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Interpretation of Λ hyperon spin polarization measurements

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Abstract. A new global polarization measurement method based on the projection of the polarization vector on the properly defined system's orbital angular momentum in the Λ hyperon's rest frame is briefly described. The advantages of the new method over the standard procedure are discussed.

1 Introduction

The recent measurements of the spin polarization of particles emitted in non-central relativistic heavy-ion collisions [1–4] are quite often listed among the most interesting experimental findings in contemporary high-energy physics [5–9]. The quantum nature of these phenomena provides new insights into the properties of the hot and dense strongly-interacting matter and triggers many new theoretical developments [10–25]. Apart from theoretical challenges, the experimental procedures employed to measure the new phenomena also require better understanding [26]. In this work, we discuss the issues connected with relativistic character of the spin measurement of Λ hyperons in heavy-ion collisions and propose a new method which may improve its interpretation.

2 The weak-decay law

The magnitude P' and orientation $\hat{\boldsymbol{P}}'$ of the spin polarization vector $\boldsymbol{P}' = P'\hat{\boldsymbol{P}}'$ of the Λ hyperon is usually determined through its parity-violating weak-decay, $\Lambda \to p + \pi^-$, employing the fact that in this process the daughter proton is preferentially emitted along the direction of the Λ spin. The measurement is performed in the Λ rest frame $S'(\boldsymbol{p}_\Lambda)$ which is related to the center-of-mass (COM) frame through the (canonical) Lorentz boost [27, 28]. In the $S'(\boldsymbol{p}_\Lambda)$ frame the angular distribution of emitted protons is given by the expression

$$\frac{dN_p^{\rm pol}}{d\Omega'} = \frac{1}{4\pi}\left[1 + \boldsymbol{P}'\cdot\hat{\boldsymbol{p}}'_p\right] = \frac{1}{4\pi}\left[1 + \alpha_\Lambda P'\left(\cos(\Phi' - \phi'_p)\sin\theta'_p\sin\Theta' + \cos\theta'_p\cos\Theta'\right)\right], \quad (1)$$

where $\hat{\boldsymbol{p}}'_p = \left(\sin\theta'_p\cos\phi'_p, \sin\theta'_p\sin\phi'_p, \cos\theta'_p\right)$ and $\hat{\boldsymbol{P}}' = (\sin\Theta'\cos\Phi', \sin\Theta'\sin\Phi', \cos\Theta')$, see Fig. 1. The quantity $\alpha_\Lambda = 0.732$ in Eq. (1) is the Λ decay constant.

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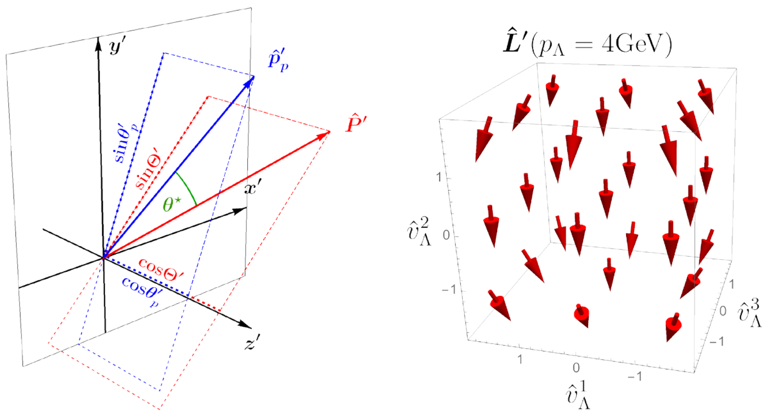

Figure 1. Left: The polarization vector $\boldsymbol{P}'$ and the proton momentum direction $\hat{\boldsymbol{p}}'_p$ as viewed in the Λ rest frame $S'(\boldsymbol{p}_\Lambda)$. The weak decay law $\Lambda \to p + \pi^-$ depends on $\cos\theta^* \equiv \cos(\Phi' - \phi'_p)\sin\theta'_p \sin\Theta' + \cos\theta'_p \cos\Theta'$, where θ^* is the angle between the two vectors, see also Eq. (1). Right: Visualization of the vectors $\boldsymbol{L}'$ for Λ with different velocity directions.

Using Eq. (1) the polarization vector can be fully determined from the averaged values of the proton's three-momentum components

$$\boldsymbol{P}' = \frac{3}{\alpha_\Lambda}\left(\langle \hat{p}'_{p,x}\rangle, \langle \hat{p}'_{p,y}\rangle, \langle \hat{p}'_{p,z}\rangle\right), \tag{2}$$

where we have used the notation $\langle \hat{p}'_{p,i}\rangle = \int \left(\frac{dN_p^{\rm pol}}{d\Omega'}\right) \hat{p}'_{p,i} \sin\theta'_p \, d\theta'_p \, d\phi'_p$.

From theoretical point of view, a particularly interesting quantity, also known as *global polarization*, is

$$P_H = \frac{8}{\pi\alpha_\Lambda}\langle \sin\phi'_p\rangle, \tag{3}$$

with $P_H = P' \sin\Theta' \sin\Phi'$ being the y-component of the polarization three-vector as seen in the Λ rest frame. This quantity has a straightforward interpretation in the non-relativistic limit (small momenta of Λ's) since, in this case, the y direction of the Λ rest frame agrees with the y direction of the COM frame, which in turn defines the direction of the global orbital angular momentum vector of the colliding system, namely

$$\hat{\boldsymbol{L}} = \frac{\boldsymbol{L}}{L} = (0, -1, 0). \tag{4}$$

Due to angular momentum conservation, the final particle-averaged angular momentum of emitted Λ's connected with spin is then expected to be directly related to the initial orbital angular momentum of the matter produced in an event. Obviously, for relativistic Λ's the interpretation of Eq. (3) is complicated by the fact that y direction seen in the Λ rest frame and the y direction in COM frame are different. The latter fact constitutes a significant problem because the measurement requires usually averaging over Λ's within a wide momentum range.

3 Correlation with global orbital angular momentum

As shown in Ref. [26] the measurement method discussed above may be considerably improved by noticing that the angular momentum vector components $L^k = -\frac{1}{2}\epsilon^{kij}L^{ij}$ are given by the spatial components of an (antisymmetric) orbital angular momentum tensor $L^{\mu\nu}$ whose Lorentz transformation properties are similar to that of the Faraday tensor [27]. In consequence, instead of the projection $\hat{\boldsymbol{L}} \cdot \boldsymbol{P}'$ one should calculate the quantity

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = \left(1 - (\boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}})^2\right)^{-1/2} \left(\hat{\boldsymbol{L}} \cdot \boldsymbol{P}' - \frac{\gamma_\Lambda}{\gamma_\Lambda + 1} \boldsymbol{v}_\Lambda \cdot \boldsymbol{P}' \;\; \boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}}\right), \tag{5}$$

which is the projection of the polarization vector along the direction of the total angular momentum of the system as seen in the rest frame of the Λ with three-momentum $\boldsymbol{p}_\Lambda = \boldsymbol{v}_\Lambda E_\Lambda = m_\Lambda \gamma_\Lambda \boldsymbol{v}_\Lambda$ in COM.

The main advantage of the formula (5) compared to $\hat{\boldsymbol{L}} \cdot \boldsymbol{P}'$ is that the spin vector of each Λ, irrespectively of its three-momentum $\boldsymbol{p}_\Lambda$ in COM, is projected on the same physical axis corresponding to the orbital angular momentum in COM. Hence, Eq. (5) is in our opinion the better estimate of the global polarization of the system and can be used to study the correlation between the polarization of emitted particles with the system's global angular momentum.

4 Numerical estimates

In order to quantify possible numerical differences resulting from the measurement based on Eq. (5) as compared to $\hat{\boldsymbol{L}} \cdot \boldsymbol{P}'$ one has to perform the average over the Λ's with different momenta. For that purpose we consider a simplified scenario where $\boldsymbol{P}' = P'\hat{\boldsymbol{L}}$. In this case one obtains

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = P' \left(1 - v_2^2\right)^{-1/2} \left(1 - \frac{v_2^2}{1 + \sqrt{1 - v^2}}\right) \equiv P' F_P(\boldsymbol{v}). \tag{6}$$

where (v_1, v_2, v_3) are the components of the Λ velocity in COM and $v = \sqrt{v_1^2 + v_2^2 + v_3^2}$. In the following we also assume the Fermi-Dirac form of the velocity distribution of Λ's, namely $F_T(v) = N\left[\exp\left(m_\lambda/(T_{\rm eff}\sqrt{1 - v^2})\right) + 1\right]^{-1}$, where $T_{\rm eff}$ is an effective temperature and N is an (irrelevant) normalization constant. The quantity

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{m-n} = P' \; \frac{\int_{v_{(m)}}^{v_{(n)}} dv \int d\Omega \, F_P(\boldsymbol{v}) F_T(v)}{\int_{v_{(m)}}^{v_{(n)}} dv \int d\Omega \, F_T(v)}, \tag{7}$$

where $v_{(n)} = \tanh\left[\sinh^{-1}\left(\frac{n\,\mathrm{GeV}}{m_\Lambda}\right)\right]$ gives average value of the formula (5) for Λ's with the momentum in the range (m,n) GeV. The numerical estimates using $T_{\rm eff} = 150$ MeV give: $\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{2-3} = 0.97\, P'$, $\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{3-4} = 0.94\, P'$, $\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{4-5} = 0.92\, P'$, and $\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{5-6} = 0.90\, P'$. In consequence, we expect that the effects accounted for by the newly proposed method may reach 10% for the most energetic Λ's studied at STAR.

5 Conclusions

In this work we have discussed possible interpretation issues connected with the current global spin polarization measurements in heavy-ion collisions resulting from relativistic character of the measured particles. Using the Lorentz transformation properties of the global angular momentum we have proposed a new measurement procedure which may serve as a possible solution to these problems. We have shown that the new observable may lead to quantitative differences of up to 10% of the currently measured signal.

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