The structure of $0^+$ states in $^{16}$O using real-time evolution method

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Abstract. $\alpha + ^{12}$C clustering in $^{16}$O has been vigorously studied. In the 2000s, a new picture was proposed that the Hoyle state, $^{12}$C($0^+_2$), is a Bose-Einstein condensate of three $\alpha$ particles by the so-called THSR framework. As a next step, many researchers are interested in $4\alpha$ condensate state in $^{16}$O. In this work, a microscopic calculation named the real-time evolution method (REM) was first applied to a $4\alpha$ system. As a result, the $0^+$ states in $^{16}$O up to $4\alpha$ condensate state were expected to be reproduced simultaneously for the first time.

1 Introduction

At the beginning of this century, a new picture was proposed that the Hoyle state is regarded as the Bose-Einstein condensate of three $\alpha$ particles. It provides us a unique opportunity to study the condensate of bosons composed of four fermions. Up to now, the $3\alpha$ condensate is almost established, and thus we focus on the $4\alpha$ system. Several works for $^{16}$O, $4\alpha$ Orthogonality Condition Model (OCM) [1], and extended Tohsaki-Horiuchi-Schuck-Röpke (eTHSR) [2] have been performed. In these studies, the $0^+_5$ state was theoretically concluded as a candidate of the $4\alpha$ condensate [1, 2]. However, the former treats an $\alpha$ particle as a boson and the latter imposes the restrictions on the symmetry of the system resulting in the lack of $0^+_5$ state in the $4\alpha$ OCM. Therefore we employ a microscopic model, REM [3].

2 Theoretical Framework

The Hamiltonian for the $N\alpha$ systems composed of $4N$ nucleons is the same as in the original paper of the REM [3] except the effective nucleon-nucleon 2(3)-body interaction. For evaluating our method with others, we used the Tohsaki No. 1 effective nucleon-nucleon interaction [4]. The intrinsic wave function of the $N\alpha$ system is defined as

$$
\Phi(Z_1, \cdots, Z_N) = \mathcal{A}[\Phi_\alpha(Z_1) \cdots \Phi_\alpha(Z_N)],
$$

where $\mathcal{A}$ represents the antisymmetrization operator. The $\Phi_\alpha(Z)$ denotes the wave packet of the $\alpha$ cluster located at $Z$. The normalized GCM wave function of $^{16}$O, $\Psi^0_{16O}$, is defined as

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\[ \Psi^{0+}_{16O} = \sum_i \{ f_i \hat{P}^{0+} \Phi_i(Z_1, \ldots, Z_4) + g_i \hat{P}^{0+} \Phi_i(Z_1', \ldots, Z_4') \}, \quad (2) \]

where \( \hat{P}^{0+} \) is the angular-momentum (\( J \)) and the parity (\( \pi \)) projection operator for \( J^\pi = 0^+ \). In the REM, the position and momentum of \( \alpha \) particles are governed by the equation-of-motion (EOM) of the Gaussian wave packets. In principle, the \( \Psi^{0+}_{16O} \) can be divided into the \( \alpha + ^{12}\text{C} \) component \( \Psi^{0+}_{\alpha+^{12}\text{C}} \) and the residual part \( \Psi^{0+}_{\text{res}} \). The squared amplitude \( |w|^2 \), where \( w \) is the coefficient of the \( \Psi^{0+}_{\alpha+^{12}\text{C}} \), can be estimated using the method in the Ref. [5].

### 3 Results

Results were obtained as follows: The intrinsic excitation energy and rebound radius in the REM are 40 MeV and 16.0 fm, respectively. The time of the EOM was evolved until 6000 fm/c by 6 fm/c units. We used the \( r^2 \)-constraint method [6]. The cutoff value is 7.0 fm.

We first discuss the excitation energy spectra measured from the \( 4\alpha \) threshold and r.m.s. charge radii compared with the eTHSR [2] in Fig. 1. As for the bound states, \( 0_1^+ \) and \( 0_2^+ \), the energies are slightly lower than that of the eTHSR reflecting the size of model space. Table 2 shows the calculated and observed r.m.s. radii and monopole matrix elements from the \( 0_1^+ \) state. The monopoles can be understood as an enhancement of the clustering in the excited states. Considering the consistency of the radius and monopole with the OCM, the \( 0_2^+ \) state in the REM is expected to be a corresponding state of \( 0_2^+ \) state in the OCM. Note that the \( 0_2^+ \) state in the OCM is mainly composed of \( \alpha + ^{12}\text{C}(1^-) \) which is not described in the eTHSR due to the size of the model space. The \( 0_2^+ \) state in the eTHSR is said to be a \( 4\alpha \) condensate. As for the \( 0_3^+ \) state in the REM, though the radius is almost consistent with that of the eTHSR, the monopole is much larger than that of the eTHSR, which imply that the \( 0_3^+ \) state in the REM is a candidate of \( 4\alpha \) condensate and has a characteristic property of the clustering enhancement.

![Figure 1](image1.png) **Figure 1.** Energy spectra of \( ^{16}\text{O}(0^+) \) states measured from the \( 4\alpha \) threshold of the eTHSR [2] and REM.

![Figure 2](image2.png) **Figure 2.** Density distribution of the intrinsic wave function maximized for \( 0_7^+ \) state using Eq. (3).

We estimated the squared amplitudes of \( \alpha + ^{12}\text{C}(0^+) \) components \( |w|^2 \), which is shown in Table 1. The \( ^{12}\text{C}(0^+) \) state was made by the same process explained in the original paper of the REM [3]. The maximized overlaps defined in Eq. (3) for each \( ^{16}\text{O}(0^+) \) state and the configurations are also shown in Table 1.

\[
O = \frac{|\langle \Psi^{0+}_{16O}|P^{0+}\Phi_{\text{opt}}\rangle|^2}{\langle P^{0+}\Phi_{\text{opt}}|P^{0+}\Phi_{\text{opt}}\rangle}. \quad (3)
\]

| \( ^{16}\text{O} \) | \( |w|^2 \) | \( O \) | Configuration |
|---|---|---|---|
| \( 0_1^+ \) | 0.89 | 0.96 | Tetrahedron |
| \( 0_2^+ \) | 0.80 | 0.64 | \( \alpha(S) + ^{12}\text{C}(0^+_1) \) |
| \( 0_3^+ \) | 0.16 | 0.53 | |
| \( 0_4^+ \) | 0.02 | 0.16 | |
| \( 0_5^+ \) | 0.42 | 0.27 | \( \alpha(S) + ^{12}\text{C}(0^+_1) \) |
| \( 0_6^+ \) | 0.16 | 0.47 | |
| \( 0_7^+ \) | 0.02 | 0.20 | |

Table 1. \( \alpha + ^{12}\text{C}(0^+) \) components \( |w|^2 \), the maximized overlaps in Eq. (3) and the main components.
Table 2. r.m.s. charge radii $R_{\text{rms}}$ and monopole matrix elements from the ground state $M(E0; 0^+_1 \rightarrow 0^+_j)$ of the experiment [7] and the 4$\alpha$ OCM [1], eTHSR [2], and REM calculations in $^{16}$O.

<table>
<thead>
<tr>
<th>$E0$</th>
<th>$0^+_1$</th>
<th>$R_{\text{rms}}$ [fm]</th>
<th>$M(E0; 0^+_1 \rightarrow 0^+_j)$ [efm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP.</td>
<td>2.71(0.02)</td>
<td>2.7</td>
<td>2.7</td>
</tr>
<tr>
<td>OCM</td>
<td>3.0</td>
<td>3.2</td>
<td>3.1</td>
</tr>
<tr>
<td>eTHSR</td>
<td>3.1</td>
<td>3.3</td>
<td>3.2</td>
</tr>
<tr>
<td>REM</td>
<td>4.0</td>
<td>4.9</td>
<td>5.2</td>
</tr>
<tr>
<td>0^+_2</td>
<td>3.1</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>0^+_3</td>
<td>5.6</td>
<td>4.9</td>
<td>4.8</td>
</tr>
</tbody>
</table>

It is easily understood that the $0^+_1$ state has a large squared amplitude and the overlap resulting from the tetrahedral configuration. The $0^+_2, 5$ states have also a rather large squared amplitude, which means that these states are composed of $\alpha + ^{12}$C$(0^+_1)$. This is consistent with the 4$\alpha$ OCM [1] in which they say that the $0^+_1$ state in the 4$\alpha$ OCM is the higher nodal state of the $0^+_2$ state. Although the $0^+_3, 6$ states have overlaps about 50%, the squared amplitude of the $\alpha + ^{12}$C$(0^+_1)$ component is small, which implies that these states are composed of the other types of configurations; i.g. $\alpha + ^{12}$C$(2^+)$, $\alpha + ^{12}$C$(1^-)$. Note that the $0^+_4$ state is expected as the continuum state due to the quite large radius. It is expected that the 4$\alpha$ condensate is composed of $\alpha + ^{12}$C$(0^+_1)$ resulting in the small squared amplitude of the $\alpha + ^{12}$C$(0^+_1)$ component and small overlap. Therefore the $0^+_7$ state is a candidate of the 4$\alpha$ condensate. The density distribution of the intrinsic wave function maximized for the $0^+_7$ state is shown in Fig. 2. It looks that The four $\alpha$ particles are loosely interacting with each other.

4 Summary

We focused on the $0^+$ states in $^{16}$O to investigate the 4$\alpha$ condensate state by adopting the REM analyzing the excitation energy, r.m.s. charge radius and monopole matrix element. Additionally, we showed the squared amplitude of the $\alpha + ^{12}$C$(0^+_1)$ component As a result, a candidate of the 4$\alpha$ condensate state was obtained. The $^{16}$O$(0^+)$ states up to the 4$\alpha$ condensate state were expected to be microscopically described simultaneously for the first time. We will calculate the $\alpha + ^{12}$C$(0^+_2)$ component to conclude the $0^+_7$ state as the 4$\alpha$ condensate state.

Acknowledgments

The numerical calculations were carried out on the high performance computing server at RCNP in Osaka University and on XC40 at YITP in Kyoto University.

References