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Quadrics for Structuring Invariant Space-Time Wave Packets

Pierre Béjot[1] and *Bertrand* Kibler[1,*]

[1]Laboratoire Interdisciplinaire Carnot de Bourgogne, UMR6303 CNRS-UBFC, 21000 Dijon, France

Abstract. We provide a general approach for structuring invariant 3D+1 optical wave packets in both bulk and structured dispersive media, through a simple engineering of phase-matched space-time frequencies on quadric surfaces.

1 Introduction

The field of light structuring has made significant progress in the design, implementation and applications [1-3], in particular, propagation-invariant or non-diffracting beams. However, the pulsed nature of light waves and the structured property of optical media like waveguides are often overlooked. We here present a full spatiotemporal approach that takes advantage of tight correlations between the spatial modes, the topological charges, and the frequencies embedded in an optical field, in order to reveal propagation-invariant (dispersion- and diffraction-free) space-time wave packets carrying orbital angular momentum (OAM) in both bulk media or multimode fibers [4]. We demonstrate that such wave packets can evolve on spiraling trajectories in both time and space (i.e., helicon wave packets), and their phase-matched space-time frequencies generally lie on quadric surfaces [5]. Besides their intrinsic linear nature, we show that such wave structures can spontaneously emerge when a rather intense ultrashort pulse propagates nonlinearly in OAM modes [4].

2 Results

We investigate space-time helicon wave packets in the framework of cylindrically symmetric waveguides. The bulk case can then be seen as the limiting case of waveguide of infinite dimension. For simplicity, we restrict our study to a scalar approach by considering the weak guidance approximation. The partial differential equation driving the linear propagation of an electric field E is then given in cylindrical coordinates by:

$$\left[\partial_z^2 + \partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\theta^2 + \frac{n^2(r,\omega)\omega^2}{c^2}\right]\tilde{E}(r,\theta,z,\omega) = 0 \quad (1)$$

where $n(r,\omega)$ is the radial-dependent refractive index. By separating transverse and longitudinal variables, the linear evolution of the field along z-axis is given in the modal basis by: $\partial_z \bar{E} = iK_z(l,p,\omega)\bar{E}$, where $K_z(l,p,\omega)$ is the propagation constant of the OAM mode (l,p) at frequency ω. Recall that l and p refer here to azimuthal and radial indices, where $l(0,\pm1,\pm2,\dots)$ is the topological charge, related to the phase front of OAM modes.

Next, one has to look for invariant fields under uniform motion but also invariant at a given rotation around the z axis. Such fields belong to the kernel of the space-time screw axis symmetry differential operator $\Pi = \partial_z + K_1\partial_t + K_l L_z - iK_0$, where K_l is an arbitrarily chosen constant and $L_z = x\partial_y - y\partial_x = \partial_\theta$ is the z-component of the angular momentum operator. This implies that the decomposition of such wave packets only embeds a family of eigenvectors whose eigenvalues respect:

$$K_z(l,p,\omega_{lp}) = K_0 + K_1\omega_{lp} + K_l l \quad (2)$$

Accordingly, any electric field built from a given family is a diffraction- and dispersion-free space-time wave packet propagating at the group velocity $1/K_1$ whose intensity continuously rotates around the propagation axis with a spatial period $2\pi/K_l$. By considering only second-order dispersion around a given arbitrarily chosen frequency ω_0, we show that the propagation constant of a waveguide can be well approximated as:

$$K_z \simeq k_0 + k_1\Omega + \frac{k_2}{2}\Omega^2 + \Gamma(l,p,\Omega) \quad (3)$$

with Γ is a three-dimensional second-order polynomial. It then follows that spatiotemporal helicon wave packets are contained in surfaces whose algebraic expression is a quadric, whose surface depends on the considered dispersion regime (normal or anomalous).

Below, a simple and analytical case at the frontier between bulk media and fibers is studied, namely a dispersive medium of finite transversal dimension (radius $R = 100$ µm). In the paraxial approximation, the propagation constants can be approximated by:

$$K_z(l,p,\omega) \simeq k_0 + k_1\Omega + \frac{k_2}{2}\Omega^2 - \frac{\alpha_{lp}^2}{2k_0R^2} \quad (4)$$

where α_{lp} is the p^{th} root of l^{th} Bessel function of first kind J_l. Using the formalism described above, with a second-order polynomial fit of α_{lp}^2, one can find that the

* Corresponding author: bertrand.kibler@u-bourgogne.fr

quadric surface associated to spatiotemporal helicon wave packets in finite bulk media has a rank of 3 [5]. Corresponding typical examples are shown in Fig. 1 in both dispersion regimes of a fused silica rod.

Figure 1(a) shows the three-dimensional pattern (i.e., quadrics) of phase-matching obtained in the normal dispersion regime, when considering a central wavelength $\lambda_0 = 2\pi c/\omega_0 = 800$ nm, and the arbitrary choice of the constants $K_1 = k_1$ and $K_0 = k_0$ (i.e., propagation constants of the wave packet are those of ω_0). The rotation period was fixed to $2\pi/|K_l| = 1$ cm. The linear construction of a helicon wave packet then results from the superposition of the calculated frequencies Ω_{lp}. Figure 1(b) presents an example corresponding to the particular case of selected modes with various radial and angular indices ($0 \le l \le 8$, and $1 \le p \le 8$), and with equal spectral amplitudes and phases (see circles in Fig. 1a). The successive subplots display the iso-surface of the spatiotemporal intensity at half-maximum at distinct propagation distances. The helicon wave packet looks like a corkscrew in space-time coordinates, the points of high intensity being extremely localized in both space and time since all frequencies are in phase. This ultrashort structure does not disperse and simply rotates during its propagation around the z- and t-axis because of the inherent invariant nature of the constructed wave packet. Another example of quadric obtained in the anomalous dispersion regime is shown in Fig. 1(c), when considering the central wavelength at 1800 nm, and the constants $K_1 = k_1$ and $K_0 = k_0$. The rotation period is now fixed equal to 1.25 cm. Figure 1(d) presents an example of constructed helicon wave packet obtained from the superposition of modes: $0 \le l \le 22$, and $1 \le p \le 2$, and with equal spectral amplitudes and phases (see circles in Fig. 1c). Again we retrieve the typical invariant spiraling trajectory of the intensity pattern, with a more complex spatiotemporal pattern made of three consecutive high-intensity sub-pulses.

Finally, we will demonstrate numerically that spiraling wave packets can naturally emerge from the nonlinear propagation of rather intense ultrashort pulses [4]. As for conical waves emerging during filamentation in bulks, the formalism of helicon wave packets brings a proper understanding of the phenomena taking place when intense pulses embedding different OAM beams propagate in bulks and waveguides. Given the current strong interest of both OAM-carrying beams and fiber modes, we expect that this work will stimulate further research in the field.

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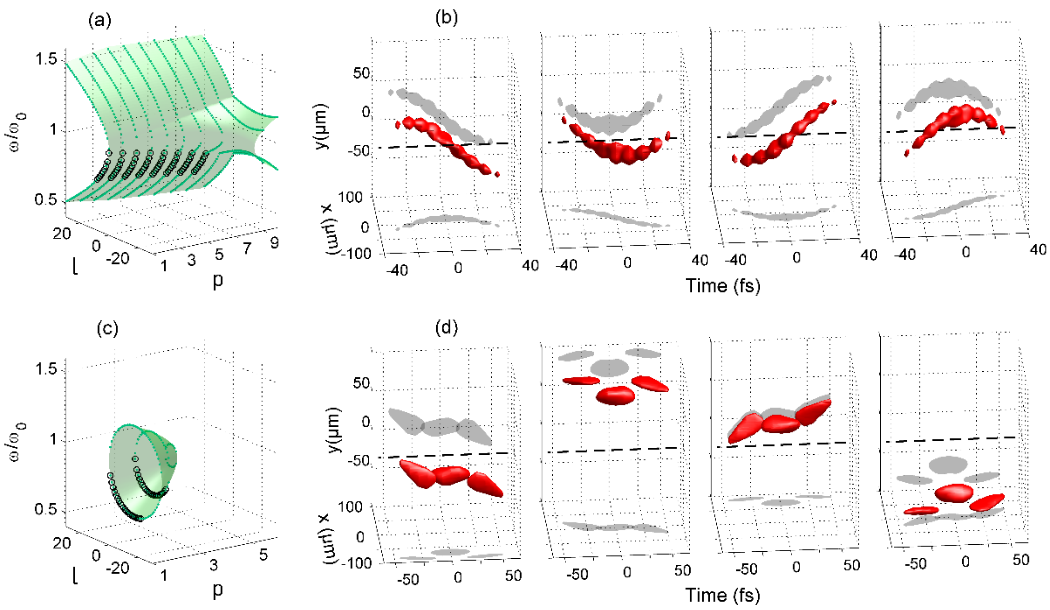

Fig. 1. Examples of helicon wavepackets constructed from phase-matching quadrics of spatiotemporal components in a fused silica rod with radius $R = 100$ μm. (a,c) Quadric surfaces obtained both in normal and anomalous dispersion regimes. Black circles correspond to the selected modes used for the linear construction of helicon wavepackets depicted in (b,d) through iso-surfaces of their spatiotemporal intensity pattern at half-maximum. The successive subplots obtained every $\pi/(2|K_l|)$ confirm the invariant nature of helicon wavepackets over propagation. The dashed line indicates the origin $(x = 0, y = 0)$. Projections on planes (shadow plots) are provided for a clear observation of rotation.