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Ray Transfer Matrix for Onion-Type GRIN Lenses

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Abstract. We present the computation of an ABCD matrix for onion-type GRIN lenses. By applying a differential approximation of the layer thickness, the matrix product of a high number of matrices is synthetized into a single matrix where the elements are integrals. The difference between this ABCD matrix and a homogeneous lens matrix is one integration term in element C, which is the GRIN contribution to the lens power. In the case of the crystalline lens, the analytical approximation to the GRIN lens power provides an accurate and concise solution in terms of Gaussian hypergeometric functions.

1 Introduction

Gradient index (GRIN) exists in a wide range of natural and artificial optical systems. There are different approaches to ray tracing computation in GRIN media. Paraxial ray tracing is essential as it determines fundamental properties of optical systems, such as cardinal elements and lens power. Ray transfer matrices are a concise and straightforward way to describe each element of an optical system.

In the particular case of the crystalline lens, there are two main approaches [1] later used to formulate the ABCD matrix. They consider either a slab-type [2] lens or a shell-type lens [3]. The latter involves the product of a high number of matrices.

Our goal is to formulate the ABCD transfer matrix for onion-type GRIN lenses. We apply a differential approximation where the layer thickness tends to zero, and the matrix product becomes a sum; then, each matrix element is computed as an integral. When calculating the power, each layer's contribution is determined by the product of its surface curvature and refractive index increment.

2 Methods

2.1 Ray Transfer Matrix of a GRIN Lens

We formulate the paraxial ray tracing through a GRIN lens with an index distribution $n(r,\theta,z)$ in cylindrical coordinates. The paraxial index distribution depends only on the z coordinate, $n \approx n(z)$. In onion-type GRIN lenses each iso-indicial surface (IIS) can be realized as a refracting optical surface. For a layer k crossing the axis at a distance z from the anterior surface, the transfer matrix is the product of a translation and a reflection matrix.

$$L_k(z_k) = \begin{bmatrix} 1 & -\Delta z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{\Delta n}{(n+\Delta n)R} & \frac{\Delta n}{n+\Delta n} \end{bmatrix} \tag{1}$$

Where $\Delta n = n'\Delta z$, $n' = dn/dz$, $n' = n'(z)$, and $R = R(z)$. With some manipulation and simplification, we obtain a matrix which is the addition of the identity matrix, a new matrix A, independent of Δz, and a residue with higher-order terms of $(\Delta z)^2$.

$$L_k(z_k) = I + A \cdot \Delta z + \mathcal{O}(\Delta z)^2 \tag{2}$$

Where

$$A(z_k) = \begin{bmatrix} 0 & -1 \\ \frac{n\prime(z)}{n(z)R(z)} & -\frac{n\prime(z)}{n(z)} \end{bmatrix} \tag{3}$$

The whole lens transfer matrix can be computed as the matrix product of all k layers from the back to the front of the lens. We approximate the product to a sum by neglecting higher-order terms of Δz.

$$M = \prod_{k=1}^{K} L(z_k) \approx I + \sum_{k=1}^{K} A(z_k)\Delta z + \mathcal{O}(\Delta z)^2 \tag{4}$$

In the continuous limit, $\Delta z \to 0$, and we convert the sum into an integral and replace Δz with dz.

$$M \approx \begin{bmatrix} 1 & -t \\ \int_0^t \frac{n\prime(z)}{n(z)R(z)} dz & 1 + \log\left(\frac{n(0)}{n(t)}\right) \end{bmatrix} \tag{5}$$

The anterior lens vertex is at $z = 0$, and t is the lens thickness. Matrix M represents the ray propagation inside the GRIN medium. When this formulation is applied to a biconvex homogeneous lens, the power is equivalent to that of a thin lens where the refractive indices are replaced with their natural logarithms.

The index changes abruptly along the surface between the lens and the external medium, so we formulate the refraction at each external surface independently. The complete lens transfer matrix comprises three matrices:

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$L = S_p M S_a$, where S_p and S_a correspond to pure refraction at the posterior and anterior surfaces.

Element L_{21} of matrix L is the most relevant, as it determines the lens power. When comparing L with the transfer of a homogeneous lens, all elements are the same except an extra term in element L_{21}, which accounts for the GRIN contribution to the power.

$$\phi_G = n_0 \int_0^t \frac{n\prime(z)}{n(z)R(z)} dz \tag{6}$$

2.2 Application to the Crystalline Lens

The human crystalline lens is a paradigmatic example of gradient index (GRIN) optics. Its onion-like inner structure can be pictured as a succession of thin meniscus lenses with their refractive power given by curvature and refractive index increment.

Most advanced lens models replicate the lens index distribution: a greatly inhomogeneous cortex with an increasing refractive index gradient towards the centre and a much more homogeneous nucleus [4-6].

For our formulation [7], we consider a sort of cemented doublet in which the anterior and posterior parts join at a central interface but are treated independently. The axial thickness is the sum of the anterior and posterior thicknesses, $t = t_a + t_p$. The refractive index distributions are [5-7] approximated to power laws of the normalized axial distances to the anterior and posterior external surfaces, z_0.

The external lens geometry–radii of curvature, R_a and R_p, and conic constants, Q_a and Q_p–will determine the shape of the iso-indicial surfaces.

2.2.1 Analytical Transfer Matrix

When we apply Eq. (6) to the crystalline lens geometry and index distribution, the integral cannot be computed in terms of known functions. We worked out a highly accurate approximation for the GRIN contribution to the power using Gaussian hypergeometric functions.

$$\phi_{Ga} \simeq -\frac{n_s \delta n / n_c}{R_a - G(Q_a+1)t_a} {}_2F_1 \left(\begin{matrix} 1, p \\ p+1 \end{matrix} \middle| -\frac{G(Q_a+1)t_a}{R_a - G(Q_a+1)t_a} \right) + \frac{n_s(-\delta n/n_c)^2}{2(R_a - G(Q_a+1)t_a)} {}_2F_1 \left(\begin{matrix} 1, 2p \\ 2p+1 \end{matrix} \middle| -\frac{G(Q_a+1)t_a}{R_a - G(Q_a+1)t_a} \right) \tag{7}$$

Here n_s and n_c are the refractive indices at the lens surface and core, respectively, G is the curvature gradient [7], $\delta n = n_s - n_c$ is the index gradient, and ${}_2F_1$ represents the Gaussian hypergeometric function. There is an equivalent expression for the posterior region of the lens.

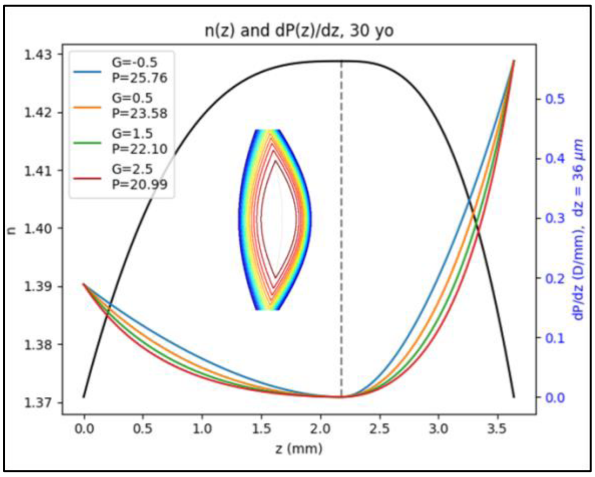

Fig. 1. Axial index distribution n and power differential dP/dz for a 30-year-old model. The label specifies the curvature gradient index G and the resulting refractive power P.

3 Results

To check the validity of our approximation, we compute the crystalline lens power for different values of G both numerically and analytically and compare the results. The average error is 0.14% between analytical and numerical calculations. Compared with a thin lens approximation of the integral, $\int n'(z)dz/R(z)$, the lens power difference with a GRIN lens is 2.13%.

4 Conclusion

The GRIN transfer matrix is like a single-layer transfer matrix, with its elements replaced by integrals, allowing an affordable and straightforward computation.

Matrix L differs from the matrix of a homogeneous lens only in one integration term in element L_{21} that represents the GRIN contribution to the power.

In the case of the crystalline lens, the analytical approximation to the GRIN lens power provides an accurate solution that relies on Gaussian hypergeometric functions, available in both commercial and open-source computing environments.

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