Low-energy three-body collisions between an antiproton $\bar{\mathrm{p}}$ and muonic hydrogen atom $\mathbf{H}_\mu$

Renat A. Sultanov[1,*] and *S. K.* Adhikari[2,**]

[1]Odessa College, Department of Mathematics, 201 W. University Blvd., Odessa, TX 79764, USA
[2]Instituto de Física Teórica, UNESP − Universidade Estadual Paulista, 01140 São Paulo, SP, Brazil

Abstract. A few-body treatment is performed for two low-energy three-body collisions with participation of heavy charge particles such as an antiproton ($\bar{\mathrm{p}}$), deuterium (D), tritium (T) and a negative muon (μ^-). Specifically, the following reactions are considered: $\bar{\mathrm{p}} + (\mathrm{D}\mu^-)_{1\mathrm{s}} \to (\bar{\mathrm{p}}\mathrm{D})_\alpha + \mu^-$ and $\bar{\mathrm{p}} + (\mathrm{T}\mu^-)_{1\mathrm{s}} \to (\bar{\mathrm{p}}\mathrm{T})_\alpha + \mu^-$. The final state antinucleon-nucleon ($\bar{\mathrm{N}}\mathrm{N}$) interaction is included in these calculations and its influence on the cross sections and rates is estimated.

1 Introduction

Since $\bar{\mathrm{p}}$ is an anti-baryonic particle with the baryonic number $B = -1$, it would be interesting to discover the strong nuclear interaction between, for example, $\bar{\mathrm{p}}$ and a proton, i.e. the $\bar{\mathrm{p}}\mathrm{p}$ interaction in protonium (Pn) atom — a bound state of the particles: $\bar{\mathrm{p}}$ and p. This two-body system is also called as anti-protonic hydrogen. Additionally, it would be very useful and extremely interesting to consider, study and compare results between the following atomic systems: $\bar{\mathrm{p}}\mathrm{D}$ and $\bar{\mathrm{p}}\mathrm{T}$, where $\mathrm{D}\equiv{}^2\mathrm{H}^+$ is the deuterium nucleus and $\mathrm{T}\equiv{}^3\mathrm{H}^+$ is tritium. Perhaps it should be even worthier to obtain these atoms in their ground ($1s$) and close to ground states ($2s$, $2p$) when the systems are compact and nuclear interaction is effective.

It would be possible to prepare such atomic systems with the use of muons, for example, in the following three-body reaction:

$$\bar{\mathrm{p}} + (\mathrm{p}\mu^-) \to (\bar{\mathrm{p}}\mathrm{p})_\alpha + \mu^-, \tag{1}$$

where μ^- is a negative muon and α is the final atomic state of Pn. At low energy collisions $\alpha = 1s$, $2s$ or $2p$. This reaction has been considered in papers [1, 2]. It was found that the nuclear interaction in the final state is important [1]. Therefore, in the current paper the following three-body reactions are of our next special interest in the study of the antimatter-matter nuclear forces: $\bar{\mathrm{p}} + (\mathrm{D}\mu^-) \to (\bar{\mathrm{p}}\mathrm{D})_\alpha + \mu^-$, and $\bar{\mathrm{p}} + (\mathrm{T}\mu^-) \to (\bar{\mathrm{p}}\mathrm{T})_\alpha + \mu^-$.

In the next section we provide a brief description of our few-body approach based on the two-component Faddeev-Hahn-type equation formalism. We discuss the inclusion of the strong interaction in the framework of this equation. Sec. 3 represents our numerical results and Sec. 4 conclusions. Muonic atomic units (m.a.u.) are used in this work, i.e. $m_\mu = e^- = \hbar = 1$, where m_μ is the muon mass, e^- is the charge of an electron and $\hbar$ is Planck's constant.

*e-mail: rsultanov@odessa.edu; r.sultanov2@yahoo.com
**e-mail: sk.adhikari@unesp.br; adhikari44@yahoo.com

2 Few-body equations

In this work a detailed few-body approach is applied to the following three-charge-particle reactions at low energy collisions:

$$\bar{\mathrm{p}} + (\mathrm{D}\mu^-)_{1\mathrm{s}} \rightarrow (\bar{\mathrm{p}}\mathrm{D})_\alpha + \mu^-, \tag{2}$$

$$\bar{\mathrm{p}} + (\mathrm{T}\mu^-)_{1\mathrm{s}} \rightarrow (\bar{\mathrm{p}}\mathrm{T})_\alpha + \mu^-. \tag{3}$$

The method is based on the reduction of the total three-body wave function Ψ onto two Faddeev-type components [3]:

$$|\Psi\rangle = \Psi_1(\vec{\rho}_1, \vec{r}_{23}) + \Psi_2(\vec{\rho}_2, \vec{r}_{13}). \tag{4}$$

Eq. (4) reflects a fact that at low energy collisions, that is before the break-up threshold, only two asymptotic spatial configurations are possible in reactions (2) and (3). For example, in (2) the component $\Psi_1(\vec{\rho}_1, \vec{r}_{23})$ represents the $\bar{\mathrm{p}}$+(Dμ) input channel, and the component $\Psi_2(\vec{\rho}_2, \vec{r}_{13})$ represents the output channel, i.e. $(\bar{\mathrm{p}}\mathrm{D})_\alpha + \mu^-$. Vectors $(\vec{\rho}_k, \vec{r}_{ij})$ are so called Jacobi coordinates in the three-body system. These two configurations are shown in Fig. 1. Therefore, from works [4, 5] one can determine the Faddeev components by writing down the following set of two coupled equations:

$$\begin{cases} \left(E - \hat{T}_{\rho_1} - \hat{h}_{23}(\vec{r}_{23})\right)\Psi_1(\vec{r}_{23}, \vec{\rho}_1) = \left(V_{23}(\vec{r}_{23}) + V_{12}(\vec{r}_{12})\right)\Psi_2(\vec{r}_{13}, \vec{\rho}_2), \\ \left(E - \hat{T}_{\rho_2} - \hat{h}^{\bar{N}N}_{13}(\vec{r}_{13})\right)\Psi_2(\vec{r}_{13}, \vec{\rho}_2) = \left(\tilde{\mathbf{V}}_{13}(\vec{r}_{13}) + V_{12}(\vec{r}_{12})\right)\Psi_1(\vec{r}_{23}, \vec{\rho}_1). \end{cases} \tag{5}$$

Here: $V_{ij}(\vec{r}_{ij})$ are Coulomb potentials between the particles ($i \neq j$ = 1, 2, 3), $\tilde{\mathbf{V}}_{13}(\vec{r}_{13}) = V_{13}(\vec{r}_{13}) + v^{\bar{N}N}_{13}(\vec{r}_{13})$ and the two-particle target hamiltonians $\hat{h}_{23}(\vec{r}_{23}) = \hat{T}_{\vec{r}_{23}} + V_{23}(\vec{r}_{23})$ and $\hat{h}^{\bar{N}N}_{13}(\vec{r}_{13}) = \hat{T}_{\vec{r}_{13}} + V_{13}(\vec{r}_{13}) + v^{\bar{N}N}_{13}(\vec{r}_{13})$ with additional strong $\bar{\mathrm{p}}$D (or $\bar{\mathrm{p}}$T) nuclear potentials - $v^{\bar{N}N}_{13}(\vec{r}_{13})$. It is shown explicitly in these equations. $\hat{T}_{\vec{r}_{ij}}$ and $\hat{T}_{\vec{\rho}_k}$ are kinetic energy quantum-mechanical operators. The Faddeev decomposition avoids the over-completeness problems, because the subsystems are treated in an equivalent way in the framework of the two-coupled equations [6–8]. The correct asymptotes are guaranteed. The Faddeev-components are smoother functions of the coordinates than the total wave function $|\Psi\rangle$. The system of Eqs. (5) is equivalent to the Schrödinger equation – this is very important.

In order to solve Eqs. (5) a modified close-coupling approach is used. It leads to an expansion of the system's wave function components $|\Psi_1\rangle$ and $|\Psi_2\rangle$ into eigenfunctions $\varphi^{(1)}_n(\vec{r}_{23})$ and $\varphi^{(2)\bar{N}N}_{n'}(\vec{r}_{13})$ of the subsystem (target) Hamiltonians:

$$\begin{cases} \Psi_1(\vec{r}_{23}, \vec{\rho}_1) \approx \left(\displaystyle\int + \sum\right)_n f^{(1)}_n(\vec{\rho}_1)\varphi^{(1)}_n(\vec{r}_{23}), \\ \Psi_2(\vec{r}_{13}, \vec{\rho}_2) \approx \left(\displaystyle\int + \sum\right)_{n'} f^{(2)}_{n'}(\vec{\rho}_2)\varphi^{(2)\bar{N}N}_{n'}(\vec{r}_{13}). \end{cases} \tag{6}$$

This procedure brings a set of coupled one-dimensional integral-differential equations after the partial-wave projection [8]. The set of coupled integral-differential equations for $f^{(1)}_n(\vec{\rho}_1)$ and $f^{(2)}_{n'}(\vec{\rho}_2)$ can be solved in the framework of different close-coupling approximations, such as $2\times1s$, $2\times(1s+2s)$, $2\times(1s+2s+2p)$ etc. Symbol "2×" indicates that we use two independent sets of the target expansion functions.

The full potentials between $\bar{\mathrm{p}}$ and D and $\bar{\mathrm{p}}$ and T are more complex, because their second parts, $v^{\bar{N}N}_{13}(\vec{r}_{13})$, possess asymmetric $\bar{\mathrm{N}}$-N nuclear interactions [9–12]. In this work we did not

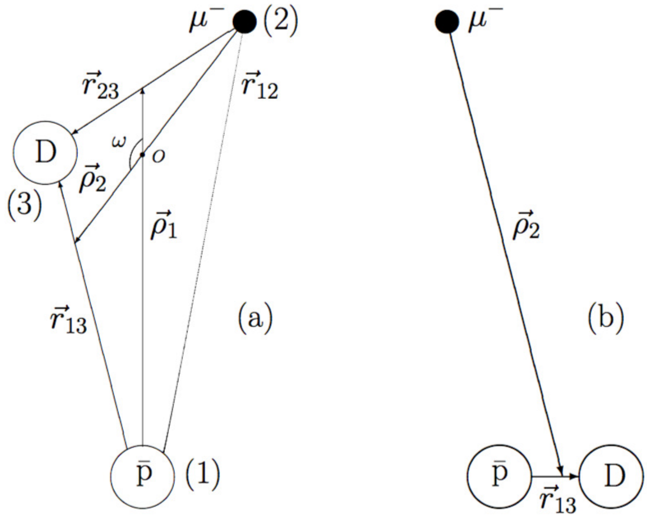

Figure 1. Two spatial coordinate configurations of the Coulomb three-body system p̄-μ-D. Here: (1) is an antiproton p̄, (2) is a negative muon μ^-, and (3) is the deuterium nucleus D. Vectors $\{\vec{r}_{23},\ \vec{\rho_1}\}$ and $\{\vec{r}_{13},\ \vec{\rho_2}\}$ are few-body Jacobi coordinates of the input and output channels of the reaction (2) correspondingly, O is the center of mass of the three-body system and ω is the angle between the Jacobi vectors $\vec{\rho_1}$ and $\vec{\rho_2}$.

explicitly include the strong interaction in our calculations, which is why in the case of the target p̄D and p̄T eigenfunctions we use pure two-body Coulomb (atomic) wave functions.

$$\varphi_{n'}^{(2)\bar{N}N}(\vec{r}_{13}) \approx \sum_{l'm'} R_{n'l'}^{(2)\bar{N}N}(r_{13}) Y_{l'm'}(\hat{r}_{13}) \approx \sum_{l'm'} R_{n'l'}^{(2)}(r_{13}) Y_{l'm'}(\hat{r}_{13}), \tag{7}$$

where $Y_{l'm'}(\hat{r}_{13})$ are spherical harmonics which depend on angular variables of $\vec{r}_{13}$, $\{n', l', m'\}$ are quantum atomic numbers and $R_{n'l'}^{(2)}(r_{13})$ are radial parts of the hydrogen-like atom wave function [13], for instance, the anti-protonic hydrogen atom. Nonetheless, the strong p̄D and p̄T interactions are approximately taken into account through the eigenstates $\mathcal{E}_{n'}$ which have shifted values from the original Coulomb levels $\varepsilon_{n'}$ [14], that is:

$$\mathcal{E}_{n'} \approx \varepsilon_{n'} + \Delta E_{n'}^{\bar{N}N} = -\mu_2/2n'^2 + \Delta E_{n'}^{\bar{N}N}, \tag{8}$$

where μ_2 is the reduced mass of the targets p̄D or p̄T in the output channels of the reactions (2) and (3).

In this work we apply Eqs. (5)-(8) with the use of an energy shift $\Delta E_{n'}^{\bar{N}N}$ in the eigenstate of p̄D and p̄T. The energy shifts can be computed, for example, with the use of the well-known Deser-Goldberger-Baumann-Thirring formula[14]:

$$\Delta E_{n'}^{\bar{N}N} = -\frac{4}{n'}\frac{a_s}{B_{\bar{p}H}}\varepsilon_{n'}, \tag{9}$$

where a_s is the pure nuclear strong interaction scattering length in the p̄+H collision (where H is D or T), i.e. without inclusion of the Coulomb interaction between the particles, $B_{\bar{p}H}$ is the Bohr radius of deuteronium, i.e. the p̄D atom or the p̄T one. Computational details of the Faddeev-Hahn equation formalism can be found in work [8].

Table 1. Total p̄D-atom formation cross sections $\sigma_{tr}(\varepsilon_{coll})$ and rates λ_D in the three-body reaction (2), where $\alpha = 1s$ and ε_{coll} is the collision energy (eV). The rate is a product of the formation cross sections and the corresponding center-of-mass velocities v_{cm} between p̄ and D_μ in the input channel. The results are represented in the framework of different close-coupling approximation models, Eq. (6). The cross section σ_{tr} is given in cm^2 and λ_D in m.a.u. Results with inclusion of the strong nuclear interaction between p̄ and D are represented only in the six-state model. Our rates λ_D are multiplied by factor of 5.

	$1s$	$1s$-$2s$	$1s$-$2s$-$2p$		(p̄-D Nucl.)	
ε_{coll}	σ_{tr}	σ_{tr}	σ_{tr}	λ_D	$\sigma_{tr}^{\bar{p}D}$	$\lambda_D^{\bar{p}D}$
0.001	2.82E-20	2.50E-20	1.22E-19	0.225	3.30E-19	0.612
0.01	8.93E-21	7.92E-21	3.84E-20	0.225	1.04E-19	0.611
0.1	2.83E-21	2.50E-21	1.20E-20	0.223	3.24E-20	0.601
0.5	1.26E-21	1.12E-21	5.18E-21	0.215	1.35E-20	0.560
1.0	8.91E-22	7.90E-22	3.50E-21	0.205	8.80E-21	0.515
2.0	6.30E-22	5.57E-22	2.26E-21	0.188	5.30E-21	0.439

In the literature one can find other theoretical formulas to compute $\Delta E_{n'}^{\bar{N}N}$, for instance, in papers [15, 16]. In future calculations it would be interesting to apply some of these theories together, for example, with the inclusion of relativistic effects in the heavy p̄D and p̄T atoms [17, 18].

Since muon is ~207 times heavier than e^-, the muonic hydrogen atom H_μ has a very small size. Therefore, in reactions (2)-(3) antiproton can very closely approach H_μ. However, annihilation between p̄ and D/T will be significantly prevented because of the μ^- screening effect and a strong p̄ and μ^- Coulomb repulsion. The quantum-mechanical p̄ tunneling through the muonic-atomic orbit H_μ is also significantly suppressed. This fact can be seen from the following quantum-mechanical tunneling probability formula [13]:

$$B = \exp\left\{-\frac{2}{\hbar}\int_0^{\rho_0} \sqrt{2M(U(r)-E)}dr\right\}, \tag{10}$$

where B is the probability, E is the total energy in the three-body system, M is the p̄-H_μ reduced mass, and $U(r)$ is the interaction potential between p̄ and H_μ:

$$U(r) = \left(\frac{1}{r} + \mu_0\right)e^{-2\mu_0 r}, \tag{11}$$

μ_0 is the muonic hydrogen reduced mass, i.e. $\mu_0 \approx 207 m_e$, where m_e is the electron mass. The integration in the Eq. (10) can be done up to $\rho_0 \approx 10$ m.a.u. One can compute the integral (10) and show that the argument of the exponent in Eq. (10) is a large number. Therefore, in the first-order approximation p̄ tunneling can be neglected. In the case of similar atomic system, where one has an electron e^- instead of muon, p̄ can easily penetrate through the light e^- cloud and annihilate with the hydrogen isotopes before the three-body reaction occurs.

3 Results

This section represents our results for reactions (2) and (3). As in paper [1], in the current work we carried out numerical calculation for the transitions to the ground states of the antiprotonic atoms, i.e. $\alpha = 1s$ in Eqs. (2)-(3). In this state the size of the p̄D and p̄T atoms

Table 2. Total p̄T-atom formation cross sections $\sigma_{tr}(\varepsilon_{coll})$ and rates λ_T in the three-body reaction (2), where $\alpha = 1s$ and ε_{coll} is the collision energy (eV). The rate is a product of the formation cross sections and the corresponding center-of-mass velocities v_{cm} between p̄ and T_μ in the input channel. The results are represented in the framework of different close-coupling approximation models, Eq. (6). The cross section σ_{tr} is given in cm^2 and λ_T in m.a.u. Results with inclusion of the strong nuclear interaction between p̄ and T are represented only in the six-state model. Our rates are multiplied by factor of 25.

	$1s$	$1s$-$2s$	$1s$-$2s$-$2p$		(p̄-T Nucl.)
ε_{coll}	σ_{tr}	σ_{tr}	σ_{tr}	λ_T	$\lambda_T^{\bar{p}T}$
0.001	2.23E-20	2.64E-20	7.80E-20	0.685	1.315
0.01	7.04E-21	8.34E-21	2.45E-20	0.685	1.316
0.1	2.23E-21	2.64E-21	7.82E-21	0.687	1.322
0.5	9.95E-22	1.18E-21	3.55E-21	0.698	1.352
1.0	7.03E-22	8.36E-22	2.56E-21	0.711	1.391
2.0	4.96E-22	5.93E-22	1.89E-21	0.742	1.473

is about ∼10 fm. Therefore, one can expect that the contribution of the nuclear forces to the three-body scattering cross sections and rates will be significant. These reactions, in our opinion, can be useful to study nuclear matter-antimatter interactions. Table 1 shows our results for reaction (2). Specifically, the table depicts our p̄-transfer cross-sections $\sigma_{tr}(E)$ and corresponding reaction rates λ_D at low energy collisions. The results are shown in the framework of the $2\times1s$, $2\times(1s$+$2s)$ and $2\times(1s$+$2s$+$2p)$ close-coupling approximation. One can see that the contribution of the $2p$ atomic states is large−slow p̄ can approach the Dμ atom particularly close and significantly polarize it. Therefore, the inclusion of the polarization channels in Eq.(6) is important. Finally, taking into account the nuclear interaction between p̄ and D increased the cross-section almost three times. This result can be seen from Table 1: from the cross section column, $\sigma_{tr}^{\bar{p}D}$, and from the corresponding reaction rate one, i.e. $\lambda_D^{\bar{p}D}$.

The inclusion of this interaction has been done within the framework of Eqs. (7)-(8) and (9). From Ref. [19] (Table 5.2) one can find the results of the p̄D atom ground-state Coulomb energy shifts. The averaged experimental result has the following value:

$$\Delta E^{\bar{p}D}_{n'=1s} = 1050 \text{ eV}. \tag{12}$$

This result was adopted in this work. However, the result for $\Delta E^{\bar{p}D}_{n'=1s}$ differs quite significantly from the theoretical calculations based on the three-body Faddeev theory [12] and another older work [11]. With the use of the given $\Delta E^{\bar{p}D}_{n'=1s}$ (12), one can estimate the p̄+D scattering length:

$$a_s^{\bar{p}D} = -\frac{B_{\bar{p}\mathrm{H}}}{4\varepsilon_{n'=1s}}\Delta E^{\bar{p}D}_{n'=1s} = 0.682 \text{ fm}, \tag{13}$$

and compute the Coulomb energy shift for 2s and 2p states ($n' = 2$):

$$\Delta E^{\bar{p}D}_{n'=2} = 131.25 \text{ eV}. \tag{14}$$

All these values have been included in the Eqs. (5) in our calculation of the reaction (2).

Table 2 shows our results for the collision (3). This is a very attractive and special three-charge-particle reaction with participation of tritium (T), which is a radioactive hydrogen isotope. It would be extremely interesting to investigate the influence of the strong interaction between p̄ and T on the rate of the reaction (3). With the use of this reaction it should be possible to study nuclear interaction between p̄ and radioactive T.

In order to carry out calculations of the process (3) one needs the nuclear energy shifts of the (p̄T) Coulomb levels as an input. However, to our best knowledge, this data are not available in the literature so far. Therefore, at this time it is not possible to carry out quality and strict calculations for this reaction. Nonetheless, we will depict our preliminary results for the reaction rates of (3) with the use of a model approach. Table 2 shows our reaction cross-sections $\sigma_{tr}(E)$ and rates λ_T. One can see that the inclusion of the $2p$ atomic states is important for the transfer channel. Also, the transfer cross section is smaller than the corresponding transfer cross-section in the reaction (2). This result agrees with the results of previous paper [2].

In regard to the nuclear interaction in (3) we also use Eqs. (7)-(9) in order to estimate the effect of the strong p̄T interaction. However, as we mentioned above, there are no results for the p̄T ground-state Coulomb level energy shifts, i. e. no results for $\Delta E^{\bar{p}T}_{n'=1s}$. We have come across only a paper where we found information about p̄+T scattering [20]. Therefore, in the current paper, we apply a model approach for the Coulomb level energy shift $\Delta E^{\bar{p}T}_{n'}$. Note that in the case of the two-particle system (p̄p) the energy shift is $\Delta E^{\bar{p}p}_{n'=1s} = 540$ eV. In the case of the three-nuclear-particle system p̄D: $\Delta E^{\bar{p}D}_{n'=1s} = 1050$ eV. Therefore, we assume that in the case of the four-particle system, i.e. p̄T, the ground state Coulomb level energy shift due to the nuclear interaction is $\Delta E^{\bar{p}T}_{n'=1s} \approx 1575$ eV.

We adopt this value and use it in order to complete our calculation of the rate of the reaction (3) with the inclusion of the strong p̄T interaction in the final state. As a preliminary treatment, the energy range for this calculation was only: 10^{-3} eV $\leqslant E \leqslant 2$ eV. We can see from Table 2 that the nuclear p̄T interaction effect is large. In order to compare our results with the results of paper [2] our p̄+(Tμ^-) rates λ_T and $\lambda_T^{\bar{p}T}$ were multiplied by 25.

4 Conclusions

The main goal of this paper is to carry out calculations of the charge-transfer reactions (2) and (3) at low energies, and investigate the influence of the strong nuclear interaction on the final states of these reactions, i.e. the influence of the p̄D and p̄T short-range nuclear forces on the output scattering parameters. A set of coupled two-component Faddeev-Hahn-type equations has been applied together with a modified close-coupling approximation technique. The main advantage of this three-body method over other few-body adiabatic approaches is that we utilize an independent formulation of the two-body targets in the reactions (2) and (3): for example, the (Dμ) atom in the input channel and the (p̄D) atom in the output channel. This distinctive property of the Eqs. (5)-(6) allows us to avoid the over-completeness problems and provide accurate three-body asymptotes for $\Psi_1(\vec{r}_{23}, \vec{\rho}_1)$ and $\Psi_2(\vec{r}_{13}, \vec{\rho}_2)$. The set of coupled Eqs. (5) is equivalent to the Schrödinger equation. We treat the Coulomb three-body systems as systems with arbitrary masses, i.e. the masses of the charged particles are taken as they are. We do not apply any type of adiabatic approximation, when the dynamics of heavy and light parts of a system are separated.

In the case of the reaction (2) it is shown that the effect of the final-state nuclear interaction is significant - up to 275%. Therefore, one could conclude that the three-body reaction could be considered as a possible candidate for future experiments with participation of low-energy antiprotons p̄$'s$ and muons in order to produce the antiprotonic hydrogen atom (p̄D) and study the nuclear interaction between p̄ and D at low energies. In regard to the very interesting (p̄T) system we carried only preliminary estimations of the influence of the strong p̄T forces on the reaction outputs. It was also found that the contribution is large.

In conclusion, we would like to make a cautious assumption, that the low-energy collisions between antiprotons and muonic atoms could also be useful to study a quite old problem in nuclear physics concerning the long-range part of the strong N̄N interaction [21].

Acknowledgements

We wish to acknowledge the support from CNPq and FAPESP of Brazil.

References

[1] R.A. Sultanov, D. Guster, S.K. Adhikari, Atoms **6**, 18 (2018).
[2] A. Igarashi, N. Toshima, Eur. Phys. J. D **46** (2008) 425.
[3] L.D. Faddeev, Zh. Eksp. Teor. Fiz. **39**, 1459 (1960) [Sov. Phys. JETP **12**, 1014 (1961)].
[4] Y. Hahn, K. Watson, Phys. Rev. A **5**, 1718 (1972).
[5] Y. Hahn, Nucl. Phys. A**389**, 1 (1982).
[6] R.A. Sultanov, S.K. Adhikari, Phys. Rev. A **61**, 227111 (2000); **62**, 022509 (2000).
[7] R.A. Sultanov, D. Guster, J. Comp. Phys. **192**, 231 (2003).
[8] R.A. Sultanov, D. Guster, J. Phys. B: At. Mol. Opt. Phys. **46**, 215204 (2013).
[9] C.B. Dover, J.-M. Richard, Phys. Rev. C **25**, 1952 (1982).
[10] J.-M. Richard, M.E. Sainio, Phys. Lett. B **110**, 349 (1982).
[11] S. Wycech, A.M. Green, J.A. Niskanen, Phys. Lett. B **152**, 308 (1985).
[12] G.P. Latta, P.C. Tandy, Phys. Rev. C **42**, 1207 (1990).
[13] L.D. Landau, E.M. Lifshitz, *Quantum-Mechanics (Non-relativistic Theory), Course of Theoretical Physics, Vol. 3* (3rd Ed., Butterworth-Heinemann, 2003).
[14] S. Deser, M.L. Goldberger, K. Baumann, W. Thirring, Phys. Rev. **96**, 774 (1954).
[15] T. L. Trueman, Nucl. Phys. **26**, 57 (1961).
[16] V.S. Popov, A.E. Kudryavtsev, V.D. Mur, Sov. Phys. JETP **50**, 865 (1979).
[17] T. Ueda, Prog. Theor. Phys. **62**, 1670 (1979).
[18] J. Thaler, J. Phys. G: Nucl. Phys. **9**, 1009 (1983); **11**, 201 (1985).
[19] E. Klempt, F. Bradamante, A. Martin, J.-M. Richard, Phys. Rep. **368**, 119 (2002).
[20] L.A. Kondratyuk, M. Z. Shmatikov, Sov. J. Nucl. Phys. **38**, 216 (1983).
[21] W.W. Buck, C.B. Dover, J.-M. Richard, Ann. Phys. **121**, 47 (1979).