Dynamics of passive modelocking in class-B lasers with saturable absorber

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Abstract. We address the problem of passive modelocking in class-B lasers with saturable abosrber taking into account the fast dynamics of both gain and absorption. Our model, which is derived from a delay differential equation model, treats in a rigorous way the definition of the fast and slow times which are typically used in the master equation approach. In that way all the dynamical variables obey exact periodic boundary conditions and this makes the model suitable for analytic and numerical treatment. The model accounts for behaviours different from fundamental modelocking, such as Q-switching modelocking and harmonic modelocking.

1 Introduction

Passive modelocking (PML) is a widespread method used to generate coherent light pulses and optical frequency combs. The basic mechanism of PML has been understood since the fundamental work of Haus based on the master equation (ME) approach [1]. Yet it is known that Haus ME may fail in the common case of a class-B laser where the cavity photon lifetime is shorter than or comparable to the gain recovery time. Here we show how a sound ME for PML in class B lasers can be derived, which is valid for gain and absorption both slow and fast. We start from a well-known delay-differential equation model, transform it into a field map and pass to the continuous limit. Then we propose a rigorous definition of a two-time description of the problem, and finally split gain and absorption into slow and fast components. This procedure leads to exact periodic boundary conditions and it is essential to keep the problem simple, both analytically and numerically.

2 From the delay-differential equation model to the master equation

The use of the standard Haus ME for PML in class-B lasers is particularly problematic when the absorber is slow, as it happens for instance in dyes and semiconductors. In the recent years the problem has been addressed [2, 3] starting from a delay-differential equation (DDE) model for PML [4]. The DDE governs the dynamics of the slowly varying amplitude of the laser field at a reference plane as

$$\gamma^{-1}\dot{E}(t) + E(t) = \sqrt{K}\mathrm{e}^{M(t-T_\mathrm{c})}E(t-T_\mathrm{c}), \tag{1}$$

where the derivative term (the overdot denotes $\mathrm{d}/\mathrm{d}t$) mimics the effect of a Lorentzian spectral filter of fixed bandwidth γ that could approximate the finite gain medium bandwidth or be due to another generic intracavity bandwidth limiting element, K is the power attenuation factor per cavity roudtrip due to nonresonant losses, $M = (1-\mathrm{i}\alpha_g)G/2-(1-\mathrm{i}\alpha_q)Q/2$ describes the net amplification per roundtrip due to the saturated gain G and absorption Q, with $\alpha_{g,q}$ the corresponding linewidth-enhancement factors of semiconductors, and T_c is the cold-cavity roundtrip time without the filter contribution.

The ME applies in the limit of small gain and losses in which one can assume that the system changes little over any cavity roundtrip [1]. In that limit it is possible to make the approximation $\mathrm{e}^M \simeq 1 + M$ and the DDE model can be written as [5]

$$\gamma^{-1}\dot{E} = -E + [1-\ell+M(t-T_\mathrm{c})]\,E(t-T_\mathrm{c}), \tag{2a}$$
$$\dot{G} = \gamma_g\,[G_0 - G(1+I)]\,, \tag{2b}$$
$$\dot{Q} = \gamma_q\,[Q_0 - Q(1+sI)]\,, \tag{2c}$$

with all (dimensionless) laser variables E, G, and Q evaluated at time t except when a different argument is explicitly written. $I(t) \equiv |E(t)|^2$ is the field intensity scaled to saturation intensity of the amplifier $I_{g,\mathrm{sat}}$, and the saturation parameter is defined as $s \equiv I_{g,\mathrm{sat}}/I_{q,\mathrm{sat}}$, where $I_{q,\mathrm{sat}}$ is the saturation intensity of the absorber. The decay rates $\gamma_{g,q}$ respectively are the inverses of the gain and absorber recovery times. Finally $\ell \equiv 1-\sqrt{K}$ represents linear losses, and G_0 and Q_0 are the unsaturated gain and absorption parameters. The initial assumption of small gain and losses is verified if $G, Q, \ell \ll 1$.

Our procedure for the derivation of the ME can be summarised as follows. First, we transform Eq. (2a) into a map relating the field at two consecutive roundtrips. Next, we introduce as it is customary a slow time T' which grows unbounded, and a fast time τ' restricted to the in-

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terval $[0, T_R]$, where T_R is the cavity roundtrip time normalised to take into account the effect of the spectral filter. However, as outlined in [6] our methodology differs from the usual one in that we introduce the new times $T = T' + \tau'$, $\tau = \tau'$ in such a way that all dynamical variables obey *exact* periodic boundary conditions of the form $X(T, \tau) = X(T, \tau + T_R)$. Another key point of our approach

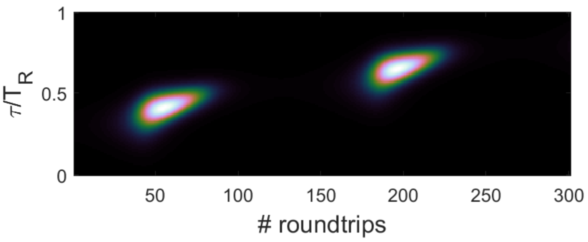

Figure 1. Example of Q-switched modelocking (QSML). The pulse is modulated from zero to a maximum value with a period of about 150 roundtrip times.

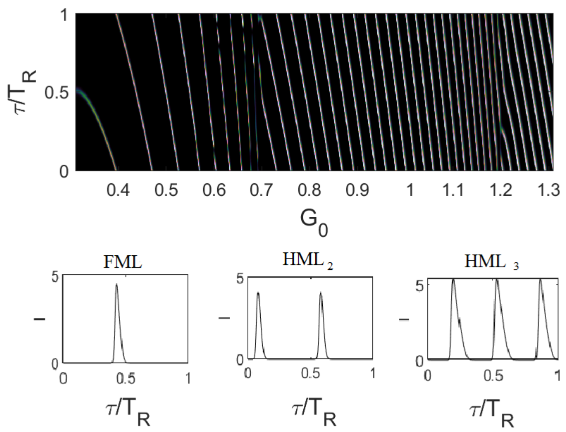

Figure 2. Example of transition from fundamental modelocking (FML) to harmonic modelocking with two pulses (HML_2) and harmonic modelocking with three pulses (HML_3).

consists in splitting the gain and absorption variables into slow and fast components [6]. The slow variables are defined as averages over one roundtrip time and are functions only of T, whereas the fast ones are defined as the difference between the total variables and the averages and can be regarded as functions only of τ.

3 Numerical results

Numerical simulations show different dynamical behaviours besides fundamental modelocking (FML). In the choice of the parameters we are guided by [2]. In all simulations we kept fixed the unsaturated absorption parameter $Q_0 = 0.3$ and the cavity losses $\ell = 0.005$, so that the (single mode) lasing threshold is $G_{0,th} = Q_0 + 2\ell = 0.31$. We also assumed that the amplifier is much slower than the absorber ($\gamma_g \ll \gamma_q$).

In Fig. 1 we considered a short cavity ($T_c = 2\gamma_q^{-1}$), relatively large gain bandwidth ($\gamma = 40\gamma_q$), linewidth enhancement factors different from zero ($\alpha_g = 1.5$, $\alpha_q = 1$), $\gamma_g = 0.01\gamma_q$, $s = 0.3$, and $G_0 = 0.5$. With these parameters both the stationary solution and FML are unstable. The long-term solution is a regime of Q-switched mode locking (QSML) with one pulse circulating in the cavity whose intensity is modulated from almost zero to a maximum value with a period of about 150 roundtrip times.

In a second simulation, shown in Fig. 2, we considered a longer cavity ($T_c = 30\gamma_q^{-1}$), relatively small gain bandwidth ($\gamma = 10\gamma_q$), null linewidth enhancement factors ($\alpha_g = \alpha_q = 0$), $\gamma_g = 0.05\gamma_q$, and $s = 0.15$. In this case we varied adiabatically the gain parameter G_0 from the lasing threshold 0.31 to 1.31. At threshold the laser already switches to the FML solution. At $G_0 \simeq 0.7$, FML is replaced by a modelocked solution with two pulses circulating in the cavity (HML_2) and finally at $G_0 \simeq 1.2$ we observe the transition to the modelocked solution with three pulses (HML_3). The transitions from one regime to the next are not abrupt, in between there are small intervals of G_0 for which no stable pulses exist. The different slope of the traces indicates that the pulses travel faster as their number increases.

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