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The structure of 0^+ states in ^{16}O using real-time evolution method

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Abstract. α + ^{12}C clustering in ^{16}O has been vigorously studied. In the 2000s, a new picture was proposed that the Hoyle state, ^{12}C(0_2^+), is a Bose-Einstein condensate of three α particles by the so-called THSR framework. As a next step, many researchers are interested in 4α condensate state in ^{16}O. In this work, a microscopic calculation named the real-time evolution method (REM) was first applied to a 4α system. As a result, the 0^+ states in ^{16}O up to 4α condensate state were expected to be reproduced simultaneously for the first time.

1 Introduction

At the beginning of this century, a new picture was proposed that the Hoyle state is regarded as the Bose-Einstein condensate of three α particles. It provides us a unique opportunity to study the condensate of bosons composed of four fermions. Up to now, the 3α condensate is almost established, and thus we focus on the 4α system. Several works for ^{16}O, 4α Orthogonality Condition Model (OCM) [1], and extended Tohsaki-Horiuchi-Schuck-Röpke (eTHSR) [2] have been performed. In these studies, the 0_6^+ state was theoretically concluded as a candidate of the 4α condensate [1, 2]. However, the former treats an α particle as a boson and the latter imposes the restrictions on the symmetry of the system resulting in the lack of 0_5^+ state in the 4α OCM. Therefore we employ a microscopic model, REM [3].

2 Theoretical Framework

The Hamiltonian for the $N\alpha$ systems composed of $4N$ nucleons is the same as in the original paper of the REM [3] except the effective nucleon-nucleon 2(3)-body interaction. For evaluating our method with others, we used the Tohsaki No. 1 effective nucleon-nucleon interaction [4]. The intrinsic wave function of the $N\alpha$ system is defined as

$$\Phi(\boldsymbol{Z}_1, \cdots, \boldsymbol{Z}_N) = \mathcal{A}\{\Phi_\alpha(\boldsymbol{Z}_1) \cdots \Phi_\alpha(\boldsymbol{Z}_N)\}, \tag{1}$$

where $\mathcal{A}$ represents the antisymmetrization operator. The $\Phi_\alpha(\boldsymbol{Z})$ denotes the wave packet of the α cluster located at $\boldsymbol{Z}$. The normalized GCM wave function of ^{16}O, $\Psi^{0^+}_{^{16}\mathrm{O}}$, is defined as

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$$\Psi^{0^+}_{^{16}\mathrm{O}} = \sum_i \{ f_i \hat{P}^{0^+} \Phi_i(\boldsymbol{Z}_1, \cdots, \boldsymbol{Z}_4) + g_i \hat{P}^{0^+} \Phi_i(\boldsymbol{Z}^*_1, \cdots, \boldsymbol{Z}^*_4) \}, \tag{2}$$

where $\hat{P}^{0^+}$ is the angular-momentum (J) and the parity (π) projection operator for $J^\pi = 0^+$. In the REM, the position and momentum of α particles are governed by the equation-of-motion (EOM) of the Gaussian wave packets. In principle, the $\Psi^{0^+}_{^{16}\mathrm{O}}$ can be divided into the $\alpha + {}^{12}$C component $\Psi^{0^+}_{\alpha+^{12}\mathrm{C}}$ and the residual part $\Psi^{0^+}_{\mathrm{res}}$. The squared amplitude $|w|^2$, where w is the coefficient of the $\Psi^{0^+}_{\alpha+^{12}\mathrm{C}}$, can be estimated using the method in the Ref. [5].

3 Results

Results were obtained as follows: The intrinsic excitation energy and rebound radius in the REM are 40 MeV and 16.0 fm, respectively. The time of the EOM was evolved until 6000 fm/c by 6 fm/c units. We used the r^2-constraint method [6]. The cutoff value is 7.0 fm.

We first discuss the excitation energy spectra measured from the 4α threshold and r.m.s. charge radii compared with the eTHSR [2] in Fig. 1. As for the bound states, 0^+_1 and 0^+_2, the energies are slightly lower than that of the eTHSR reflecting the size of model space. Table 2 shows the calculated and observed r.m.s. radii and monopole matrix elements from the 0^+_1 state. The monopoles can be understood as an enhancement of the clustering in the excited states. Considering the consistency of the radius and monopole with the OCM, the 0^+_6 state in the REM is expected to be a corresponding state of 0^+_5 state in the OCM. Note that the 0^+_5 state in the OCM is mainly composed of $\alpha + {}^{12}\mathrm{C}(1^-)$ which is not described in the eTHSR due to the size of the model space. The 0^+_V state in the eTHSR is said to be a 4α condensate. As for the 0^+_7 state in the REM, though the radius is almost consistent with that of the eTHSR, the monopole is much larger than that of the eTHSR, which imply that the 0^+_7 state in the REM is a candidate of 4α condensate and has a characteristic property of the clustering enhancement.

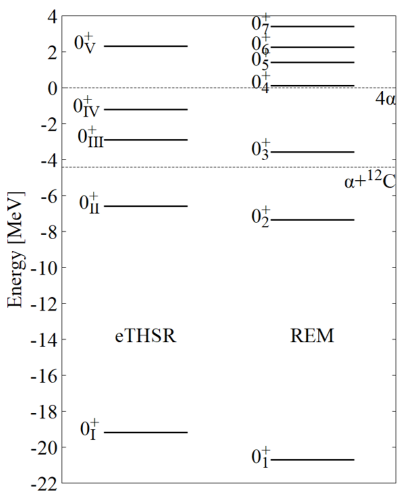

Figure 1. Energy spectra of ^{16}O(0^+) states measured from the 4α threshold of the eTHSR [2] and REM.

Table 1. $\alpha + {}^{12}\mathrm{C}(0^+_1)$ components $|w|^2$, the maximized overlaps in Eq. (3) and the main components.

^{16}O	$\lvert w\rvert^2$	$\mathcal{O}$	Configuration
0^+_1	0.89	0.96	Tetrahedron
0^+_2	0.80	0.64	$\alpha(S) + {}^{12}\mathrm{C}(0^+_1)$
0^+_3	0.16	0.53	
0^+_4	0.02	0.16	
0^+_5	0.42	0.27	$\alpha(S) + {}^{12}\mathrm{C}(0^+_1)$
0^+_6	0.16	0.47	
0^+_7	0.02	0.20	

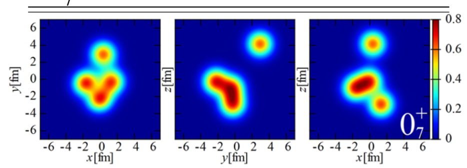

Figure 2. Density distribution of the intrinsic wave function maximized for 0^+_7 state using Eq. (3).

We estimated the squared amplitudes of $\alpha + {}^{12}\mathrm{C}(0^+_1)$ components $|w|^2$, which is shown in Table 1. The ${}^{12}\mathrm{C}(0^+_1)$ state was made by the same process explained in the original paper of the REM [3]. The maximized overlaps defined in Eq. (3) for each ^{16}O(0^+) state and the configurations are also shown in Table 1.

$$\mathcal{O} = |\langle \Psi^{0^+}_{^{16}\mathrm{O}} | P^{0^+} \Phi_{\mathrm{opt}} \rangle|^2 / \langle P^{0^+} \Phi_{\mathrm{opt}} | P^{0^+} \Phi_{\mathrm{opt}} \rangle. \tag{3}$$

Table 2. r.m.s. charge radii $R_{\rm rms}$ and monopole matrix elements from the ground state $\mathcal{M}(E0; 0_1^+ \to 0_f^+)$ of the experiment [7] and the 4α OCM [1], eTHSR [2], and REM calculations in ^{16}O.

	$R_{\rm rms}$[fm]				$\mathcal{M}(E0; 0_1^+ \to 0_f^+)[e\mathrm{fm}^2]$			
REM	EXP.	OCM	eTHSR	REM	EXP.	OCM	eTHSR	REM
0_1^+	2.71(0.02)	2.7	2.7	2.7				
0_2^+		3.0	3.2	3.1	3.55	3.9	5.9	5.8
0_3^+		3.1	3.3	3.2	4.03	2.4	5.7	4.3
0_4^+				5.7				0.9
0_5^+		4.0	4.9	5.2		2.4	0.8	1.1
0_6^+		3.1		3.1	3.3	2.6		2.9
0_7^+		5.6	4.9	4.8		1.0	0.7	2.4

It is easily understood that the 0_1^+ state has a large squared amplitude and the overlap resulting from the tetrahedral configuration. The $0_{2,5}^+$ states have also a rather large squared amplitude, which means that these states are composed of α + ^{12}C(0_1^+). This is consistent with the 4α OCM [1] in which they say that the 0_4^+ state in the 4α OCM is the higher nodal state of the 0_2^+ state. Although the $0_{3,6}^+$ states have overlaps about 50%, the squared amplitude of the α + ^{12}C(0_1^+) component is small, which implies that these states are composed of the other types of configurations; i.g. α + ^{12}C(2^+), α + ^{12}C(1^-). Note that the 0_4^+ state is expected as the continuum state due to the quite large radius. It is expected that the 4α condensate is composed of α + ^{12}C(0_2^+) resulting in the small squared amplitude of the α + ^{12}C(0_1^+) component and small overlap. Therefore the 0_7^+ state is a candidate of the 4α condensate. The density distribution of the intrinsic wave function maximized for the 0_7^+ state is shown in Fig. 2. It looks that The four α particles are loosely interacting with each other.

4 Summary

We focused on the 0^+ states in ^{16}O to investigate the 4α condensate state by adopting the REM analyzing the excitation energy, r.m.s. charge radius and monopole matrix element. Additionally, we showed the squared amplitude of the α + ^{12}C(0_1^+) component As a result, a candidate of the 4α condensate state was obtained. The ^{16}O(0^+) states up to the 4α condensate state were expected to be microscopically described simultaneously for the first time. We will calculate the α + ^{12}C(0_2^+) component to conclude the 0_7^+ state as the 4α condensate state.

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