Scattering parameters of a non-reciprocal magneto-optical integrated coupler used as an isolator

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Abstract. We theoretically investigate a non-reciprocal magneto-optical integrated slab directional coupler. The scattering parameters of the structure are derived in the frame of the Coupled-Mode Theory (CMT). By properly designing the coupler, it is possible to achieve a perfect non-reciprocity (with 100\% contrast) between the two directions of propagation. Other operating points can also be defined, especially since modal dispersion in the spectral domain is naturally taken into consideration.

1 Introduction

Couplers are important components of photonic circuits. Using magneto-optical materials as constituents of such systems allows one to design non-reciprocal devices, such as nonreciprocal Mach-Zehnder interferometers or nonreciprocal directional isolators [1, 2]. In this communication, we discuss the principle and the operating conditions of such an isolator based on two coupled magneto-optical waveguides with opposite directions of magnetization.

2 Magneto-optical coupler

The coupler consists of two parallel slab waveguides of equal thickness \( h \) made of a bigyrotropic magnetic medium \( B \). Both the spacer of thickness \( d \) separating the waveguides and the surrounding space are made of the same isotropic, non-magnetic medium \( A \) (Fig. 1). Propagation occurs over a coupling length \( L \) along the \( z \)-axis. Time dependence is taken as \( \exp(+i\omega t) \). Both guides are magnetized at saturation along the \( y \)-axis, in an “anti-parallel” transverse configuration.

In the frame of the Coupled-Mode Theory, the coupling constant \( \chi \) between two phase-matched slab waveguides can be retrieved by evaluating the modal propagation constants \( \beta_e \) and \( \beta_o \) of the \textit{even} (slow) and \textit{odd} (fast) super-modes, since \( \chi = (\beta_e - \beta_o)/2 \).

According to previous calculations reported in [3], a system made of coupled bigyrotropic slab waveguides in the anti-parallel transverse magnetic configuration becomes non-reciprocal: the two waveguides remain perfectly phase-matched, but the coupling constant depends on the direction of propagation. Let us call \( \chi^+ \) and \( \chi^- \) the coupling constants along the \( z \)-axis from the left to the right or from the right to the left, respectively. In a non-dissipative system, they are real and positive, with \( \chi^+ < \chi^- \).

Besides, considering the spectral effects of modal dispersion, we derive an expression for \( \chi^+ \) and \( \chi^- \) that can be linearized around a given operating wavelength.

3 Non-reciprocal S-parameters

3.1 S-parameters and signal flow-graph

The scattering matrix expresses output fields \((b_p)\) in terms of input fields \((a_p)\). The signal flow graph is the graphical representation of the same connections (Fig. 2).

In the general case, Input/output fields at port \( p \) are denoted \((a_p, b_p)\).
Since the coupler is directional, no reflection (either direct or cross) is experienced:

\[ S_{11} = S_{22} = S_{33} = S_{44} = 0, \quad (1a) \]
\[ S_{13} = S_{31} = S_{24} = S_{42} = 0. \quad (1b) \]

The remaining S-parameters are: from the left to the right, \( S_{21} = S_{43} = r^+ \), \( S_{41} = S_{32} = k^+ \); from the right to the left, \( S_{32} = S_{43} = r^- \), \( S_{41} = S_{32} = k^- \). We get eventually:

\[
\begin{pmatrix}
 b_1 \\
 b_2 \\
 b_3 \\
 b_4
\end{pmatrix} = \begin{pmatrix}
 r^+ & k^+ & a_1 \\
 k^+ & r^- & a_2 \\
 r^- & k^- & a_3 \\
 k^- & r^+ & a_4
\end{pmatrix}
\]

with

\[
\begin{align}
 r^+ &= \cos(\chi^+L) \exp(-i \beta^+L), \quad (3a) \\
 k^+ &= -i \sin(\chi^+L) \exp(-i \beta^+L), \quad (3b) \\
 r^- &= \cos(\chi^-L) \exp(-i \beta^-L), \quad (3c) \\
 k^- &= -i \sin(\chi^-L) \exp(-i \beta^-L), \quad (3d)
\end{align}
\]

where \( \beta^+ = (\beta^+ + \beta^-)/2 \) stands for the average propagation constant from the left to the right; similarly, from the right to the left, \( \beta^- = (\beta^+ - \beta^-)/2 \). Numerical calculations show that the values of \( \beta^+ \) and \( \beta^- \) are almost identical, but anyway, they appear only as a global phase term.

It is possible to design the waveguide width \( h \) and/or the gap \( d \) (as well as the interaction length \( L \)) in order to ensure that, for a given wavelength \( \lambda_0 \), \( \chi^+L \) and \( \chi^-L \) are adjusted at will, so that

\[
\frac{\chi^+}{\chi^-} = \frac{2q}{2q+1}, \quad \text{with } q \in \mathbb{N},
\]

which ensures that the coupler behaves as a perfect directional isolator.

3.2 Perfect directional isolator

For GGG and Bi:YIG as materials A and B, the condition expressed by Eq. (4) can for instance be satisfied (and perfect isolation be achieved) at \( \lambda_0 = 1.5 \mu m \) with \( h = 0.62 \mu m, \quad d = 0.155 \mu m, \quad \text{and} \quad L = 206.5 \mu m \), so that:

\[
(\chi^+L) = 10\pi, \quad (\chi^-L) = 10.5\pi.
\]

As a result, the system becomes highly non-reciprocal, as schematically illustrated in Fig. 3, since

\[
[S_{21}]^2 = |S_{34}|^2 = 1, \quad |S_{43}|^2 = |S_{24}|^2 = 0, \quad (6a)
\]
\[
[S_{12}]^2 = |S_{42}|^2 = 0, \quad |S_{13}|^2 = |S_{32}|^2 = 1. \quad (6b)
\]

3.3 Chromatic dispersion and consequences

Once the coupler is designed as a perfect isolator for a given operating wavelength \( \lambda_{op} \), the knowledge of chromatic dispersion (both modal and material) enables one to assess the bandwidth over which the behaviour remains close to ideal. Around \( \lambda_{op} \), we write:

\[
\chi^\pm = \chi^\pm(\lambda_{op}) + (\lambda - \lambda_{op})(\partial \chi^\pm/\partial \lambda),
\]

since both coupling constants \( \chi^+ \) and \( \chi^- \) exhibit roughly the same value of \( \partial \chi^\pm/\partial \lambda \). Let us start from:

\[
\begin{align}
\chi^+L &= q\pi + \theta \quad (q \in \mathbb{N}), \quad (8a) \\
\chi^-L &= q\pi + \pi/2 + \theta \quad (8b) \\
\theta &= (\lambda - \lambda_{op})(\partial \chi^\pm/\partial \lambda)L. \quad (8c)
\end{align}
\]

Then we get:

\[
[S_{21}]^2 = \cos^2\theta, \quad |S_{13}|^2 = \sin^2\theta. \quad (9)
\]

If we strive to maintain \( \sin^2\theta < 0.01 \) (which means \( |\theta| < 0.1 \)), then the corresponding bandwitdth \( \Delta \lambda \) is:

\[
\Delta \lambda = \frac{0.2}{(\partial \chi^\pm/\partial \lambda)} \quad (10)
\]

Assuming \( \partial \chi^\pm/\partial \lambda = 0.07 \mu m^{-2}, \quad \lambda_{op} = 1.5 \mu m \) and \( L = 206.5 \mu m \), we get \( \Delta \lambda = 14 \mu m \) (or, in frequency terms, \( \Delta \nu = 1.8 \times 10^{12} \text{ Hz} \approx 1.8 \text{ THz} \)).

The same element can also be seen as a building-block to be inserted into more complex structures, such as a non-reciprocal Mach-Zehnder interferometer.

Fig. 3. Illustration of high-contrast non-reciprocity: the material parameters of the coupler can be adjusted in such a way that, for a given operating wavelength, the structure works as a perfect isolator.

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References