Cooperative Spontaneous Four-wave Mixing in Single-channel and Dual-channel Sequences of Side-coupled Ring Resonators

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Abstract. Cooperative photon pair generation by Spontaneous Four-Wave Mixing (SFWM) process in single-channel and dual-channel side-coupled ring resonator sequences is investigated. Our analysis shows that super-linear growth of generation rate with respect to the number of rings is possible even in presence of loss. Experimental evidence of super-SFWM is provided by comparing individual and collective generation rates obtained from a dual-channel ring resonator sequence. The results are in good agreement with theory and suggest that high photon pair generation rates can be achieved from integrated silicon ring resonator sequences without initiating nonlinear absorption processes.

1 Introduction

Despite its strong third order susceptibility, the rate of photon pair emission in silicon resonators is limited by nonlinear absorption at telecommunication wavelengths. Recently, it has been found that a process analogous to atomic super-radiance, *i.e.*, the enhancement of spontaneous emission from a collection of emitters [1], can occur in nonlinear parametric processes [2]. The authors considered photon pair generation by Spontaneous Four-Wave Mixing (SFWM) in a single-channel (SC) chain of N loss-less ring resonators, predicting that the rate would increase by a factor N^2 compared to that of the individual resonators. Hence, through super-radiant assisted SFWM, one can achieve a high pair generation rate without increasing the field amplitude in individual rings. However, linear absorption and scattering of the light in a real device limit the quadratic scaling of the generation rate. Here, we study the impact of linear loss mechanisms on the performance of ring resonators sequences with SC and dual-channel (DC) configurations. The DC configuration provides individual access to every emitter which is crucial for experimentally examining the cooperative emission. We provide experimental results to demonstrate the cooperative emission of ring resonators in a DC configuration.

Table 1: Transmission (T) and Field Enhancement (F)

Configuration	T	F
All-pass	$\frac{\sigma-e^{-\gamma l}}{1-\sigma e^{-\gamma l}}$	$\frac{j\kappa}{1-\sigma e^{-\gamma l}}$
Add-drop	$\frac{-e^{-\gamma l/2}\kappa^2}{1-\sigma^2 e^{-\gamma l}}$	$\frac{j\kappa}{1-\sigma^2 e^{-\gamma l}}$

2 Results

Figure 1(a) and Fig. 1(b) respectively show a SC and a DC ring resonator sequence. In the lossless case, the SC configuration has an all-pass frequency response with unity transmission. In presence of loss, transmission drops at the resonant frequencies and limits the efficiency of emission due to absorption of both the pump and the generated photon pairs, named the signal and the idler. The DC configuration behaves as a cascade of band-pass filters, hence the pump power decreases more slowly compared to the lossy SC sequence. But successive spectral filtering of the generated photon pairs can still limit the generation rate. To theoretically investigate the differences of the two configurations, we follow the approach described in [2, 3]. Field enhancement F and transmission functions T for these configurations are summarized in Table 1. Here, l is the circumference of the rings, $\gamma = \alpha + jk(\omega)$ is the complex propagation constant, and we assume an energy-conserving point coupler for each resonator with σ being its self-coupling coefficient and $j\kappa$ being its cross-coupling coefficient, satisfying $\sigma^2 + \kappa^2 = 1$. For a continuous-wave excitation, the generation rate is calculated from [2]

$$|\beta|^2 \propto \frac{\omega_p^2}{v^2(\omega_p)} \int d\omega \frac{\omega\omega'}{v(\omega)v(\omega')} \left|J(\omega, \omega', \omega_p, \omega_p)\right|^2, \quad (1)$$

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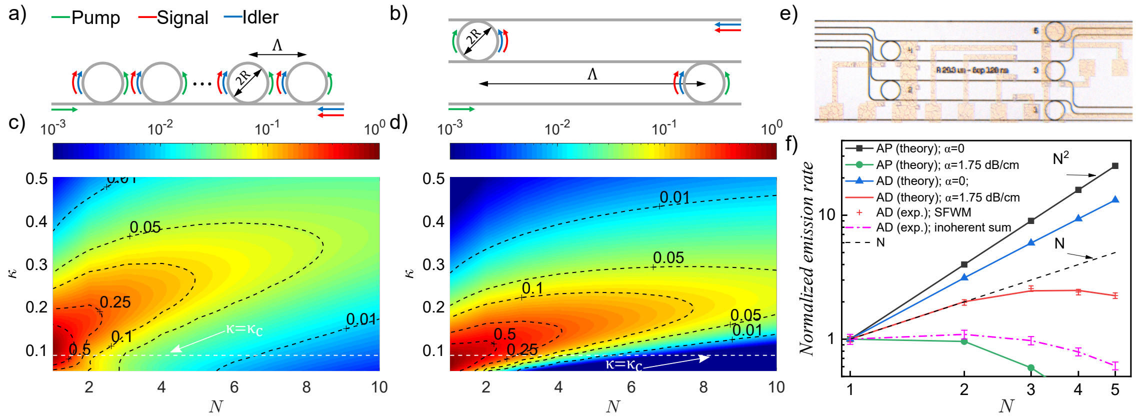

Figure 1: Schematic of a SC all-pass (a) and a DC add-drop (b) ring resonator sequence. Radii of the rings are R and the rings are separated by Λ. Asymptotic-in fields for pump, signal and idler waves are respectively shown by green, red and blue arrows. Photon pair generation rate $|\beta|^2$ for all-pass (c) and add-drop (d) configurations in presence of loss normalized to that of the sequence with the highest generation rate, calculated for different coupling coefficients (κ) and sequence lengths (N). (e) Optical microscope image of the fabricated DC ring resonator sequence. The heater layer (gold) is overlaid to that of the waveguides (black). (f) Theoretical and experimental normalized generation rate of the all-pass (AP) and the add-drop (AD) configurations for loss-less and lossy cases.

where ω and $\omega' = 2\omega_p - \omega$ span respectively over the frequency bandwidth of the signal and the idler waves, ω_p is the angular frequency of the pump and $v(\omega)$ is the group velocity. Here

$$J(\omega_1,\omega_2,\omega_3,\omega_4) = J_0 F(\omega_1)F(\omega_2)F(\omega_3)F(\omega_4)\frac{\sinh(N\mu/2)}{\sinh(\mu/2)} \times e^{-N(j\theta_T(\omega_1)+j\theta_T(\omega_2)+\alpha_T(\omega_1)+\alpha_T(\omega_2))-N(\gamma(\omega_1)+\gamma(\omega_2))\Lambda+(-N+1)\mu/2}$$

is the overlap integral of the asymptotic-in fields for the waves involved in the four-wave mixing process over all rings. The propagation direction of the asymptotic-in fields is schematically shown in Fig. 1(a,b). $T(\omega) = e^{-\alpha_T(\omega)-j\theta_T(\omega)}$, $\mu = j(\theta_T(\omega_3)+\theta_T(\omega_4)-\theta_T(\omega_1)-\theta_T(\omega_2)) + \alpha_T(\omega_3)+\alpha_T(\omega_4)-\alpha_T(\omega_1)-\alpha_T(\omega_2)$, and J_0 incorporates the remaining terms. Figure 1(c) and Fig. 1(d) respectively show the generation rate calculated for the lossy SC and DC configurations for different coupling coefficients (κ) and number of resonators (N). Here, we normalized $|\beta|^2$ to that of the sequence which shows the highest generation rate. Radii of the rings are $R = 29.3\ \mu m$, the spacing between resonators is $\Lambda = 500\ \mu m$, and the effective mode index and group index are 2.56 and 4.08, respectively. We assumed $\alpha = 1.75\ dB/cm$. Signal, pump, and idler resonant wavelengths are respectively around 1570 *nm*, 1560 *nm*, and 1550 *nm*. All these values correspond to the ones measured in the experiment discussed later. We see that for each value of κ, there is an optimum length N_{opt} that provides the highest generation rate. For under-coupled or critically-coupled resonators $N_{opt} = 1$. Figure 1(f) compares the scaling behavior of the generation rate of both configurations under loss-less and lossy scenarios. Here coupling coefficient is assumed to be $\kappa^2 = 0.03$, *i.e.*, its value in the experiment. For the SC configuration and in absence of loss (black), the scaling is quadratic with the number of resonators, as demonstrated earlier [2]. When losses are introduced, the generation rate quickly drops after two resonators (green). For the lossless DC configuration (blue), the normalized rate can be shown to grow as $N^{\frac{3}{2}}$. The different scaling with that of the all-pass configuration is set by the spectral filtering imparted to the photon pairs at each drop event in the sequence. When experimental value of loss is introduced in the calculations (red), the generation rate initially grows linearly with N, then it reaches a maximum and after that it decreases. We experimentally tested a DC sequence whose microscope image is shown in Fig. 1(e), and found a scaling law (red points) that agrees very well with the theoretical predictions. To prove the existence of cooperative emission, we independently recorded the coincidence rate of each individual ring in the sequence, and used them to predict the normalized rate that would be observed at the end of the sequence if their emission were incoherent. This is shown in Fig. 1(f) (magenta). The clear discrepancy with the case where all rings were coherently pumped demonstrates the presence of cooperative emission from the array, which enhances the pair generation probability.

3 Conclusions

In conclusion, we investigated SFWM in single- and dual-channel ring resonator sequences. We found that the super-radiant enhancement originally predicted in a loss-less chain of all-pass rings is severely affected by the presence of loss. On the other hand, the DC configuration is more loss-tolerant, and a super-linear scaling can be achieved even with moderate loss. We also provided experimental evidence of cooperative emission of photon pairs from an array of microresonators on a silicon photonic chip whose behaviour is in very good agreement with our theory.

References

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