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Static Multi-Vortex Structures in Nonlinear Optical Media

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Abstract. We demonstrate through numerical simulations the existence of a new type of nonlinear waves in optical media: structures of vortex solitons that remain static in certain configurations, which depend on their relative positions and topological charges. Several examples are presented to illustrate this surprising behavior.

1 Introduction

Optical vortices are singularities on the phase distribution of a light beam with an integer number l of windings around a point where the amplitude of the field vanishes. The integer l is usually called the *topological charge* of the vortex, although we must point out that this definition is only valid for boundless fields, where the charge is a conserved quantity, as it is the case of the present work.

The interactions amongst optical vortices contained in the same coherent field have been studied since long ago [1] and it is well known that different types of dynamics take place depending on the relative values of l. In this work we present for the first time numerical evidence of the existence of static configurations of optical vortices embedded in a constant background, that can be formed in nonlinear materials with a self-defocusing nonlinearity of the Kerr type.

Our results show that intricate phase interactions yield exotic distributions of the phase dislocations that remain stable for long propagations, without changes in the amplitude profile of the light distribution, which is therefore static. On the other hand, there is a complex phase structure that remains stationary as the beam propagates. This surprising behavior can be heuristically understood as a combination of topological optical "forces" that arrange themselves in such a way that a complex structure is formed that supports itself by the phase gradient profile generated by the sum of all the vortices of the structure. If any of the vortices were removed, the entire structure would rapidly unravel.

We must stress that the solutions we present here are very different from the vortex lattices or clusters obtained in similar systems like Bose-Einstein condensates in rotating traps[2]. First of all, no trap exists in our case. Moreover, the existence of the static structures described here depend on a delicate combination of vortices with different topological charges (in particular, vortices and antivortices must be present). Finally, these structures are certainly not the minimal energy configuration for a particular set of conserved quantities and thus, they are, at most, metastable. However, our numerical simulations show that they can survive for long propagations, in fact surviving in some cases for our whole simulation time.

2 Theoretical model

Paraxial beam propagation in a nonlinear Kerr-type optical material is described by a nonlinear Schrödinger equation, which in its adimensional form reads as:

$$i\frac{\partial\psi}{\partial z} + \frac{1}{2}\nabla_\perp^2\psi + |\psi|^2\psi = 0 \tag{1}$$

being $\nabla_\perp^2$ the nabla operator in the transverse plane x, y and z the propagation distance. All the spatial dimensions are normalized by the wavelength in the material and $|\psi|^2$ in average gives the beam irradiance divided by the second-order nonlinear refractive index.

Within the previous model we use as an input function for our simulations a continuous light background with N phase dislocations located in different positions with coordinates x_j, y_j ($j = 1\dots N$), yielding an input beam of the form $\psi(x, y, 0) = \prod_{j=1}^{N}\psi_j(r_j)e^{il_j\theta_j}$ being $l_j = \pm1, \pm2, \dots$ the topological charge of each vortex, $r_j = \sqrt{(x-x_j)^2 + (y-y_j)^2}$ the distance to each phase dislocation and $\theta_j = \arctan((y-y_j)/(x-x_j))$ the polar coordinate in the transverse plane for each vortex. The $\psi_j(r_j)$ are the amplitude profiles for the vortices of topological order j.

It is well known [1, 3] that if a vortex dislocation is placed within a fluid flow, it simply moves along it. Thus, when several vortices coexist in the same background, each of them induces a particular motion on each other. Then, static configurations can be engineered in cases in which all the induced velocities induced on each vortex by all the other vortices sum up to zero, assuming that the separation between vortices is much larger that the core size of the vortices themselves. As we will show in detail elsewhere, this type of configurations can be achieved

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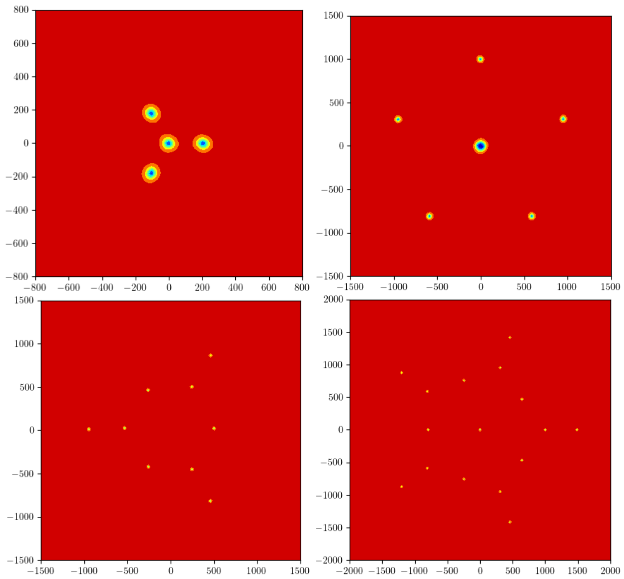

Figure 1. Transverse intensity distributions in the (x, y) plane of several vortex structures that remain unaltered during propagation. See the text for the description of each configuration.

by using ansatze with Z_n symmetry, where Z_n is a subgroup of the $U(1)$ rotation group around a particular center. Namely, we use regular n-polygons with the same center, each of which with vortices of a particular l on their vertices. Then, we look for relations between their sizes and the aforementioned topological charges to fulfil the staticity condition. It turns out that families of solutions exist, which, in particular, must satisfy the Diophantine equation:

$$\left(\sum_{i=1}^{N} l_i\right)^2 = \sum_{i=1}^{N} l_i^2 \tag{2}$$

This shows that the l_i cannot all of them have the same sign. It is also worth remarking that Eq. (2) can be derived by using a Pohozaev identity[4].

3 Numerical results

The numerical simulations show that the previous calculations yield several possible structures in the form of vortex clusters with different configurations and topological charges that display a static intensity configuration as can be seen in fig. 1. In this figure we plot transverse intensity distributions in the (x, y) plane of several vortex clusters that remain unaltered during propagation. From top-left to right bottom is show: a central $l = -1$ vortex surrounded by three $l = 1$ phase dislocations forming an equilateral triangle (Z_3 symmetry), a central $l = 2$ vortex surrounded by five $l = -1$ singularities located in the vertices of a regular pentagon (Z_5 symmetry), an arrangement of nine single-charged vortices of which the six that form the inner core (which is almost a hexagon) are positive and the external three are negative (Z_3 symmetry) and finally an arrangement of 16 singly-charged vortices, where the central and the external vortices are positively charged, whereas the singularities forming the internal pentagons are negative (Z_5 symmetry). Notice that all of these structures of course satisfy Eq. (2).

It must be also underlined that the static configuration of the intensity distribution is obtained within a dynamical equilibrium in the phase space, which is evolving in time as can be seen in 2.

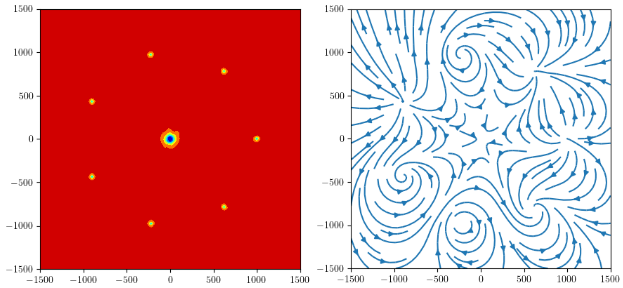

Figure 2. Transverse sections in the (x, y) plane showing the intensity (left) and phase gradient lines (right) of a vortex soliton structure with a central $l = 3$ singularity surrounded by an heptagonal regular distribution of negative single-charged vortices (Z_7 symmetry).

4 Conclusions

We have demonstrated through numerical simulations the existence of static structures of vortex solitons that remain unaltered during propagation for certain configurations, which depend on their relative positions and topological charges. Several examples have been presented illustrating that this surprising behavior can be achieved in very different configurations.

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