Carbon burning rates on the compound nucleus formation

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Abstract. The $^{12}\text{C}+^{12}\text{C}$ reaction rates based on the compound nucleus formation seem to be concordant with the standard rates. The resonant contribution in $^{12}\text{C}+^{12}\text{C}$ is also discussed. To put the rates on firm ground, the resonances below $E_{\text{c.m.}} = 3$ MeV will have to be studied further.

1 Introduction

The $^{12}\text{C}+^{12}\text{C}$ fusion reaction is one of the key reactions to understand the evolution of massive stars and various explosive scenarios. However, precise measurements of cross sections below $E_{\text{c.m.}} = 3$ MeV are difficult because of the tiny amplitudes caused by the Coulomb barrier. At present, the direct measurements have been performed in $E_{\text{c.m.}} = 2.1$–$2.5$ MeV [1]. The indirect measurements have been studied with $^{24}\text{Mg}(\alpha,\alpha')$ [2] and Trojan horse method [3]. The derived rates [3] are much faster than CF88 [4], due to the resonant states at $E_{\text{c.m.}} \approx 1.5$ MeV, which may have the $^{12}\text{C}+^{12}\text{C}$ molecule-like structure. The nuclear fusion for $^{12,13}\text{C}+^{12}\text{C}$ have also been discussed experimentally to understand C+C comprehensively [5].

In this presentation, I use a barrier penetration model (BPM), and I show the calculated results of isotope dependence of fusion cross sections and reaction rates in C+C. The transmission coefficients are given by the WKB approximation, semi-classically, and the potentials used in the present work are calculated from a single-folding model [6] with [7, 8]. I also discuss the contribution from the resonances in $^{12}\text{C}+^{12}\text{C}$ by comparing the result of BPM with a schematic calculation of the coupled-channels multi-level R-matrix [9].

2 Compound nucleus formation

Before moving on to the results, let me recall the compound nucleus (CN) formation, to understand the reaction mechanism in C+C. The $^{12}\text{C}+^{12}\text{C}$ potential obtained from the studies of elastic scattering has predicted the sequences of the rotational excitation in $^{24}\text{Mg}$ [7, 10]. These resonances are the excited states with the $^{12}\text{C}+^{12}\text{C}$ molecule-like structure in $^{24}\text{Mg}$. However, the potential resonances at $E_{\text{c.m.}} \approx 0$ are dispersed easily, because of the couplings to reaction channels. Although the inelastic channels are closed at $E_{\text{c.m.}} = 4.44$ MeV, other reaction channels are open, and they work as absorption to the entrance channel. Accordingly, their fragments are distributed around the original energy positions. In fact, many fragments of $J^\pi = 2^+, 4^+$ resonances have been observed, in addition to $0^+$ [3]. Whereas most of flux are consumed by Coulomb scattering, a small amount of flux is captured into the long-living fragment levels, and exits through the proton, neutron, and $\alpha$ channels after forming a compound nucleus. Under the circumstance, the reactions should be described statistically.

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and the emitted nuclei have to be treated as evaporation products. Therefore, I adopt BPM and $R$-matrix based on the CN formation in the present study.

In BPM, the energy-averaged fusion cross sections are given by $\sigma_F = (\pi/k^2) \sum_{L}(2L + 1)\langle |S_{CN}|^2 \rangle$. $k$ is the wavenumber; $L$ is the angular momentum between nuclei. $\langle |S_{CN}|^2 \rangle$ are the transmission coefficients $T_L$, given by WKB approximation for $E_{c.m.} < E_B$: $T_L = [1 + \exp \left(2 \int_{R_1}^{R_2} K(R) dR \right)]^{-1}$, $K(R) = (2\mu)/\hbar^2 [\hat{U}_L(R) - E_{c.m.}]^{1/2}$. $E_B$ is the barrier height energy. $\mu$ is the reduced mass. $R_1$ and $R_2$ are the inner and outer turning points of effective potentials $\hat{U}_L$. The nuclear potentials in $\hat{U}_L$ are calculated recursively from the single-folding model [6–8]. The cross sections are also given by $R$-matrix theory, $\sigma_F = (\pi/k^2) \sum_{L}(2L + 1) S_{c,0}^L$. $S_{c,0}^L$ is the $S$-matrix deduced from the $R$-matrix with the resonance parameters in [3]. The reduced width of $^{12}\text{C}+^{12}\text{C}$ is statistically assumed to be a constant $\gamma_{il}^2 = 0.001\gamma_w^2$ for all levels, based on the CN formation. $\gamma_w^2$ is the Wigner limit. In [5], the transmission coefficients are calculated from an approximation using the unitarity relation of $S$-matrix. To display the cross sections, the $S^*$ factors are defined as $S^* = \sigma_F E_{c.m.} \exp (87.21 E_{c.m.}^{-1/2} + 0.46 E_{c.m.})$, e.g. [3].

3 Results & Conclusion

The calculated $S^*$ factors with BPM for $^{12}\text{C}+^{12}\text{C}$ are shown by the solid curve in Fig. 1(A), and they appear to give the trend of the energy variation in the experimental data [1, 11]. The derived reaction rates are shown in ratio to CF88. (Fig. 1(B)) They seem consistent with CF88. For $^{12,13}\text{C}+^{13}\text{C}$ and $^{12}\text{C}+^{14,15}\text{C}$, the present calculations of BPM (solid curves) reproduce the experimental data [5, 12], consistently, as shown in Figs. 1(C) – 1(F).

Figure 2(A) illustrates the isotope dependence of $S^*$ obtained from BPM. In BPM, the $^{12}\text{C}+^{12}\text{C}$ $S^*$ factors below $E_{c.m.} = 2$ MeV become the largest, so the derived reaction rates are the fastest below $T_9 = 0.8$ (Fig. 2(B)). In addition, the $S^*$ factors are found to be enhanced at the sub-barrier energies as the number of neutrons increases. Particularly, those of $^{12}\text{C}+^{15}\text{C}$ are enhanced larger. The barrier radius $R_B$ and barrier height energy $E_B$ in the present calculations are shown in Figs. 2(C) and 2(D), as a function of the mass number. $R_B$ ($E_B$) becomes large (small) as the number of neutrons increases. Especially, $R_B$ suddenly becomes large at $^{15}\text{C}$. This is caused by the weakly-bound $s$-wave neutron in $^{15}\text{C}$. Therefore, the corresponding $S^*$ and reaction rates are expected to be enhanced more by the sharp reduction of $E_B$.

The resonant contribution in $^{12}\text{C}+^{12}\text{C}$ is shown in Fig. 3. From the result of the $R$-matrix calculation by the solid curve in Fig. 3(A), the values of $S^*$ are found to be much smaller than those of [3]. In the present calculation, I include the same 34 levels and four exit channels as
those in [3]. If $\gamma^2_{IL} = 0.05\gamma^2_W$ is used as the reduced width of $^{12}\text{C}+^{12}\text{C}$ at $E_{c.m.} \approx 1.5$ MeV, the reaction rates would increase like those in [3]. The carbon burning rates are sensitive to the reduced width of $^{12}\text{C}+^{12}\text{C}$. In addition, the reaction rates estimated from the $R$-matrix extrapolation are confirmed to be reduced from the result of BPM. However, the derived rates at $T_9 = 0.6$ still seem to be consistent with CF88. To put the rates on firm ground, the resonances below $E_{c.m.} = 3$ MeV will have to be scrutinized further.

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References

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