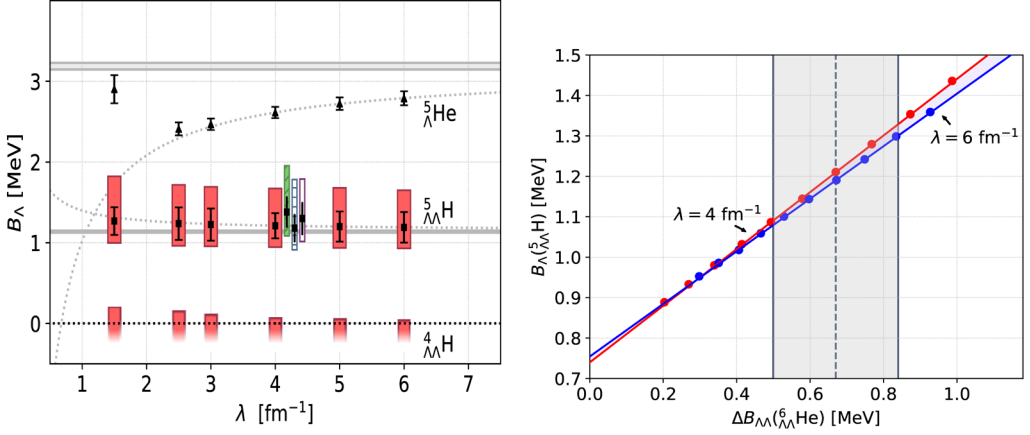**Figure 3.** Left: $B_\Lambda(^{5}_{\Lambda}$He, $^{4}_{\Lambda\Lambda}$H, $^{5}_{\Lambda\Lambda}$H) from πEFT calculations [39]. Black error bars reflect given uncertainties in $^{3}_{\Lambda}$H, $^{4}_{\Lambda}$H, $^{4}_{\Lambda}$H*, $^{6}_{\Lambda\Lambda}$He B_Λ input data, red rectangles arise by varying $a_{\Lambda\Lambda}$ between -0.5 to -1.9 fm. Thin dotted lines show extrapolations to $\lambda \to \infty$ limits marked by gray horizontal bands. Right: Tjon lines calculations of $B_\Lambda(^{5}_{\Lambda\Lambda}$H) vs. $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}$He), with vertical straight lines marking the experimental uncertainty of $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}$He).

Λ separation energy values $B_\Lambda(^{5}_{\Lambda\Lambda}$H) from πEFT calculations [39] are shown in the left panel of Fig. 3. Several representative values of the $\Lambda\Lambda$ scattering length were used from a broad range of values suggested by analyses of $\Lambda\Lambda$ correlations derived recently in relativistic heavy-ion collisions and by analyzing the KEK-PS E522 [41] invariant mass spectrum in the reaction ^{12}C$(K^-, K^+)\Lambda\Lambda X$ near the $\Lambda\Lambda$ threshold; see Ref. [39] for references. Here the choice of $a_{\Lambda\Lambda}$ determines the one $\Lambda\Lambda$ low-energy constant (LEC) required at LO, while the $\Lambda\Lambda N$ LEC was fitted to the $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}$He)=0.67$\pm$0.17 MeV datum [38]. The reported calculations used several ΛN scattering-length combinations, demonstrating that the ΛN model dependence is weak when it comes to double-Λ hypernuclei, provided B_Λ values of single-Λ hypernuclei for $A < 5$ are fitted to generate the necessary ΛNN LECs. With values of $B_\Lambda(^{5}_{\Lambda\Lambda}$H)$\sim$1 MeV, calculated over a broad range of cutoff values λ, it is clear that the particle stability of $^{5}_{\Lambda\Lambda}$H is robust, in contrast to $^{4}_{\Lambda\Lambda}$H for which $B_\Lambda(^{4}_{\Lambda\Lambda}$H) comes out negative over most of the permissible parameter space in these calculations. Finally, Tjon-line correlations [42] between $B_\Lambda(^{5}_{\Lambda\Lambda}$H) and $B_\Lambda(^{6}_{\Lambda\Lambda}$He) are demonstrated in the right panel of Fig. 3. We conclude that the isodoublet $^{5}_{\Lambda\Lambda}$H–$^{5}_{\Lambda\Lambda}$He marks the onset of $\Lambda\Lambda$ hypernuclear binding. J-PARC Experiment E75 will search for $^{5}_{\Lambda\Lambda}$H [43].

Acknowledgments

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