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Including Fresnel reflection losses in freeform lens design

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Abstract. We present an inverse method for optical design that compensates local Fresnel reflections. We elaborate this method for a point source and far-field target. We modify an existing design algorithm based on the least-squares method. This is done in such a way that the shape of the transmitted intensity is as desired.

1 Introduction

With the introduction of LED lighting, a lot of research has been done on freeform optics. Specifically inverse methods are of interest due to their relatively short computation time. Often the geometrical optics approximation is used, ignoring some physical phenomena like Fresnel reflections. However, the arbitrary shapes of freeform lenses can lead to varying angles of incidence and thus varying Fresnel losses. We present an inverse method for designing a freeform lens for shaping a point source to a far-field distribution while compensating for Fresnel losses. For this, we adapt the least-squares solver developed by Romijn et al [1]. A more complete overview is given in [2].

2 Mathematical model

Our goal is to construct a lens with one freeform surface to shape light from a point source to a far-field target. Light is emitted by the point source with each ray defined by its unit direction vector $\hat{\boldsymbol{s}} = (s_1, s_2, s_3)^\mathsf{T}$. We assume that the first surface is spherical, centered around the point source. The second surface is freeform, defined by the radial distance to the point source $u(\hat{\boldsymbol{s}})$. After refraction, the direction of a ray is given by the unit vector $\hat{\boldsymbol{t}} = (t_1, t_2, t_3)^\mathsf{T}$; see Figure 1. At each optical surface, a part of the flux is transmitted and a part is reflected, depending on the incident angle [3, Sec. 4.6]. We assume that the reflected light

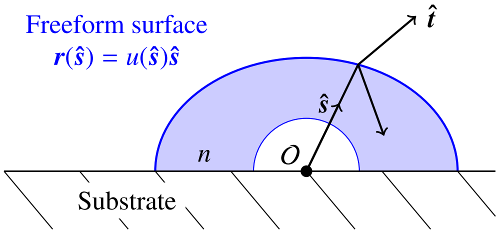

Figure 1: Intersection of a lens with the x, z-plane.

ends up on the substrate and is absorbed there. We have a given source distribution $f = f(\hat{\boldsymbol{s}})$ and a desired target distribution $g = g(\hat{\boldsymbol{t}})$, both with total flux equal to 1. Note that g is hypothetical, since we lose part of the flux from f. The source domain $\mathcal{S}$ is given as the support of f, and similarly $\mathcal{T}$ is the support of g. We parameterize $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{t}}$ by the stereographic projections $\boldsymbol{x} = (x_1, x_2)^\mathsf{T}$ and $\boldsymbol{y} = (y_1, y_2)^\mathsf{T}$, respectively, and introduce $\mathcal{X} = \boldsymbol{x}(\mathcal{S})$ and $\mathcal{Y} = \boldsymbol{y}(\mathcal{T})$.

The goal is to compute the surface shape, given by u. For that, we first find a mapping $\boldsymbol{m}$, with $\boldsymbol{y} = \boldsymbol{m}(\boldsymbol{x})$. We can derive an equation of the form

$$\nabla_{\boldsymbol{x}} c(\boldsymbol{x}, \boldsymbol{y}) - \nabla u_1(\boldsymbol{x}) = \boldsymbol{0}; \tag{1}$$

see [1]. The function u_1 is given by $u_1(\boldsymbol{x}) = \log(\hat{\boldsymbol{s}}(\boldsymbol{x}))$ and c is the so-called cost function, given by $c(\boldsymbol{x}, \boldsymbol{y}) = -\log(n - \hat{\boldsymbol{s}}(\boldsymbol{x}) \cdot \hat{\boldsymbol{t}}(\boldsymbol{y}))$. Substituting $\boldsymbol{y} = \boldsymbol{m}(\boldsymbol{x})$ and differentiating w.r.t. $\boldsymbol{x}$ gives

$$\boldsymbol{C}\mathrm{D}\boldsymbol{m} = \mathrm{D}^2 u_1 - \mathrm{D}_{\boldsymbol{xx}} c =: \boldsymbol{P}, \tag{2}$$

where $\boldsymbol{C} = \mathrm{D}_{\boldsymbol{xy}} c$, the matrix containing mixed second order derivatives.

We now have a description of the propagation of transmitted rays, but not of the flux at the target. To take that into account, we consider conservation of energy in relation with Fresnel coefficients. These coefficients are functions of the incident and transmitted angles [3]. We can express these in terms of $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{t}}$ [2]. We write T_1 and $T_2 = T_2(\hat{\boldsymbol{s}}, \hat{\boldsymbol{t}})$ for the transmittance at the first and second surface, respectively. Note that the transmittance at the spherical first surface is constant. Let g_t be the transmitted intensity, we then have for any $\mathcal{A} \subset \mathcal{S}$

$$\int_{\mathcal{A}} T_1 T_2(\hat{\boldsymbol{s}}, \hat{\boldsymbol{t}}) f(\hat{\boldsymbol{s}}) \, \mathrm{d}S(\hat{\boldsymbol{s}}) = \int_{\hat{\boldsymbol{t}}(\mathcal{A})} g_\mathrm{t}(\hat{\boldsymbol{t}}) \, \mathrm{d}S(\hat{\boldsymbol{t}}). \tag{3}$$

We can choose g_t to be a scaling of g, i.e., a distribution with the same shape as g, but a total flux equal to the transmitted fraction of the source flux. We write $g_\mathrm{t} = \beta g$ for a $\beta \in (0, 1)$. Setting $\mathcal{A} = \mathcal{S}$ in Eq. (3) gives

$$\beta(\boldsymbol{m}) = T_1 \int_{\mathcal{S}} T_2(\hat{\boldsymbol{s}}, \hat{\boldsymbol{t}}(\boldsymbol{m})) f(\hat{\boldsymbol{s}}) \, \mathrm{d}S(\hat{\boldsymbol{s}}). \tag{4}$$

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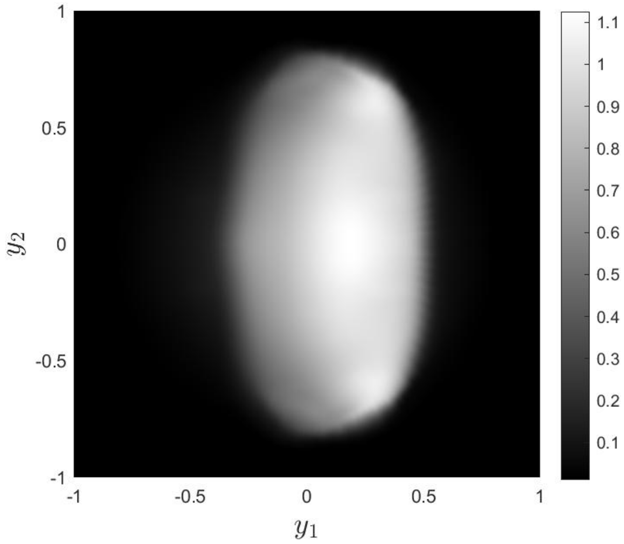

Figure 2: Target distribution in stereographic coordinates

Writing Eq. (3) in differential form in terms of stereographic coordinates gives us the generalized Monge-Ampère equation

$$\det(\mathrm{D}\boldsymbol{m}) = T_1 T_2(\boldsymbol{x}, \boldsymbol{m}(\boldsymbol{x}))\frac{J(\boldsymbol{x})}{J(\boldsymbol{m}(\boldsymbol{x}))}\frac{f(\boldsymbol{x})}{g_t(\boldsymbol{m}(\boldsymbol{x}))} \\ =: F(\boldsymbol{x}, \boldsymbol{m}(\boldsymbol{x})), \tag{5}$$

where J is the Jacobian resulting from the stereographic coordinate transformation. We slightly abuse notation by writing for example $f(\boldsymbol{x})$ instead of $f(\hat{\boldsymbol{s}}(\boldsymbol{x}))$. The boundary condition for this problem is given by $\boldsymbol{m}(\partial\mathcal{X}) = \partial\mathcal{Y}$.

Summarizing, we compute a mapping $\boldsymbol{m}$ from Eq. (2), where $\boldsymbol{P}$ satisfies $\det(\boldsymbol{P}) = F\det(\boldsymbol{C})$ due to Eq. (5). The mapping is subject to $\boldsymbol{m}(\partial\mathcal{X}) = \partial\mathcal{Y}$. Subsequently, we calculate u_1 from Eq (1) and from this the freeform surface.

We solve both problems iteratively in a least-squares sense. For more details on this method, see [2].

3 Numerical results

We use our method to design a lens for a typical street lamp. We consider a point source with a Gaussian light distribution on the positive half-sphere. The target is an intensity typically used by street lamps [1]; see Fig 2. With the algorithm, we obtain a so-called peanut lens. Calculating the reflectance coefficients on the lens (see Fig. 3) shows us quite large variations, with a loss of up to 20 percent in some places. This indicates that ignoring Fresnel losses in the algorithm would lead to a transmitted intensity significantly different from the hypothetical g. To stress this point, we perform a raytrace on the resulting lens from the algorithm with and without considering Fresnel losses. To compare the results, we scale the raytraced intensities to the same total flux as g. A comparison along the line $y_1 = 0$ is shown in Fig. 4. We see that the result of the algorithm with Fresnel is close to the desired intensity, while the result of the unadapted algorithm is clearly too low in some spots and too high in others. The areas with reduced flux correspond to the peaks in the reflectance as shown in Fig. 3.

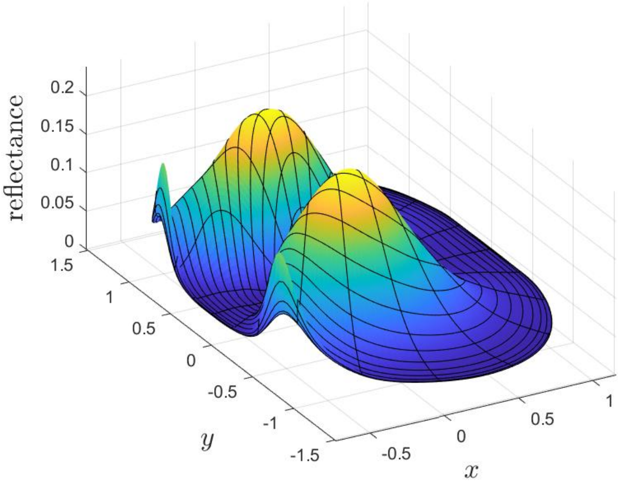

Figure 3: The reflectance on the freeform surface resulting from our algorithm as function of Cartesian x, y-coordinates.

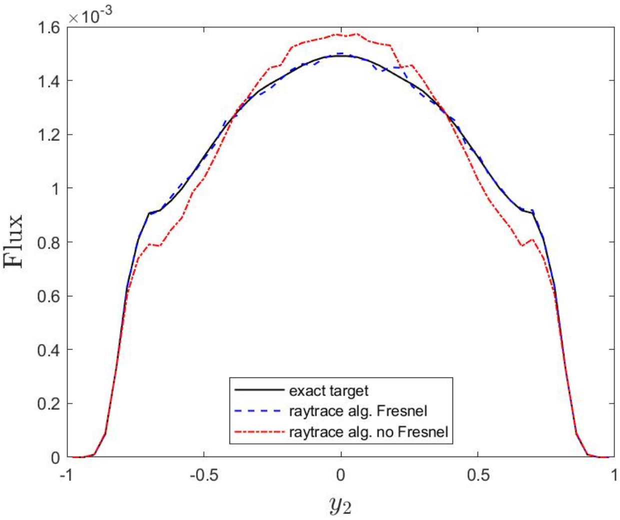

Figure 4: A comparison between the desired output intensity and the results of the algorithm with and without Fresnel reflection scaled to have the same total flux.

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